# Lecture 21: Discrete Mathematics

**Course Title:** Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

## Objectives

The main aim of the lecture is to

- *define graphs and multigraphs*
- *define different components of graphs with examples.*

## References:


- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

**Graphs:**

A *graph* $G$ consists of two things:

(i) A set $V = V(G)$ whose elements are called *vertices*, *points*, or *nodes* of $G$.
(ii) A set $E = E(G)$ of unordered pairs of distinct vertices called *edges* of $G$.

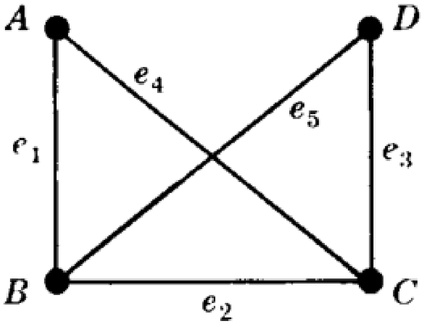
We denote such a graph by $G(V, E)$ when we want to emphasize the two parts of $G$.

For example, see the graph given on right. Here

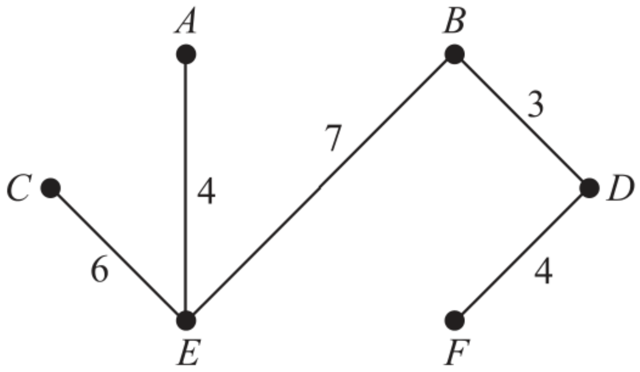
$V = \{A, B, C, D\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$ such that

$e_1 = \{A, B\}$, $e_2 = \{B, C\}$, $e_3 = \{C, D\}$, $e_4 = \{A, C\}$, $e_5 = \{B, D\}$.

In fact, we will usually denote a graph by drawing its diagram rather than explicitly listing its vertices and edges.

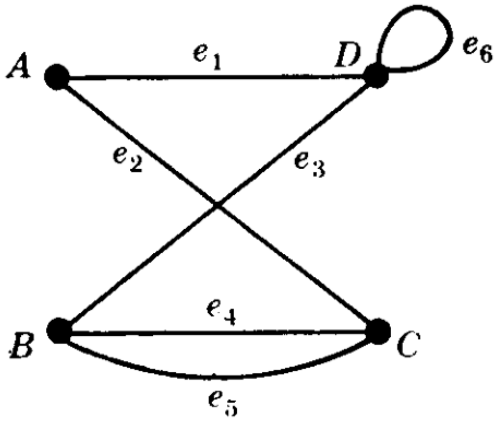


Vertices *u* and *v* are said to be ***adjacent*** or ***neighbors*** if there is an edge *e* = {*u*,*v*}. In such a case, *u* and *v* are called the ***endpoints*** of *e*, and *e* is said to ***connect*** *u* and *v*. Also, the edge *e* is said to be ***incident on*** each of its endpoints *u* and *v*.



## Multigraphs

Consider the diagram given on right. The edges $e_4$ and $e_5$ are called multiple edges since they connect the same endpoints, and the edge $e_6$ is called a loop since its endpoints are the same vertex. Such a diagram is called a ***multigraph***; the formal definition of a graph permits neither multiple edges nor loops.




(b) Multigraph

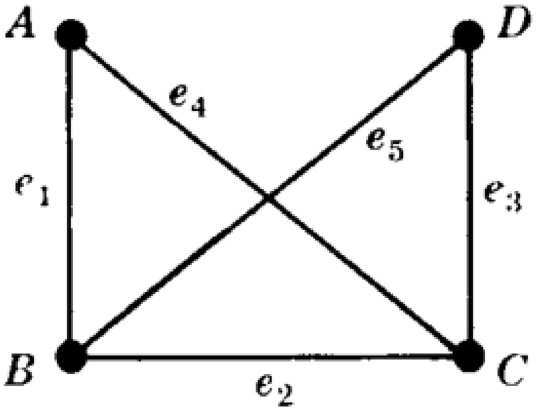
**Remark:** Some texts use the term graph to include multigraphs and use the term simple graph to mean a graph without multiple edges and loops.

**Degree of a Vertex**

The degree of a vertex *v* in a graph *G*, written deg (*v*), is equal to the number of edges in *G* which connect *v*, that is, which are incident on *v*.

Example:

Consider the following graph:



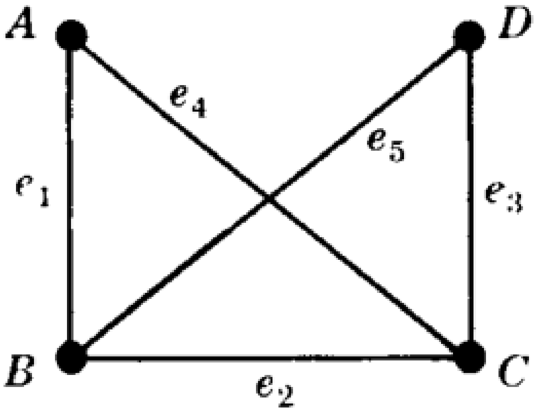

deg(*A*) = 2,

deg(*B*) = 3,

deg(*C*) = 3,

deg(*D*) = 2.

Since each edge is counted twice in counting the degrees of the vertices of *G*, we have the following simple but important result.



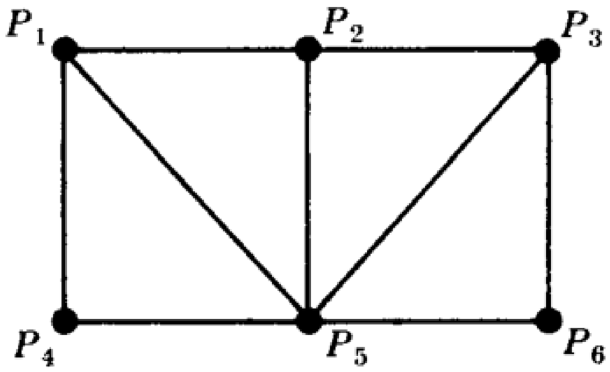

**Theorem:**
The sum of the degrees of the vertices of a graph *G* is equal to twice the number of edges in *G*.

The sum of the degrees equals 10 which, as expected, is twice the number of edges.

Above theorem is also holds for multigraphs.

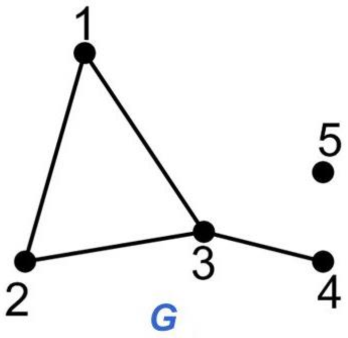
**Even or Odd Vertices:**
A vertex is said to be *even* or *odd* according as its degree is an even or an odd number.

*For exmple:*
$P_1$ is odd as its degree is odd number 3.
$P_4$ is even as its degree is even number 2.



**Isolated Vertex:**
A vertex of degree zero is called an isolated vertex.

*For example:*
Vertex 5 of the graph *G* is isolated.




*Thanks for your attention.*