Lecture 06: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to discuss about

- *Propositional functions with more than one variable*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Propositional Functions with more than One Variable

A propositional function (of n variables) defined over a product set $A = A_1 \times A_2 \times \ldots \times A_n$ is an expression $p(x_1, x_2, \ldots, x_n)$, which has the property that $p(a_1, a_2, \ldots, a_n)$ is true or false for any n-tuple $(a_1, a_2, \ldots, a_n)$ in A. For example,

$$x + 2y + 3z < 18$$

is a propositional function on $\mathbf{N}^3 = \mathbf{N} \times \mathbf{N} \times \mathbf{N}$. Such a propositional function has no truth value. However, we do have the following:

Basic Principle: A propositional function preceded by a quantifier for each variable, for example,

$$\forall x \exists y, p(x, y) \qquad \text{or} \qquad \exists x \, \forall y \, \exists z, p(x, y, z)$$

denotes a statement and has a truth value.

Examples

Let $B = \{1, 2, 3, \ldots, 9\}$ and let $p(x, y)$ denote "$x + y = 10$." Then $p(x, y)$ is a propositional function on $A = B^2 = B \times B$.

(a) The following is a statement since there is a quantifier for each variable:

$\forall x \exists y, p(x, y)$, that is, "For every x, there exists a y such that $x + y = 10$"

This statement is true. For example, if $x = 1$, let $y = 9$; if $x = 2$, let $y = 8$, and so on.

(b) The following is also a statement:

$\exists y \forall x, p(x, y)$, that is, "There exists a y such that, for every x, we have $x + y = 10$"

No such y exists; hence this statement is false.

Negating Quantified Statements with more than One Variable

Quantified statements with more than one variable may be negated by successively applying following theorem:

Theorem (DeMorgan):

(a) $\neg(\forall x \in A)p(x) \equiv (\exists\, x \in A)\neg p(x)$ (b) $\neg(\exists x \in A)p(x) \equiv (\forall x \in A)\neg p(x)$.

Thus each ∀ is changed to ∃ and each ∃ is changed to ∀ as the negation symbol ¬ passes through the statement from left to right. For example,

$$\neg[\forall x\ \exists y\ \exists z,\ p(x, y, z)]$$

$$\equiv \exists x\ \neg[\exists y\ \ \exists z,\ p(x, y, z)]$$

$$\equiv\ \exists x\ \forall y\ \neg[\exists z,\ p(x, y, z)$$

$$\equiv\ \exists x\ \forall y\ \forall z,\ \neg p(x, y, z).$$

Naturally, we do not put in all the steps when negating such quantified statements.

Example

(a) Consider the quantified statement:

"Every student has at least one course where the lecturer is a teaching assistant."

Its negation is the statement:

"There is a student such that in every course the lecturer is not a teaching assistant."

(b) The formal definition that L is the limit of a sequence $a_1, a_2, \ldots$ follows:

$$\forall\ \varepsilon > 0, \exists\ n_0 \in \mathbb{N}, \forall\ n > n_0 \text{ we have } |\ a_n - L| < \varepsilon .$$

Thus L is not the limit of the sequence $a1, a2, \ldots$ when:

$$\exists\ \varepsilon > 0, \forall n_0 \in \mathbb{N}, \exists\ n > n_0 \text{ such that } |\ a_n - L| \geq \varepsilon .$$

THANKS FOR YOUR ATTENTION