Lecture 17: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to

- *define Big-O notation,*
- *define little-o notation,*
- *define little-omega notation, and*
- *give examples and related results.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.
- https://en.wikipedia.org/wiki/Big_O_notation

Big O Notation (also written as "big oh")

Definition: Let $f(x)$ and $g(x)$ be arbitrary functions defined on $\mathbb{R}$ or a subset of $\mathbb{R}$. We say "$f(x)$ is of order $g(x)$," or "$f(x)$ is Big-O of $g(x)$" if there exists a real number k and a positive constant C such that, for all $x > k$, we have

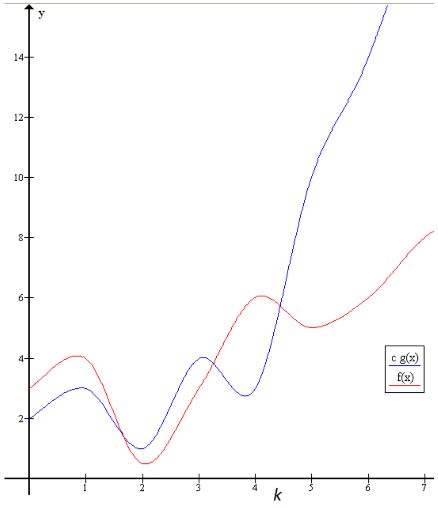
$$|f(x)| \leq C|g(x)|.$$

Notation: It is written $f(x) = O(g(x))$ or $f \in O(g)$.

For example,

$$7x^2 - 9x + 4 = O(x^2) \quad \text{and}$$

$$8x^3 - 576x^2 + 832x - 248 = O(x^3)$$



Little o Notation (also written as "little oh")

Definition: Let $f(x)$ and $g(x)$ be arbitrary functions defined on $\mathbb{R}$ or a subset of $\mathbb{R}$. We say "$f(x)$ is little-o of $g(x)$," if for every positive ε there exists a real number k such that, for all $x > k$, we have

$$|f(x)| < \varepsilon |g(x)|.$$

Notation: It is written as $f(x) = o(g(x))$ or $f \in o(g)$.

For example, one has

$$2x = o(x^2) \quad \text{and} \quad \frac{1}{x} = o(1).$$

Note that, as $g(x)$ is nonzero, or at least becomes nonzero beyond a certain point, the relation $f(x) = o(g(x))$ is equivalent to $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$ (by using the definition of limit of function).

Difference between big-O and little-o:

The difference between the earlier definition for the big-O notation and the present definition of little-o, is that while the former has to be true for *at least one* constant C, the latter must hold for *every* positive constant ε, however small. In this way, little-o notation makes a *stronger statement* than the corresponding big-O notation: every function that is little-o of g is also big-O of g, but not every function that is big-O of g is also little-o of g.

For example,

$$2x^2 = O(x^2) \text{ but } 2x^2 \neq o(x^2).$$

Little-Omega Notation

It is the inverse of the little-o notation.

Definition: Let $f(x)$ and $g(x)$ be arbitrary functions defined on $\mathbb{R}$ or a subset of $\mathbb{R}$. We say "$f(x)$ is little-omega of $g(x)$," if $g(x) = o(f(x))$ or $g \in o(f)$.

Notation: It is written as $f(x) = \omega(g(x))$ or $f \in \omega(g)$.

For example, one has

$$x^2 = \omega(x) \quad \text{and} \quad x = \omega(1).$$

Remarks: 1. Note that $f(x) = \omega(g(x))$ if and only if $g(x) = o(f(x))$.

2. Also note that if $f(x) = \omega(g(x))$ then $\lim_{x\to\infty} \dfrac{g(x)}{f(x)} = 0$.

3. The above three notations are part of the notation known as "***asymptotic notation***" in the literature.

Thanks for your attention.