Lecture 04: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to

- *define argument, valid argument, and fallacy.*
- *discuss examples and few related results.*
- *define propositional function.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

ARGUMENTS

An *argument* is an assertion that a given set of propositions $P_1, P_2, \dots, P_n$, called *premises*, yields (has a consequence) another proposition Q, called the *conclusion*.

Such an argument is denoted by $P_1, P_2, \dots, P_n \vdash Q$

The notion of a "logical argument" or "valid argument" is formalized as follows:

Valid Argument: An argument $P_1, P_2, \dots, P_n \vdash Q$ is said to be *valid* if Q is true whenever all the premises $P_1, P_2, \dots, P_n$ are true.

Fallacy: An argument which is not valid is called *fallacy*.

Examples:

(a) The following argument is valid:

$$p, p \rightarrow q \vdash q \qquad \textit{(Law of Detachment)}$$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(b) The following argument is a fallacy:

$$p \rightarrow q, q \vdash p$$

Note that propositions $P_1, P_2, \ldots, P_n$ are true simultaneously if and only if the proposition $P_1 \wedge P_2 \wedge \ldots P_n$ is true.

So we come arrive at the following theorem.

Theorem:

The argument $P_1, P_2, \ldots, P_n \vdash Q$ is valid if and only if the proposition $(P_1 \wedge P_2 \ldots \wedge P_n) \rightarrow Q$ is a tautology.

Example: A fundamental principle of logical reasoning states:

"If *p* implies *q* and *q* implies *r*, then *p* implies *r*"

p	q	r	$[(p$	$\rightarrow$	$q)$	$\wedge$	$(q$	$\rightarrow$	$r)]$	$\rightarrow$	$(p$	$\rightarrow$	$r)$
T	T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	F	F	T	T	F	F
T	F	T	T	F	F	F	F	T	T	T	T	T	T
T	F	F	T	F	F	F	F	T	F	T	T	F	F
F	T	T	F	T	T	T	T	T	T	T	F	T	T
F	T	F	F	T	T	F	T	F	F	T	F	T	F
F	F	T	F	T	F	T	F	T	T	T	F	T	T
F	F	F	F	T	F	T	F	T	F	T	F	T	F
Step			1	2	1	3	1	2	1	4	1	2	1

That is, the following argument is valid: $p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$ ***(Law of Syllogism)***

This fact is verified by the above truth table which shows that the following proposition is a tautology:

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r).$$

For example, consider the following argument:

S_1 : If a man is a bachelor, he is unhappy.

S_2 : If a man is unhappy, he dies young.

Conclusion: S : Bachelors die young

Here the statement S below the line denotes the conclusion of the argument, and the statements S_1 and S_2 above the line denote the premises. We claim that the argument $S_1, S_2 \vdash S$ is valid. For the argument is of the form

$$p \rightarrow q, q \rightarrow r \vdash p \rightarrow r,$$

where p is "He is a bachelor," q is "He is unhappy" and r is "He dies young".
and by "Law of Syllogism" argument S is valid.

Propositional Functions

Let A be a given set. *A propositional function* (or an *open sentence* or *condition*) defined on A is an expression $p(x)$, which has the property that $p(a)$ is true or false for each $a \in A$.

The set A is called the *domain* of $p(x)$, and the set T_p of all elements of A for which $p(a)$ is true is called the *truth set* of $p(x)$. In other words,

$$T_p = \{x \mid x \in A, p(x) \text{ is true}\} \text{ or } T_p = \{x \mid p(x)\}$$

Example: Consider propositional function $p(x)$ defined on the set **N** of positive integers.

(a) Let $p(x)$ be "$x + 2 > 7$." Its truth set is {6, 7, 8, . . .} consisting of all integers greater than 5.

(b) Let $p(x)$ be "$x + 5 < 3$." Its truth set is the empty set. That is, $p(x)$ is not true for any integer in **N**.

(c) Let $p(x)$ be "$x + 5 > 1$." Its truth set is **N**. That is, $p(x)$ is true for every element in **N**.

THANKS FOR YOUR ATTENTION