Lecture 13: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to define the notion of

- *equivalence relation,*
- *partial ordering relation,*
- *n-array relations and*
- *related problems.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Equivalence Relation:

Consider a nonempty set S. A relation R on S is an *equivalence relation* if R is reflexive, symmetric, and transitive. That is, R is an equivalence relation on S if it has the following three properties:

(1) For every $a \in S$, aRa. (2) If aRb, then bRa. (3) If aRb and bRc, then aRc.

The general idea behind an equivalence relation is that it is a classification of objects which are in some way "alike." In fact, the relation "$=$" of equality on any set S is an equivalence relation; that is:

(1) $a = a$ for every $a \in S$. (2) If $a = b$, then $b = a$. (3) If $a = b$, $b = c$, then $a = c$.

EXAMPLE

(a) Let L be the set of lines and let T be the set of triangles in the Euclidean plane.

  (i) The relation "is parallel to or identical to" is an equivalence relation on L.

  (ii) The relations of congruence and similarity are equivalence relations on T.

(b) The relation $\subseteq$ of set inclusion is not an equivalence relation. It is reflexive and transitive, but it is not symmetric since $A \subseteq B$ does not imply $B \subseteq A$.

(c) Let m be a fixed positive integer. Two integers a and b are said to be *congruent modulo m*, written

$$a \equiv b \pmod{m}$$

if m divides $a - b$. For example, for the modulus $m = 4$, we have

$$11 \equiv 3 \pmod{4} \quad \text{and} \quad 22 \equiv 6 \pmod{4}$$

since 4 divides $11 - 3 = 8$ and 4 divides $22 - 6 = 16$. This relation of congruence modulo m is an important equivalence relation.

Partial Ordering Relations:

A relation R on a set S is called a *partial ordering* or a *partial order* of S if R is reflexive, antisymmetric, and transitive. A set S together with a partial ordering R is called a *partially ordered set* or *poset*.

EXAMPLE

(a) The relation $\subseteq$ of set inclusion is a partial ordering on any collection of sets since set inclusion has the three desired properties. That is,

(1) $A \subseteq A$ for any set A.
(2) If $A \subseteq B$ and $B \subseteq A$, then $A = B$.
(3) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

(b) The relation $\leq$ on the set **R** of real numbers is reflexive, antisymmetric, and transitive. Thus $\leq$ is a partial ordering on **R**.

(c) The relation "a divides b," written $a \mid b$, is a partial ordering on the set **N** of positive integers. However, "a divides b" is not a partial ordering on the set **Z** of integers since $a \mid b$ and $b \mid a$ need not imply $a = b$. For example, $3 \mid -3$ and $-3 \mid 3$ but $3 \neq -3$.

n-Ary Relations

All the relations discussed above were binary relations. By an *n-ary relation*, we mean a set of ordered n-tuples. For any set S, a subset of the product set S^n is called an n-ary relation on S. In particular, a subset of S^3 is called a *ternary relation* on S.

EXAMPLE

(a) Let L be a line in the plane. Then "betweenness" is a ternary relation R on the points of L; that is, $(a, b, c) \in R$ if b lies between a and c on L.

(b) The equation $x^2 + y^2 + z^2 = 1$ determines a ternary relation T on the set $\mathbf{R}$ of real numbers. That is, a triple (x, y, z) belongs to T if (x, y, z) satisfies the equation, which means (x, y, z) is the coordinates of a point in $\mathbf{R}^3$ on the sphere S with radius 1 and center at the origin $O = (0, 0, 0)$.

Problems:

See the first reference given on start of the presentation (Chapter 2: Page 34)
Please consider the following solved problems:

2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.9, 2.10, 2.14, 2.15, 2.18, 2.19

Also see the *Supplementary Problems* at page 40.