Chapter No. 7

Important Definitions:

Kinematics: The branch of mechanics concerned with the motion of objects without reference to the forces which cause the motion.

Dynamics: The branch of mechanics concerned with the motion of objects with reference to the forces which cause the motion.

Displacement: Shortest distance between two points is called displacement, and it is a vector quantity.

Velocity: Time derivative of displacement is called velocity or time rate of change of displacement.

Acceleration: Time derivative of velocity or time rate of change of velocity.

Example: A particle is moving in such a way that its position at any time t is specified by

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$$\bar{r} = (t^3 + t^2)\hat{\imath} + (\cos t + \sin^2 t)j + (e^t + \log t)k$$

Find its velocity and acceleration.

Solution:

$$\bar{v} = \frac{d\bar{r}}{dt} = (3t^2 + 2t)i + (2\sin t\cos t - sint)j + (e^t + \frac{1}{t})\, k$$

$$\bar{v} = (3t^2 + 2t)i + (\sin 2\text{t} - sint)j + (e^t + \frac{1}{t})k$$

$$\bar{a} = \frac{d\bar{v}}{dt} = (6t + 2)i + (2\cos 2t - \cos t)j + \left(e^t - \frac{1}{t^2}\right)k$$

Cartesian Components of Velocity and Acceleration:

In plane cartesian coordinates displacement of any point can be written as

$$\bar{r} = xi + yj \qquad (1)$$

Where i & j are the unit vectors along x and y axis.

$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d}{dt}(xi + yj) = \frac{dx}{dt}i + \frac{dy}{dt}j$$

$$\bar{v} = \frac{dx}{dt} i + \frac{dy}{dt} j \qquad (2)$$

In eq(2) $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are the cartesian component of velocity along x-axis and y-axis respectively.

The magnitude of the velocity is given by

$$v = |\bar{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

Now,

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left(\frac{dx}{dt} i + \frac{dy}{dt} j\right) = \frac{d^2x}{dt^2} i + \frac{d^2y}{dt^2} j \qquad (3)$$

In eq(3) $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$ are the cartesian component of acceleration along x-axis and y-axis respectively.

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Example: At any time, the position of a particle in a plane, can be specified by $(a\cos\omega\, t, a\sin\omega t)$, where a & ω are constants. Find components of velocity and acceleration.

Solution:

Along x-axis component of $\bar{r}$ is

$$x = a\cos\omega t$$

Component of velocity along x-axis

$$\frac{dx}{dt} = -a\omega\sin\omega t$$

Component of acceleration along x-axis

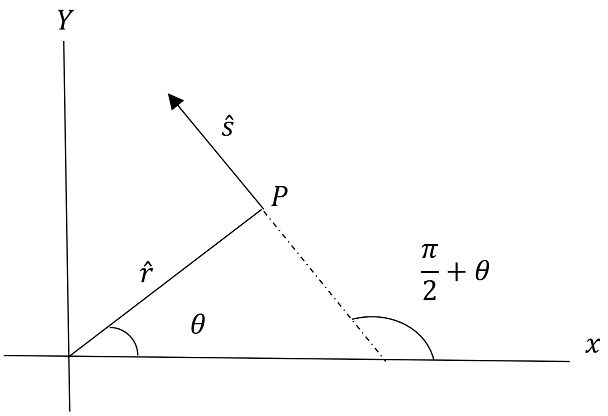
$$\frac{d^2x}{dt^2} = -a\omega^2\cos\omega t$$

Similarly component of velocity and acceleration along y-axis are
$a\,\omega\cos\omega t \quad and \quad -a\omega^2\sin\omega t$ respectively.

Radial and Transverse Components:

In polar coordinates the position of a particle is specifed by radius vector $\bar{r}$ and polar angle θ. The direction of the radius vector is known as the *radial direction* and that perpendicular to it in the direction of increassing θ is called the *transverse direction.*

Let $\hat{r}, \hat{s}$ be the unit vector in the radial and transverse direction respectively as shown in fig.



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In polar coordinates the relation between x, y, r and θ is as

$$x = r\,cos\theta$$

$$y = r \sin\theta$$

We know in cartesian coordinates

$$\bar{r} = xi + yj$$

$$\Rightarrow \qquad \bar{r} = r\cos\theta\, i + r\sin\theta\, j$$

$$\Rightarrow \qquad \frac{\bar{r}}{r} = \cos\theta\, i + \sin\theta\, j$$

$$\Rightarrow \qquad \hat{r} = \cos\theta\, i + \sin\theta\, j \qquad (1)$$

Similarly for transverse components

$$\hat{s} = \cos\left(\frac{\pi}{2} + \theta\right) i + \sin\left(\frac{\pi}{2} + \theta\right) j$$

$$\Rightarrow \qquad \hat{s} = -\sin\theta\, i + \cos\theta\, j \qquad (2)$$

Now

$$\frac{d\hat{r}}{dt} = -\sin\theta\frac{d\theta}{dt}i + \cos\theta\frac{d\theta}{dt}j = (-\sin\theta\ i + \cos\theta\ j)\frac{d\theta}{dt}$$

By using (2)

$$\Rightarrow \quad \frac{d\hat{r}}{dt} = \hat{s}\dot{\theta} \qquad (3)$$

$$\frac{d\hat{s}}{dt} = -\cos\theta\frac{d\theta}{dt}i - \sin\theta\frac{d\theta}{dt}j = -(\cos\theta i + \sin\theta\ j)\frac{d\theta}{dt}$$

By using (1)

$$\Rightarrow \quad \frac{d\hat{s}}{dt} = -\hat{r}\dot{\theta} \qquad (4)$$

Components of Velocity

$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d}{dt}(\hat{r}r)$$

$$\Rightarrow \quad \bar{v} = r\frac{d\hat{r}}{dt} + \frac{dr}{dt}\hat{r}$$

by using (3)

$$\bar{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{s}$$

So radial component of velocity $v_r = \dot{r}$, transverse component is $v_\theta = r\dot{\theta}$

Components of acceleration

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left(\dot{r}\hat{r} + r\dot{\theta}\hat{s}\right)$$

$$= \ddot{r}\hat{r} + \dot{r}\frac{d\hat{r}}{dt} + \dot{r}\dot{\theta}\hat{s} + r\ddot{\theta}\hat{s} + r\dot{\theta}\frac{d\hat{s}}{dt}$$

$$= \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{s} + \dot{r}\dot{\theta}\hat{s} + r\ddot{\theta}\hat{s} - r\dot{\theta}\dot{\theta}\hat{r}$$

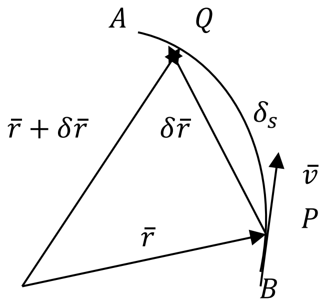
$$\bar{a} = \left(\ddot{r} - r\left(\dot{\theta}\right)^2\right)\hat{r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{s}$$

So, $a_r = \ddot{r} - r\left(\dot{\theta}\right)^2$ and $a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

Theorem:

The velocity of a particle at any point is along the tangent at that point.

Proof: Consider the motion of a particle along path AB. Suppose at times t and $t+\delta_t$ the particle travels the distance s and $s+\delta_s$ from the fixed point A to the point P and Q whose position vectors are $\bar{r}$ and $\bar{r}+\bar{\delta}_r$ respectively.



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If $\bar{v}$ is the velocity of the particle at time t.

Then

$$\bar{v}=\frac{d\bar{r}}{dt} \qquad and \qquad v=\frac{ds}{dt}$$

Now

$$\bar{v}=\frac{d\bar{r}}{dt}=\frac{d\bar{r}}{ds}\cdot\frac{ds}{dt}$$

$$\bar{v}=v\frac{d\bar{r}}{ds} \qquad (1)$$

Direction of $\frac{d\bar{r}}{ds}$:

Since

$$\frac{d\bar{r}}{ds}=\lim_{\delta_s\to 0}\left(\frac{1}{\delta s}\right)\bar{\delta}_r$$

$\therefore \quad \frac{d\bar{r}}{ds}$ is a vector along the tangent to the path at point P.

Magnitude of $\frac{d\bar{r}}{ds}$:

$$\left|\frac{d\bar{r}}{ds}\right|=\left|\lim_{\delta_s\to 0}\left(\frac{\bar{\delta}_r}{\delta s}\right)\right|=1$$

Hence $\frac{d\bar{r}}{ds}$ is unit vector along tangent.

So we can say $\frac{d\bar{r}}{ds} = \hat{t}$

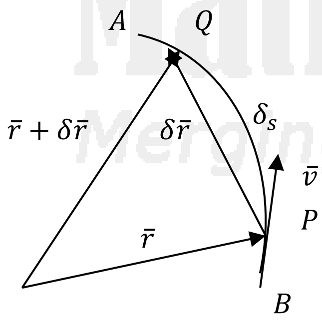
Eq (1) becomes

$$\bar{v} = v\hat{t}$$

Above equation show that velocity of a particle at point P is along the tangent at P.

Tangential and normal components of velocity and acceleration:

Consider the motion of a particle along path AB. Suppose at times t and $t + \delta_t$ the particle travels the distance s and $s + \delta_s$ from the fixed point A to the point P and Q whose position vectors are $\bar{r}$ and $\bar{r} + \bar{\delta}_r$ respectively. Clearly $\overline{PQ} = \delta\bar{r}$



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If $\bar{v}$ is the velocity of the particle at time t.

Then

$$\bar{v} = \frac{d\bar{r}}{dt} \qquad and \qquad v = \frac{ds}{dt}$$

Now

$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d\bar{r}}{ds} \cdot \frac{ds}{dt}$$

$$\bar{v} = v\frac{d\bar{r}}{ds} \qquad (1)$$

Direction of $\frac{d\bar{r}}{ds}$:

Since,

$$\frac{d\bar{r}}{ds} = \lim_{\delta_s \to 0} \left(\frac{1}{\delta s}\right) \bar{\delta}_r$$

$\therefore$ $\frac{d\bar{r}}{ds}$ is a vector along the tangent to the path at point P.

Magnitude of $\frac{d\bar{r}}{ds}$:

$$\left|\frac{d\bar{r}}{ds}\right| = \left|\lim_{\delta_s \to 0} \left(\frac{\bar{\delta}_r}{\delta s}\right)\right| = 1$$

Hence $\frac{d\bar{r}}{ds}$ is unit vector along tangent.

So we can say $\frac{d\bar{r}}{ds} = \hat{t}$

Eq (1) becomes

$$\bar{v} = v\hat{t} \qquad (2)$$

Above equation show that velocity of a particle at point P is along the tangent at P, by above equation we can see that velocity have no component along the normal.

<u>Components of Acceleration:</u>

If $\bar{a}$ is the acceleration of the particle at time, then

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(v\hat{t}) \qquad by\ (2)$$

$$\bar{a} = \left(\frac{dv}{dt}\right)\hat{t} + v\frac{d\hat{t}}{dt} \qquad (3)$$

Direction of $\frac{d\hat{t}}{dt}$:

Since $\hat{t}$ is a unit vector so

$$\hat{t} \cdot \hat{t} = 1$$

$$\frac{d}{dt}(\hat{t} \cdot \hat{t}) = 0$$

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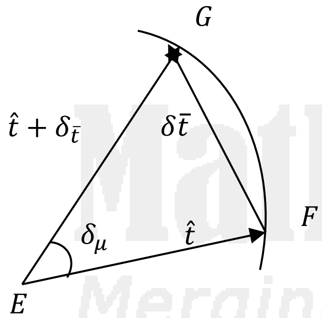
$$\hat{t} \cdot \frac{d\hat{t}}{dt} = 0$$

$$\Rightarrow \qquad \hat{t} \perp \frac{d\hat{t}}{dt}$$

Hence $\frac{d\hat{t}}{dt}$ is along the normal. Thus if $\hat{n}$ is a unit vector along the normal then

$$\frac{d\hat{t}}{dt} = \left|\frac{d\hat{t}}{dt}\right| \hat{n} \qquad (4)$$



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Magnitude of $\frac{d\hat{t}}{dt}$:

Suppose at times t and $\delta_t + t$ the unit tangents at P and Q are $\hat{t}$ and $\hat{t} + \delta_{\hat{t}}$ be represented by $\overline{EF}$ and $\overline{EG}$ with angle δ_μ.

Now

$$\overline{EF} + \overline{FG} = \overline{EG}$$

$$\overline{FG} = \overline{EG} - \overline{EF}$$

$$\overline{FG} = \delta_{\hat{t}}$$

$$\left|\frac{d\hat{t}}{dt}\right| = \lim_{\delta_t \to 0} \left|\frac{\delta\hat{t}}{\delta\, t}\right| = \lim_{\substack{\delta_t \to 0 \\ \delta_\mu \to 0 \\ \delta_s \to 0}} \left|\frac{\delta\hat{t}}{\delta_\mu} \cdot \frac{\delta_\mu}{\delta_s} \cdot \frac{\delta_s}{\delta\hat{t}}\right|$$

Here $\lim_{\delta_\mu \to 0} \left|\frac{\delta\hat{t}}{\delta_\mu}\right| = 1, \quad \lim_{\delta_s \to 0} \left|\frac{\delta_\mu}{\delta_s}\right| = \kappa, \qquad \lim_{\delta_t \to 0} \left|\frac{\delta_s}{\delta\hat{t}}\right| = \frac{ds}{dt} = v$

So

$$\left|\frac{d\hat{t}}{dt}\right| = \kappa v = \frac{1}{\rho}v$$

$$\Rightarrow \quad \frac{d\hat{t}}{dt} = \frac{v}{\rho}\hat{n}$$

$$\Rightarrow \quad \bar{a} = \left(\frac{dv}{dt}\right)\hat{t} + \left(\frac{v^2}{\rho}\right)\hat{n}$$

Where $\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2x}{dy^2}}$

Example:

A particle is moving along the parabola $x^2 = 4ay$ with constant speed. Determine the tangential and normal components of its acceleration when it reaches the point whose abscissa is $\sqrt{5}a$.

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Solution:

$$x^2 = 4ay \qquad (1)$$

Put $x = \sqrt{5}a$ in (1) we get $y = \frac{5a}{4}$, thus the point will be $p\left(\sqrt{5}a\,\frac{5a}{4}\right)$,

We know $\bar{a} = \frac{dv}{dt}\hat{t} + \frac{v^2}{\rho}\hat{n}$

$a_t = \frac{dv}{dt} = 0$ v is constant.

For ρ

$$x^2 = 4ay$$

$$2x = 4a\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2a} \qquad (2)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{2a}$$

Hence

$$\left(\frac{dy}{dx}\right)_p = \frac{\sqrt{5}}{2} \quad and \quad \left(\frac{d^2y}{dx^2}\right)_p = \frac{1}{2a}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\frac{5}{4}\right]^{\frac{3}{2}}}{\frac{1}{2a}}$$

$$= 2a.\left(\frac{9}{4}\right)^{\frac{3}{2}} = \frac{27}{4}a$$

So $a_n = \frac{v^2}{\rho} = \frac{4v^2}{27a}$

Example:

A particle moves in a plane in such a way that at any time t its distance from a fixed point o, is $r = at + bt^2$ and the line connecting o and p makes an angle is $\theta = ct^{\frac{3}{2}}$ with a fixed line. Find radial and transverse components of velocity and acceleration at t=1.

Solution:

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Here

$$r = at + bt^2$$

$$\Rightarrow \quad \dot{r} = a + 2bt$$

$$\Rightarrow \quad \ddot{r} = 2b$$

$$\theta = ct^{\frac{3}{2}}$$

$$\Rightarrow \quad \dot{\theta} = \frac{3}{2}ct^{\frac{1}{2}}$$

$$\Rightarrow \quad \ddot{\theta} = \frac{3}{4}c\frac{1}{\sqrt{t}}$$

$at\ t = 1,$

$$r = a + b, \qquad \dot{r} = a + 2b, \quad \ddot{r} = 2b$$

$$\theta = c, \qquad \dot{\theta} = \frac{3}{2}c, \qquad \ddot{\theta} = \frac{3}{4}c$$

Radial and transverse Components of velocity:

$$v_r = \dot{r} = a + 2b$$

$$v_\theta = r\dot{\theta} = \frac{3}{2}c(a+b)$$

Radial and transverse Components of acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= 2b - (a+b)\left(\frac{3}{2}c\right)^2$$

$$a_r = 2b - \frac{9}{4}c^2(a+b)$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= (a+b).\frac{3}{4}c + 2(a+2b).\frac{3}{2}c$$

$$a_\theta = \frac{3}{4}c(5a+9b)$$

Exercise

Q#1 A particle starts from O at $t = 0$. Find its velocity and acceleration.

i. $\bar{r} = (t^3 + 2t)i + (5t^2 - 7)j$

$$\bar{v} = \frac{d\bar{r}}{dt} = (3t^2 + 2)i + 10tj$$

$$\bar{a} = \frac{d\bar{v}}{dt} = 6ti + 10j$$

ii. $\bar{r} = at^2 i + 4atj$

iii. $\bar{r} = a\cos t\, i + b\sin t\, j$

iv. $\bar{r} = a(t - cost\,)i + a(1 + sint)j$

Q#2 The position of a particle moving along an ellipse is given by $\bar{r} = a\cos t\;\; i + b\sin t\, j$ a>b. Find the position of the particle where its velocity has a maximum or minimum magnitude.

Solution:

$$\bar{r} = \text{a}\cos t\;\; i + b\sin t\, j$$

$$\bar{v} = \frac{d\bar{r}}{dt} = -a\sin t\, i + b\cos t\, j$$

$$v = \sqrt{(-a\,sint)^2 + (bcost)^2}$$

$$v = \sqrt{a^2\sin^2 t + b^2\cos^2 t}$$

$$v = \sqrt{a^2(1-\cos^2 t) + b^2\cos^2 t}$$

$$v = \sqrt{a^2 - (a^2 - b^2)\cos^2 t}$$

Now v is minimum if $\cos^2 t = 1$ or $\cos t = \pm 1 \;\; \Rightarrow \;\; t = 0, \pi$

Thus for min v

$$\bar{r} = \text{a}\cos 0\;\; i + b\sin 0\, j = ai$$

$$\bar{r} = \text{a}\cos\pi\;\; i + b\sin\pi\, j = -ai$$

Now v is maximum if $\cos^2 t = 0$ or $t = \frac{\pi}{2}, \frac{3\pi}{2}$

Thus for maximum v

$$\bar{r} = \text{a}\cos\frac{\pi}{2}\;\; i + b\sin\frac{\pi}{2}j = bj$$

$$\bar{r} = a\cos\frac{3\pi}{2}\ i + b\sin\frac{3\pi}{2} j = -bj$$

Q#3 A particle moving with uniform speed v along the curve $x^2y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$. Show that its acceleration has maximum value $\frac{10v^2}{9a}$.

Solution:

We know that $\bar{a} = \frac{dv}{dt}\hat{t} + \frac{v^2}{\rho}\hat{n}$

Here $\frac{dv}{dt} = 0$, because v is constant.

Let

$$x^2y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$$

$$x^2y - ax^2 = \frac{a^3}{\sqrt{5}}$$

$$y - a = \frac{a^3}{\sqrt{5}}\frac{1}{x^2}$$

Taking derivative

$$\frac{dy}{dx} = -\frac{2a^3}{\sqrt{5}x^3}$$

$$\frac{d^2y}{dx^2} = \frac{6a^3}{\sqrt{5}x^4}$$

We know

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$\Rightarrow \quad \rho = \frac{\left(1 + \frac{4a^6}{5x^6}\right)^{\frac{3}{2}}}{\frac{6a^3}{\sqrt{5}x^4}}$$

$$= \left(\frac{5x^6 + 4a^6}{5x^6}\right)^{\frac{3}{2}} \frac{\sqrt{5}x^4}{6a^3}$$

$$\rho = \frac{(5x^6 + 4a^6)^{\frac{3}{2}}}{30a^3x^5}$$

$$a_n = \frac{v^2}{\rho} = \frac{v^2 30a^3x^5}{(5x^6 + 4a^6)^{\frac{3}{2}}}$$

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For maximum acceleration check (Hint: Second Derivative Rule FSc. Part II Ex. 2.9)

Differentiation w.r.t.x

$$\frac{da}{dx} = 30a^3v^2\ \frac{\left((5x^6+4a^6)^{\frac{3}{2}}.5x^4 - x^5\frac{3}{2}(5x^6+4a^6)^{\frac{1}{2}}30x^5\right)}{(5x^6+4a^6)^3}$$

$$= 30a^3v^2(5x^6+4a^6)^{\frac{1}{2}}\left(\frac{(5x^6+4a^6)\quad 5x^4 - 45x^{10}}{(5x^6+4a^6)^3}\right)$$

$$\frac{da}{dx} = 600a^3v^2\left[\frac{x^4(a^6-x^6)}{(5x^6+4a^6)^{\frac{5}{2}}}\right]$$

Again

$$\frac{d^2a}{dt^2} = \left[\frac{4x^3(a^6-x^6)}{(5x^6+4a^6)^{\frac{5}{2}}} + \frac{x^4(-6x^5)}{(5x^6+4a^6)^{\frac{5}{2}}} + \frac{x^4(a^6-x^6)\left(-\frac{5}{2}\right)(30x^5)}{(5x^6+4a^6)^{\frac{7}{2}}}\right]$$

Taking $\frac{da}{dt} = 0$

$$600a^3v^2\left[\frac{x^4(a^6-x^6)}{(5x^6+4a^6)^{\frac{5}{2}}}\right] = 0$$

$$x^4(a^6-x^6) = 0$$

$$\Rightarrow \quad x^4 = 0\,, \quad a^6 - x^6 = 0$$

$$\Rightarrow \quad x = 0, \quad (a^2-x^2)(a^4+x^4+a^2x^2) = 0$$

$$a^2 - x^2 = 0, \quad a^4 + x^4 + a^2x^2 = 0$$

Let $a^2 - x^2 = 0$, other due to imaginary

$$x = \pm a$$

Hence

$$\left(\frac{d^2 2}{dx^2}\right)_{x=a} = 600a^3v^2\left[-\frac{6a^9}{(5x^6+4a^6)^{\frac{5}{2}}}\right] < 0$$

So acceleration will be maximum at x=a

$$a = \frac{v^2 30a^3x^5}{(5x^6+4a^6)^{\frac{3}{2}}}$$

$$a = \frac{30v^2a^8}{(9a^6)^{\frac{3}{2}}} = \frac{30v^2a^8}{(3a^3)^3} = \frac{10v^2}{9a}$$

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Q#4 Find the tangential and normal components of acceleration of a point describing the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with uniform speed V when the particle is at (0,b)

Solution:

Since

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Rightarrow \quad b^2x^2+a^2y^2=a^2b^2$$

Taking derivative

$$2b^2x+2a^2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{b^2}{a^2}\frac{x}{y}$$

$$\frac{d^2y}{dx^2}=-\frac{b^2}{a^2}\left[\frac{y-x\frac{dy}{dx}}{y^2}\right]$$

$$\left(\frac{dy}{dx}\right)_{(0,b)}=0$$

So

$$\left(\frac{d^2y}{dx^2}\right)_{(0,b)}=-\frac{b^2}{a^2}\frac{b}{b^2}=-\frac{b}{a^2}$$

$$\rho=\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$\rho=\frac{1}{\frac{-b}{a^2}}=-\frac{a^2}{b}$$

$$\bar{a}=\frac{dv}{dt}\hat{t}+\frac{v^2}{\rho}\hat{n}$$

$$\bar{a}=0-\frac{bv^2}{a}\hat{n}$$

$$\bar{a}=-\frac{bv^2}{a}\hat{n}$$

Q#5 Find the radial and transverse components of velocity of a particle moving at curve

$$ax^2+by^2=1 \qquad (1)$$

At any time t. if polar angle is $\theta=ct^2$.

Solution:

We know $x = r\cos\theta,\ \ y = r\sin\theta$ $\quad put\ in\ (1)$

$$ar^2\cos^2\theta + br^2\sin^2\theta = 1$$

$$r^2 = \frac{1}{a\cos^2\theta + b\sin^2\theta}$$

$$r = (a\cos^2\theta + b\sin^2\theta)^{-\frac{1}{2}}$$

Also $\theta = ct^2 \Rightarrow \quad \dot{\theta} = 2ct$

$$\dot{r} = \frac{ct(a-b)sin2\theta}{(a\cos^2\theta + b\sin^2\theta)^{\frac{3}{2}}}$$

$$v_r = \dot{r} = \frac{ct(a-b)sin2\theta}{(a\cos^2\theta + b\sin^2\theta)^{\frac{3}{2}}}$$

$$v_\theta = r\dot{\theta} = \frac{2ct}{(a\cos^2\theta + b\sin^2\theta)^{\frac{1}{2}}}$$

Q#6 Find the radial and transverse components of acceleration along $x^2 + y^2 = a^2$ with constant angular velocity.

Solution:

$$x^2 + y^2 = a^2 \qquad (1)$$

We know $x = r\cos\theta,\ \ y = r\sin\theta$ $\quad put\ in\ (1)$

$$r^2 = a^2$$

$$r = a, \quad and \quad \dot{\theta} = c$$

$$\Rightarrow \dot{r} = 0, \ddot{r} = 0, \qquad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -ac^2$$

$$a_\theta = a\dot{r}\dot{\theta} + r\ddot{\theta} = 0$$

Note: These notes are written for the Chapter no. 7 of the book Mechanics by Q.K. Ghori.

Written by: Umair Sabi Ullah

Head of Mathematics Department

Controller Examination

ORION COLLEGE, PAKPATTAN

For any suggestions feel free to contact

Email: umairmughal70@gmail.com **Mob: +923136504842**