

COMSATS Institute of Information Technology Attock campus

Department of Mathematics

Assignment # 02

Class: MSc-III Due Date: 09-10-2017
Subject: Real Analysis II Course Code: MTH322

Instructor: Dr. Atiq ur Rehman Marks: 05

Question #1

Suppose that $f \in R(\alpha; a, b)$ and that $g \in R(\alpha; a, b)$ for every $b \ge a$, where $f(x) \ge 0$ and $g(x) \ge 0$ for $x \ge a$. If

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = c, \text{ provided } c \neq 0,$$

then prove that $\int_{a}^{\infty} f d\alpha$ and $\int_{a}^{\infty} g d\alpha$ both converge or both diverge.

Also prove that if c = 0, then we can only conclude that convergence of $\int_{a}^{\infty} g \, d\alpha$

implies convergence of $\int_{a}^{\infty} f \, d\alpha$.

Question #2

Prove that $\lim_{x\to\infty} \frac{x^{p+2}}{e^x} = 0$ for all $p \in \mathbb{R}$.

Question #3

Show that if $\int_{a}^{\infty} f d\alpha$ and $\int_{a}^{\infty} g d\alpha$ are convergent, then

- $\int_{a}^{\infty} (f \pm g) d\alpha$ is convergent.
- $\int_{a}^{\infty} cf \, d\alpha$, where c is some constant, is convergent.

Academic Honesty Requirements:

You are encouraged to work with others in the completion of assignments but it doesn't include copying. However, in the spirit of Academic Honesty, which includes crediting others for their contribution to your work, please include one of the following statements with every submitted assignment on title page:

- 1. I worked alone on this assignment.
- 2. I worked with the following: List their full names. Include their relationship to you if they are not also a member of this class.