# Lecture 27: Discrete Mathematics

**Course Title:** Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

## Objectives

The main aim of the lecture is to

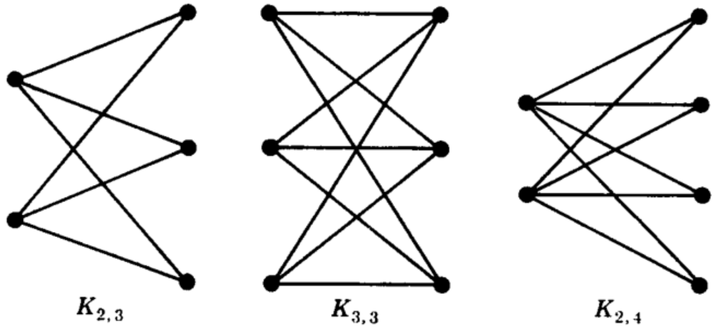
- *define bipartite graphs,*
- *define tree and forest graphs,*
- *define spanning tree.*

## References:


- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

## Bipartite Graphs

A graph *G* is said to be ***bipartite*** if its vertices *V* can be partitioned into two subsets *M* and *N* such that each edge of *G* connects a vertex of *M* to a vertex of *N*. By a ***complete bipartite*** graph, we mean that each vertex of *M* is connected to each vertex of *N*; this graph is denoted by $K_{m,n}$, where *m* is the number of vertices in *M* and *n* is the number of vertices in *N*, and, for standardization, we will assume $m \leq n$. Clearly the graph $K_{m,n}$ has *mn* edges.

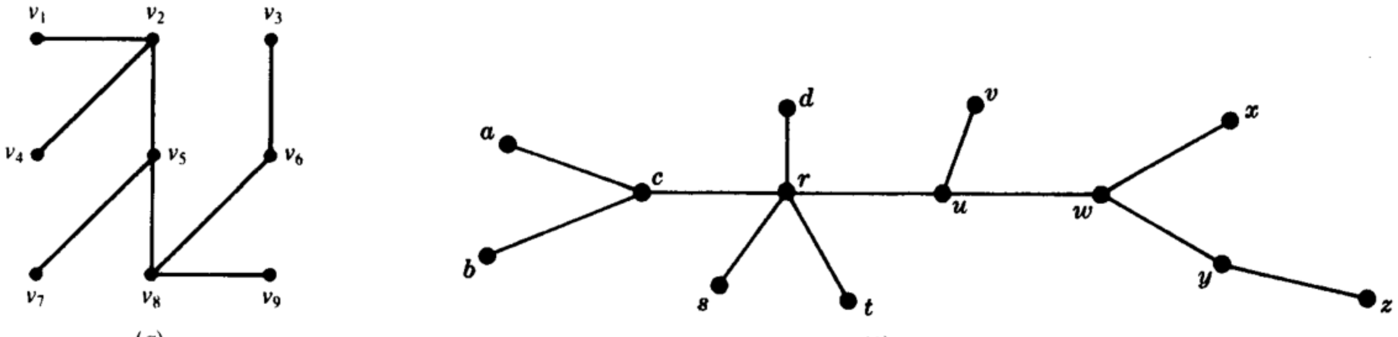


## Tree Graph

A graph $T$ is called a ***tree*** if $T$ is connected and $T$ has no cycles.

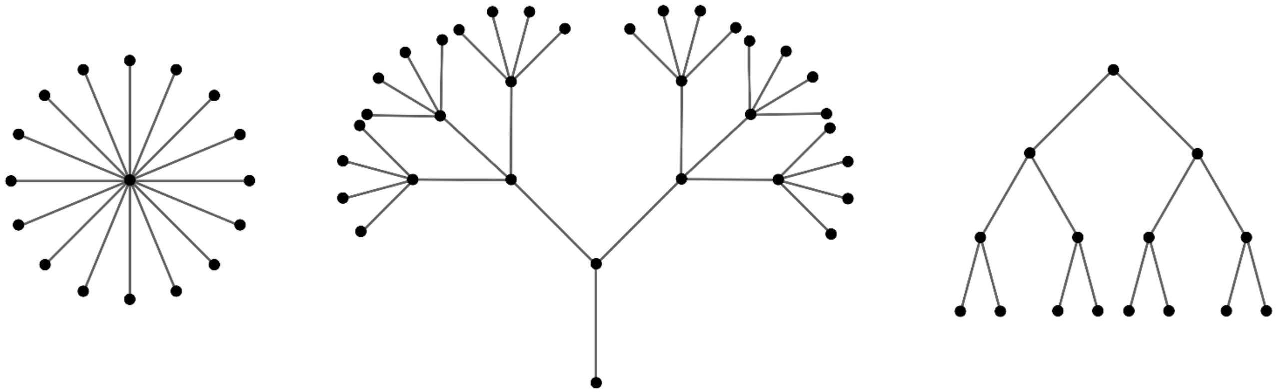
A graph without cycles is said to be ***cycle-free***. The tree consisting of a single vertex with no edges is called the ***degenerate tree***.

For example, see the following graphs:




(a) (b)

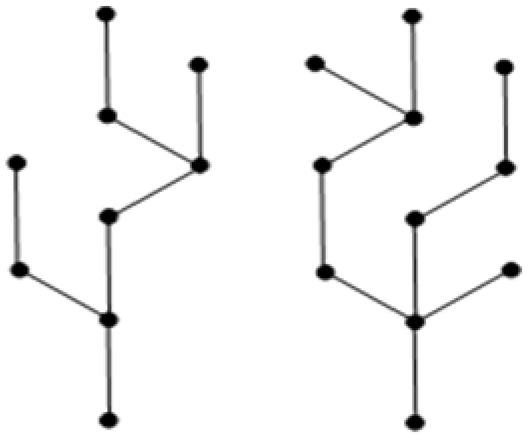
**Beautiful Example of Trees:**



## Forest

A ***forest*** *G* is a graph with no cycles and all of whose connected components are trees.

In other words, the graph consists of a disjoint union of trees.



The above graph looks like two sub-graphs, but it is a single disconnected graph. There are no cycles in the above graph. Therefore, it is a forest.

**Theorem:** Let $G$ be a graph with $n > 1$ vertices. Then the following are equivalent:

(i) $G$ is a tree.

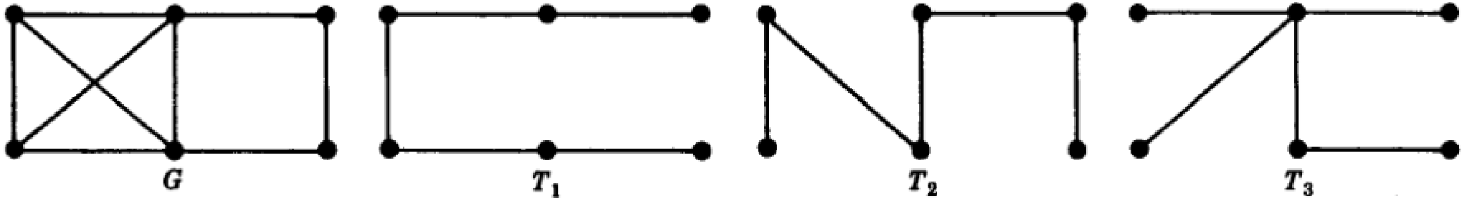
(ii) $G$ is a cycle-free and has $n - 1$ edges.

(iii) $G$ is connected and has $n - 1$ edges.

## Spanning Trees

A subgraph $T$ of a connected graph $G$ is called a spanning tree of $G$ if $T$ is a tree and $T$ includes all the vertices of $G$.

Figure below shows a connected graph $G$ and spanning trees $T_1$ , $T_2$ , and $T_3$ of $G$.




*Thanks for your attention.*