Lecture 16: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to

- *define algorithm in mathematics,*
- *give examples of algorithm,*
- *define Big oh notation.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Algorithms and Functions

An algorithm M is a finite step-by-step list of well-defined instructions for solving a problem, say, to find the output $f(X)$ for a given function f with input X. (Here X may be a list or set of values.) Frequently, there may be more than one way to obtain $f(X)$, as illustrated by the following examples. The particular choice of the algorithm M to obtain $f(X)$ may depend on the "*efficiency*" or "*complexity*" of the algorithm.

Examples of algorithms:

1st Example: To add the numbers 150 and 457, split the numbers first and then add hundreds, tens and ones.

150 = 100 + 50,   457 = 400 + 50 + 7

Now

150 + 457 = 100+400+ 50+50 + 7 = 500 + 100 + 7 = 607.

Stepwise solution:

Step 1: 150 = 100 + 50 [Write in the expanded form.]

Step 2: 457 = 400 + 50 + 7 [Write in the expanded form.]

To find 150 + 457, add the hundreds, tens, and the ones separately.

Step 3: 100+400=500 [Adding the hundreds.]

Step 4: 50+50=100 [Adding the tens.]

Step 5: 0+7=7 [Adding the tens.]

Step 6: 150 + 457 = 500 + 100 + 7 = 607 [Finally sum of hundreds + sum of tens + sum of ones.]

2nd Example

Let *a* and *b* be positive integers with, say, $b < a$; and suppose we want to find $d = \text{GCD}(a,b)$, the greatest common divisor of *a* and *b*. This can be done in the following two ways.

Direct Method:

Here we find all the divisors of *a*, say by testing all the numbers from 2 to $a/2$, and all the divisors of *b*. Then we pick the largest common divisor. For example, suppose $a = 258$ and $b = 60$. The divisors of *a* and *b* follow:

$a = 258$;   divisors: 1, 2, 3, 6, 86, 129, 258

$b = 60$;   divisors: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Accordingly, $d = \text{GCD}(258,60) = 6$.

Euclidean Algorithm:

Here we divide a by b to obtain a remainder r_1 . (Note $r_1 < b$.) Then we divide b by the remainder r_1 to obtain a second remainder r_2 . (Note $r_2 < r_1$.) Next we divide r_1 by r_2 to obtain a third remainder r_3 . (Note $r_3 < r_2$.) We continue dividing r_k by r_{k+1} to obtain a remainder r_{k+2} . Since

$$a > b > r_1 > r_2 > r_3 > \ldots$$

eventually we obtain a remainder r_m = 0. Then r_{m-1} = GCD (a, b).

For example, suppose a = 258 and b = 60. Then:

Step 1: Dividing a = 258 by b = 60 yields the remainder r_1 = 18.

Step 2: Dividing b = 60 by r_1 = 18 yields the remainder r_2 = 6.

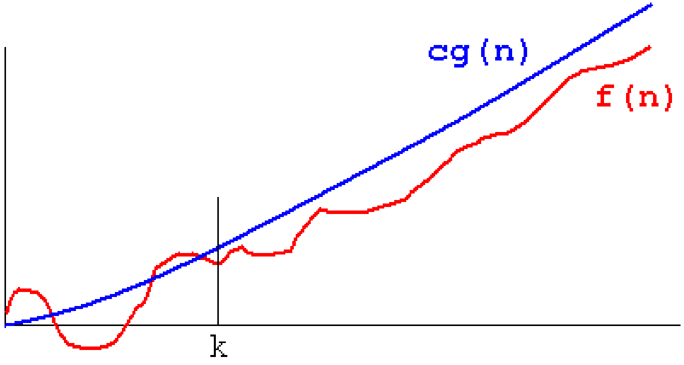
Step 3: Dividing r_1 = 18 by r_2 = 6 yields the remainder r_3 = 0.

Step 4: Thus r_2 = 6 = GCD(258,60).

Big O Notation (also written as “big oh”)

Let $f(x)$ and $g(x)$ be arbitrary functions defined on $\mathbb{R}$ or a subset of $\mathbb{R}$. We say “$f(x)$ is of order $g(x)$,” written $f(x) = O(g(x))$ if there exists a real number k and a positive constant C such that, for all $x > k$, we have

$$|f(x)| \leq C|g(x)|.$$



In other words, $f(x) = O(g(x))$ if a constant multiple of $|g(x)|$ exceeds $|f(x)|$ for all x greater than some real number k.

For example,

$$7x^2 - 9x + 4 = O(x^2) \quad \text{and} \quad 8x^3 - 576x^2 + 832x - 248 = O(x^3)$$

Thanks for your attention.