Lecture 07: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to define the notion of

- *Order pair*
- *Product of sets*
- *Relation*
- *Inverse relation*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Order Pair:

An *ordered pair* of elements a and b, where a is designated as the first element and b as the second element, is denoted by (a, b). In particular,

$$(a, b) = (c, d)$$

if and only if $a = c$ and $b = d$. Thus $(a, b) \neq (b, a)$ unless $a = b$. This contrasts with sets where the order of elements is irrelevant; for example, $\{3, 5\} = \{5, 3\}$.

Product of Sets:

Consider two arbitrary sets A and B. The set of all ordered pairs (a, b) where $a \in A$ and $b \in B$ is called the *product*, or *Cartesian product*, of A and B. A short designation of this product is $A \times B$, which is read "A cross B." By definition,

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

One frequently writes A^2 instead of $A \times A$.

EXAMPLE Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Then

$$A \times B = \{(1, a),\ (1, b),\ (1, c),\ (2, a),\ (2, b),\ (2, c)\}$$
$$B \times A = \{(a, 1),\ (b, 1),\ (c, 1),\ (a, 2),\ (b, 2),\ (c, 2)\}$$

Also, $A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

EXAMPLE $\mathbf{R}$ denotes the set of real numbers and so $\mathbf{R}^2 = \mathbf{R} \times \mathbf{R}$ is the set of ordered pairs of real numbers. The reader is familiar with the geometrical representation of $\mathbf{R}^2$ as points in the plane as in Fig. 2-1. Here each point P represents an ordered pair (a, b) of real numbers and vice versa; the vertical line through P meets the x-axis at a, and the horizontal line through P meets the y-axis at b. $\mathbf{R}^2$ is frequently called the *Cartesian plane*.

The idea of a product of sets can be extended to any finite number of sets. For any sets $A_1, A_2, \ldots, A_n$, the set of all ordered n-tuples $(a_1, a_2, \ldots, a_n)$ where $a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n$ is called the *product* of the sets $A_1, \ldots, A_n$ and is denoted by

$$A_1 \times A_2 \times \cdots \times A_n \quad \text{or} \quad \prod_{i=1}^{n} A_1$$

Just as we write A^2 instead of $A \times A$, so we write A^n instead of $A \times A \times \cdots \times A$, where there are n factors all equal to A. For example, $\mathbf{R}^3 = \mathbf{R} \times \mathbf{R} \times \mathbf{R}$ denotes the usual three-dimensional space.

Relation

Definition Let A and B be sets. A *binary relation* or, simply, *relation* from A to B is a subset of $A \times B$.

Suppose R is a relation from A to B. Then R is a set of ordered pairs where each first element comes from A and each second element comes from B. That is, for each pair $a \in A$ and $b \in B$, exactly one of the following is true:

(i) $(a, b) \in R$; we then say "a is R-related to b", written aRb.

(ii) $(a, b) \notin R$; we then say "a is not R-related to b", written $a \not{R} b$.

If R is a relation from a set A to itself, that is, if R is a subset of $A^2 = A \times A$, then we say that R is a relation *on* A.

The *domain* of a relation R is the set of all first elements of the ordered pairs which belong to R, and the *range* is the set of second elements.

EXAMPLE

(a) $A = (1, 2, 3)$ and $B = \{x, y, z\}$, and let $R = \{(1, y), (1, z), (3, y)\}$. Then R is a relation from A to B since R is a subset of $A \times B$. With respect to this relation,

$$1Ry,\ 1Rz,\ 3Ry, \quad \text{but} \quad 1\not{R}x,\ 2\not{R}x,\ 2\not{R}y,\ 2\not{R}z,\ 3\not{R}x,\ 3\not{R}z$$

The domain of R is $\{1, 3\}$ and the range is $\{y, z\}$.

(b) Set inclusion $\subseteq$ is a relation on any collection of sets. For, given any pair of set A and B, either $A \subseteq B$ or $A \nsubseteq B$.

(c) A familiar relation on the set $\mathbf{Z}$ of integers is "m divides n." A common notation for this relation is to write $m \mid n$ when m divides n. Thus $6 \mid 30$ but $7 \nmid 25$.

(d) Consider the set L of lines in the plane. Perpendicularity, written "$\perp$," is a relation on L. That is, given any pair of lines a and b, either $a \perp b$ or $a \not\perp b$. Similarly, "is parallel to," written "$\parallel$," is a relation on L since either $a \parallel b$ or $a \nparallel b$.

Inverse Relation

Let R be any relation from a set A to a set B. The *inverse* of R, denoted by R^{-1}, is the relation from B to A which consists of those ordered pairs which, when reversed, belong to R; that is,

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}$$

For example, let $A = \{1, 2, 3\}$ and $B = \{x, y, z\}$. Then the inverse of

$$R = \{(1, y), (1, z), (3, y)\} \quad \text{is} \quad R^{-1} = \{(y, 1), (z, 1), (y, 3)\}$$

Clearly, if R is any relation, then $(R^{-1})^{-1} = R$. Also, the domain and range of R^{-1} are equal, respectively, to the range and domain of R. Moreover, if R is a relation on A, then R^{-1} is also a relation on A.

THANKS FOR YOUR ATTENTION