

Hyperbolic Formulas

$$\begin{aligned} \textcircled{1} \quad \cosh^2 x & - \sinh^2 x = 1 \\ \textcircled{2} \quad \cosh^2 x & + \sinh^2 x = \cosh 2x \end{aligned}$$

Adding $\textcircled{1}$ and $\textcircled{2}$

$$\textcircled{3} \quad 2\cosh^2 x = 1 + \cosh 2x$$

Subtracting $\textcircled{2}$ from $\textcircled{1}$

$$\textcircled{4} \quad 2\sinh^2 x = \cosh 2x - 1$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$$

nth order derivatives

$$\frac{d^n}{dx^n} \sin(ax+b) = a^n \sin\left[n \frac{\pi}{2} + ax+b\right]$$

$$\frac{d^n}{dx^n} \cos(ax+b) = a^n \cos\left[n \frac{\pi}{2} + ax+b\right]$$

$$\frac{d^n}{dx^n} a^x = a^x (\ln a)^n$$

$$\frac{d^n}{dx^n} e^{ax} = a^n e^{ax}$$

$$\frac{d^n}{dx^n} \left[\frac{1}{ax+b} \right] = \frac{a^n (-1)^n n!}{(ax+b)^{n+1}}$$

$$\frac{d^n}{dx^n} e^{ax} \sin(bx+c) = [a^2 + b^2]^{\frac{n}{2}} e^{ax} \sin\left(bx+c+n \tan^{-1} \frac{b}{a}\right)$$

$$\frac{d^n}{dx^n} e^{ax} \cos(bx+c) = [a^2 + b^2]^{\frac{n}{2}} e^{ax} \cos\left(bx+c+n \tan^{-1} \frac{b}{a}\right)$$

$\tan^{-1} z = \frac{1}{2} \cos^{-1} \left(\frac{1-z^2}{1+z^2} \right)$	$\tanh^{-1} z = \frac{1}{2} \cosh^{-1} \left(\frac{1+z^2}{1-z^2} \right)$
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$$\int \frac{dx}{a+b\cos x} = \frac{1}{\sqrt{a^2-b^2}} \cos^{-1} \left(\frac{b+a\cos x}{a+b\cos x} \right) + c, \quad a > b$$

$$\int \frac{dx}{a+b\cos x} = \frac{1}{\sqrt{b^2-a^2}} \cosh^{-1} \left(\frac{b+a\cos x}{a+b\cos x} \right) + c, \quad a < b$$

$$\int \frac{dx}{a+b\cos x} = \frac{1}{a} \tan \frac{x}{2} + c, \quad a = b$$