Lecture 12: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to discuss

- *transitive relation on a set with examples.*
- *equivalence relation on a set with examples.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Transitive Relations

A relation R on a set A is transitive if whenever aRb and bRc then aRc, that is, if whenever $(a, b), (b, c) \in R$ then $(a, c) \in R$.

Thus R is not transitive if there exist $a, b, c \in R$ such that $(a, b), (b, c) \in R$ but $(a, c) \not\in R$.

Example 1

Consider the following five relations on the set $A = \{1, 2, 3, 4\}$:

$R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$

$R_2 = \{(1, 1)(1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

$R_3 = \{(1, 3), (2, 1)\}$

$R_4 = \emptyset$, the empty relation

$R_5 = A \times A$, the universal relation

The relation R_3 is not transitive since $(2, 1), (1, 3) \in R_3$ but $(2, 3) \not\in R_3$. All the other relations are transitive.

Example 2

Consider the following five relations:

(1) Relation $\leq$ (less than or equal) on the set $\mathbb{Z}$ of integers.

(2) Set inclusion $\subseteq$ on a collection C of sets.

(3) Relation $\perp$ (perpendicular) on the set L of lines in the plane.

(4) Relation $\parallel$ (parallel) on the set L of lines in the plane.

(5) Relation $|$ of divisibility on the set $\mathbb{N}$ of positive integers.

(Recall $x \mid y$ if there exists z such that $xz = y$.)

The relations $\leq$, $\subseteq$, and $|$ are transitive, but certainly not $\perp$. Also, since no line is parallel to itself, we can have $a \parallel b$ and $b \parallel a$, but $a \parallel a$. Thus $\parallel$ is not transitive.

Equivalence Relations

Consider a nonempty set S. A relation R on S is an equivalence relation if R is reflexive, symmetric, and transitive.

That is, R is an equivalence relation on S if it has the following three properties:

(1) For every $a \in S$, aRa.

(2) If aRb, then bRa.

(3) If aRb and bRc, then aRc.

Examples 3

The relation "=" of equality on any set S is an equivalence relation; that is:

(1) $a = a$ for every $a \in S$.

(2) If $a = b$, then $b = a$.

(3) If $a = b$, $b = c$, then $a = c$.

Examples 4

The relation set inclusion ⊆ on a collection *C* of sets is not an equivalence relation. It is reflexive and transitive, but it is not symmetric since A ⊆ B does not imply B ⊆ A.

Example 5

Let *m* be a fixed positive integer. Two integers *a* and *b* are said to be congruent modulo *m*, written

$$a \equiv b \pmod{m}$$

if *m* divides $a - b$. For example, for the modulus $m = 4$, we have

$$11 \equiv 3 \pmod{4} \quad \text{and} \quad 22 \equiv 6 \pmod{4},$$

since 4 divides $11 - 3 = 8$ and 4 divides $22 - 6 = 16$.

This relation of congruence modulo *m* is an important equivalence relation.

THANKS FOR YOUR ATTENTION