Lecture 15: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to

- *define floor and ceiling functions,*
- *draw graph of floor and ceiling functions,*
- *define power function.*

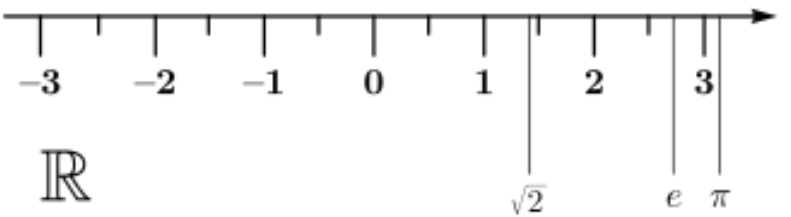
References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.
- The LibreTexts libraries; https://math.libretexts.org/

Real Numbers

In mathematics, a real number is a value of a continuous quantity that can represent a distance along a line. The real numbers include all the rational numbers, such as the integer −5 and the fraction 4/3, and all the irrational numbers, such as $\sqrt{2}$ (1.41421356..., the square root of 2) , π (3.1415…).

$\mathbb{R}$



Floor and Ceiling Functions

Let x be any real number. Then x lies between two integers called the floor and the ceiling of x. Specifically,

$\lfloor x \rfloor$, called the ***floor*** of x, denotes the greatest integer that does not exceed x.

$\lceil x \rceil$, called the ***ceiling*** of x, denotes the least integer that is not less than x.

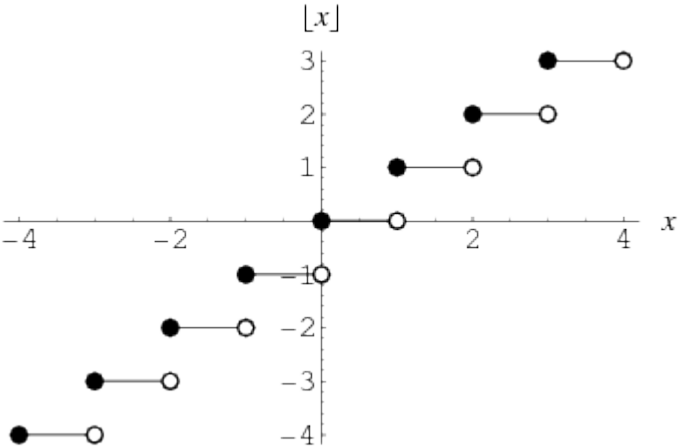
If x is itself an integer, then $\lfloor x \rfloor = \lceil x \rceil$; otherwise $\lfloor x \rfloor + 1 = \lceil x \rceil$.

Examples:

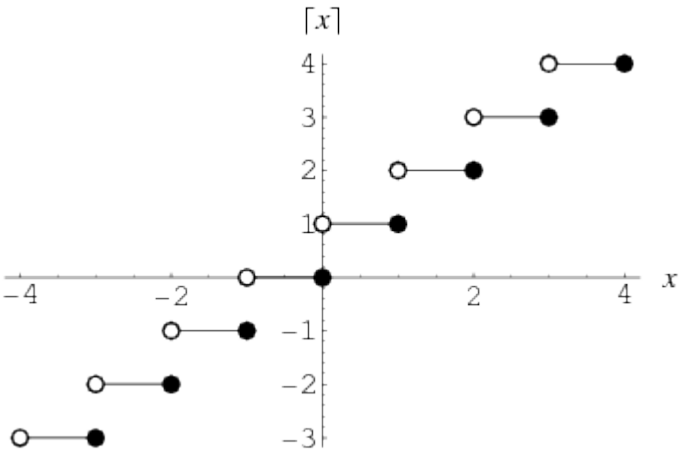
$$\lfloor 3.14 \rfloor = 3,\ \lfloor \sqrt{5} \rfloor = 2,\ \lfloor -\sqrt{5} \rfloor = -3,\ \lfloor 7 \rfloor = 7,$$

$$\lceil 3.14 \rceil = 4,\ \lceil \sqrt{5} \rceil = 3,\ \lceil -\sqrt{5} \rceil = -2,\ \lceil 7 \rceil = 7.$$

Graph of Floor Function:



Graph of Ceiling Functions:



Power Function:

A ***power function*** is a function that can be represented in the form

$$f(x) = kx^p,$$

where k and p are real numbers, and k is known as the ***coefficient***.

Examples

- The constant and identity functions are power functions because they can be written as $f(x) = x^0$, and $f(x) = x^1$ respectively.
- The quadratic and cubic functions are power functions with whole number powers $f(x) = x^2$ and $f(x) = x^3$.
- The square root and cube root functions are power functions with fractional powers because they can be written as $f(x) = x^{\frac{1}{2}}$ and $f(x) = x^{\frac{1}{3}}$.

Thanks for your attention.