

# Textbook of Algebra and Trigonometry for Class XI

(Punjab Textbook Board)

## SHORT TERM PREPARATION

***IMPORTANT MCQs***

***IMPORTANT Short Questions***

***IMPORTANT Long Questions***

## EXERCISE WISE

***M.SALMAN SHERAZI (03337727666/03067856232)***

*This document contains all the important MCQs, Short Questions and Long Questions of Mathematics HSSC-I (FSc Part 1) from the Textbook of Algebra and Trigonometry for Class XI. It has been done to help the students and teachers at no cost by M Salman Sherazi. This work (pdf) is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0.*

## EXERCISE 1.1

Tick (✓) the correct answer.

1.  $\sqrt{2}$  is a number  
 (a) Rational (b) ✓ Irrational (c) Prime (d) Natural
2. The numbers which can be written in the form of  $\frac{p}{q}$ ,  $p, q \in \mathbb{Z}$ ,  $q \neq 0$  are :  
 (a) ✓ Rational number (b) Irrational number (c) Complex number (d) Whole number
3. A decimal which has a finite numbers of digits in its decimal part is called \_\_\_\_\_ decimal.  
 (a) ✓ Terminating (b) Non-Terminating (c) Recurring (d) Non recurring
4. 5.333 .... Is  
 (a) ✓ Rational (b) Irrational (c) an integer (d) a prime number
5.  $\pi$  is  
 (a) Rational (b) ✓ Irrational (c) Natural number (d) None
6.  $\frac{22}{7}$  is  
 (a) ✓ Rational (b) Irrational (c) an integer (d) a whole number
7. Multiplicative inverse of '0' is  
 (a) 0 (b) any real number (c) ✓ not defined (d) 1
8. Golden rule of fraction is that for  $k \neq 0$ ,  $\frac{a}{b} =$   
 (a) ✓  $\frac{ka}{kb}$  (b)  $\frac{ab}{l}$  (c)  $\frac{ka}{b}$  (d)  $\frac{kb}{b}$
9. The set  $\{1, -1\}$  possesses closure property w. r. t  
 (a) ' + ' (b) ✓ ' × ' (c) ' ÷ ' (d) ' - '
10. If  $a < b$  then  
 (a)  $a < b$  (b)  $\frac{1}{a} < \frac{1}{b}$  (c) ✓  $\frac{1}{a} > \frac{1}{b}$  (d)  $a - b > 0$

### SHORT QUESTIONS

- i. Write down the "Closure Property for addition".
- ii. Does the set  $\{1, -1\}$  possess closure property with respect to  
 (a) addition (b) multiplication
- iii. Name the properties :  $1000 \times 1 = 1000$  and  $-3 < -2 \Rightarrow 0 < 1$
- iv. Prove that  $-\frac{7}{12} - \frac{5}{18} = \frac{-21-10}{36}$
- v. Simplify justifying each step :  $\frac{4+16x}{4}$

## EXERCISE 1.2

Tick (✓) the correct answer.

1. The multiplicative identity of complex number is  
 (a) (0,0) (b) (0,1) (c) ✓ (1,0) (d) (1,1)
2.  $i^{13}$  equals:

- (a)   $i$  (b)  $-i$  (c) 1 (d)  $-1$
3. The multiplicative inverse of  $(4, -7)$  is:  
 (a)  $(-\frac{4}{65}, -\frac{7}{65})$  (b)  $(-\frac{4}{65}, \frac{7}{65})$  (c)  $(\frac{4}{65}, -\frac{7}{65})$  (d)   $(\frac{4}{65}, \frac{7}{65})$
4.  $(0, 3)(0, 5) =$   
 (a) 15 (b)   $-15$  (c)  $-8i$  (d)  $8i$
5.  $(-1)^{-\frac{21}{2}} =$   
 (a)  $i$  (b)   $-i$  (c) 1 (d)  $-1$
6.  $\sqrt{-\frac{1}{4}} =$   
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{\sqrt{2}}i$  (c)   $\frac{1}{2}i$  (d)  $-\frac{1}{2}i$
7. Factorization of  $3x^2 + 3y^2$  is:  
 (a)  $(3x + 3yi)(3x - 3yi)$  (b)   $3(x + iy)(x - iy)$  (c)  $(x - iy)(x + iy)$  (d) None of these
8. The multiplicative inverse of  $(0, 0)$  is:  
 (a)  $(0, 1)$  (b)  $(1, 0)$  (c)  $(0, 0)$  (d)  Does not exist
9. The product of any two conjugate complex numbers is  
 (a)  Real number (b) complex number (c) zero (d) 1
10. Identity element of complex number is  
 (a)  $(0, 1)$  (b)  $(0, 1)$  (c)  $(0, 0)$  (d)   $(1, 0)$

### SHORT QUESTIONS

- i. Prove that "multiplicative inverse" of  $(a, b)$  is  $(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2})$ .
- ii. Find the "multiplicative inverse" of  $(1, 0)$
- iii. Factorize  $a^2 + 4b^2$  and  $9a^2 + 16b^2$
- iv. Prove that the sum as well as the product of two conjugate complex numbers is a real number.
- v. Separate into real and imaginary parts:  $\frac{i}{1+i}$
- vi. Simplify the following:  $(-1)^{-\frac{21}{2}}$  and  $i^9$
- vii. Write in terms of  $i$ :  $\sqrt{-5}$  and  $\sqrt{\frac{-16}{25}}$
- viii. Simplify the following  $(2, 6)(3, 7)$  and  $(2, 6) \div (3, 7)$

## EXERCISE 1.3

Tick () the correct answer.

1. If  $z_1$  and  $z_2$  are complex numbers then  $|z_1+z_2|$  is \_\_\_\_\_  
 (a)  $< |z_1+z_2|$  (b)   $\leq |z_1| + |z_2|$  (c)  $\geq |z_1+z_2|$  (d) None of these
2. The figure representing one or more complex numbers on the complex plane is called:  
 (a) Cartesian plane (c) Z-Plane (c) Complex plane (d)  Argand diagram
3.  $y - axis$  represents  
 (a) Real numbers (b)  Imaginary numbers (c) natural numbers (d) Rational numbers

4. If  $z = x + iy$  then  $|z| =$

- (a)  $x^2 + y^2$  (b)  $x^2 - y^2$  (c)   $\sqrt{x^2 + y^2}$  (d)  $\sqrt{x^2 - y^2}$

5. The moduli of 3 is

- (a)  3 (b) 4 (c) 5 (d) 6

6.  $z\bar{z} =$

- (a)  $z^2$  (b)  $z$  (c)  $\bar{z}$  (d)   $|z|^2$

7.  $(z - \bar{z})^2$  is

- (a) Complex number (b)  Real number (c) both (a) and (b) (d) None of these

8.  $i^{101} =$

- (a) 1 (b)  $-1$  (c)   $i$  (d)  $-i$

### SHORT QUESTIONS

i. Show that  $z^2 + \bar{z}^2$  is a real number.

ii. Prove that  $\bar{\bar{z}} = z$  iff  $z$  is real.

iii. Simplify  $(-ai)^4, a \in R$

iv. Simplify the following  $5 + 2\sqrt{-4}$

v. Simplify the following  $(a + bi)^2$

vi. State "De Moivre's theorem".

vii. Find the moduli of  $-5i$

viii. Simplify the following  $(a - bi)^3$

### EXERCISE 2.1

Tick (✓) the correct answer.

1. A set is a collection of objects which are

- (a) Well defined (b)  Well defined and distinct (c) identical (d) not defined

2. The set of odd numbers between 1 and 9 are

- (a)  $\{1,3,5,7\}$  (b)  $\{3,5,7,9\}$  (c)  $\{1,3,5,7,9\}$  (d)   $\{3,5,7\}$

3. There are \_\_\_\_\_ methods to describe a set.

- (a) 2 (b)  3 (c) 4 (d) 5

4.  $\{1,2,3\}$  and  $\{2,1,3\}$  are sets.

- (a)  Equal (b) Equivalent (c) Not equal (d) None of these

5. The sets  $N$  and  $O$  are sets.

- (a) Equal (b)  Equivalent (c) Not equal (d) None of these

6. Which of the following is true?

- (a)  $N \subset R \subset Q \subset Z$  (b)  $R \subset Z \subset Q \subset N$  (c)  $Z \subset N \subset Q \subset R$  (d)   $N \subset Z \subset Q \subset R$

7. The empty set is a subset of

- (a) Empty set (b)  Every set (c) Natural set (d) Whole set

8. Total number of subsets that can be formed from the set  $\{x, y, z\}$  is

- (a) 1 (b)  8 (c) 5 (d) 2

9. A set having only one element is called

- (a) Empty set                      (b) ✓ Singleton set                      (c) Power set                      (d) Subset

10. The set of odd integers between 2 and 4 is

- (a) Null set                      (b) Power set                      (c) ✓ Singleton set                      (d) Subset

## SHORT QUESTIONS

- i. If  $A = \{a, b\}$ , then find  $P(A)$ .
- ii. Write the following sets in "Set – builder method"  
(a) {January, June, July}    (b) {100, 101, 102,....., 400}
- iii. Write the following set in "descriptive method" and "tabular form"  
(a)  $\{x|x \in N \wedge 4 < x < 12\}$                       (b)  $\{x|x \in R \wedge x = x\}$
- iv. Write two power subsets of  $N$  and  $\{a, b, c\}$
- v. Is there any set which has no proper sub set? If so name that set.
- vi. What is the difference between  $\{a, b\}$  and  $\{\{a, b\}\}$ ?
- vii. Write down the power set of  $\{\varnothing\}$  and  $\{+, -, \times, \div\}$ .

## EXERCISE 2.2

Tick (✓) the correct answer.

1. A diagram which represents a set is called \_\_\_\_\_  
(a) ✓ Venn's                      (b) Argand                      (c) Plane                      (d) None of these
2.  $A \cup \varnothing =$   
(a)  $\varnothing$                       (b)  $U$                       (c) ✓  $A$                       (d)  $U - A$
3.  $A - U =$   
(a) ✓  $\varnothing$                       (b)  $A$                       (c)  $U$                       (d)  $U - A$
4.  $n(A \cup B) =$   
(a) ✓  $n(A) + n(B)$                       (b)  $n(A) - n(B)$                       (c)  $n(B) - n(A)$                       (d)  $n(A)n(B)$
5. If  $A \subseteq B$  then  $A \cup B =$   
(a)  $A$                       (b) ✓  $B$                       (c)  $A^c$                       (d)  $B^c$
6. Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  and  $A = \{2, 4, 6, 8, 10\}$  then  $A^c =$   
(a)  $\{1, 2, 3, 4, 5\}$                       (b)  $\{6, 7, 8, 9, 10\}$                       (c) ✓  $\{1, 3, 5, 7, 9\}$                       (d)  $\{2, 4, 6, 8, 10\}$
7. If  $A$  and  $B$  are disjoint sets then :  
(a) ✓  $A \cap B = \varnothing$                       (b)  $A \cap B \neq \varnothing$                       (c)  $A \subset B$                       (d)  $A - B = \varnothing$
8. If  $U = N$  then  
(a)  $E' = E, O' = O$                       (b)  $E' = U, O' = U$                       (c) ✓  $E' = O, O' = E$                       (d) None of these
9. If the intersection of two sets is the empty then sets are called  
(a) ✓ Disjoint sets                      (b) Overlapping Sets                      (c) Subsets                      (d) Power sets

## SHORT QUESTIONS

- i. Exhibit  $A \cup B$  and  $A \cap B$  by "Venn Diagram" in the following case :-  $B \subseteq A$
- ii. Under what conditions on  $A$  and  $B$  are the following statements true?
  - (a)  $A \cup B = B$
  - (b)  $U - A = \varphi$
- iii. Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{2, 4, 6, 8, 10\}$  find  $U^c$  and  $A^c$
- iv. Define Union of Two sets.

### EXERCISE 2.3

Tick (✓) the correct answer.

1. Commutative property of intersection is:
  - (a)  $A \cup B = B \cup A$
  - (b) ✓  $A \cap B = B \cap A$
  - (c)  $A \cap B = B \cup A$
  - (d)  $A \cup B = B \cap A$
2.  $(A \cup B)' =$ 
  - (a)  $A' \cup B'$
  - (b) ✓  $A' \cap B'$
  - (c)  $A' \cap B$
  - (d)  $A \cup B'$
3. Take any set, say  $A = \{1, 2, 3, 4, 5\}$  then  $A \cup \emptyset =$ 
  - (a) ✓  $A$
  - (b)  $\emptyset$
  - (c)  $U$
  - (d) None of these
4.  $L \cup M = L \cap M$  then  $L$  is equal to
  - (a) ✓  $M$
  - (b)  $L$
  - (c)  $\varphi$
  - (d)  $M'$
5. If  $U = \{1, 2, 3, 4, 5, \dots, 20\}$  and  $A = \{1, 3, 5, \dots, 19\}$  then  $A \cap U =$ 
  - (a)  $A$
  - (b)  $U$
  - (c) ✓  $\emptyset$
  - (d)  $A'$
6. If  $x \in L \cup M$  then
  - (a)  $x \notin L$  or  $x \notin M$
  - (b)  $x \notin L$  or  $x \in M$
  - (c)  $x \in L$  or  $x \notin M$
  - (d) ✓  $x \in L$  or  $x \in M$
7. If  $x \in (A \cup B)'$  then
  - (a)  $x \in (A \cup B)$
  - (b) ✓  $x \notin (A \cup B)$
  - (c)  $x \in A$  or  $x \in B$
  - (d)  $x \in A$  and  $x \in B$

### SHORT QUESTIONS

- i. Write down "De Morgan's Laws".
- ii. Write down "Associativity property of union".
- iii. Prove that  $A \cap B = B \cap A$
- iv. Verify "Associativity of intersection" of  $N, Z, Q$
- v. Let  $U =$  The set of English alphabet,  $A = \{x | x \text{ is a vowel}\}$  and  $B = \{y | y \text{ is consonant}\}$  Verify  $(A \cup B)' = A' \cap B'$
- vi. If  $U = \{1, 2, 3, 4, 5, \dots, 20\}$  and  $A = \{1, 3, 5, \dots, 19\}$  then verify  $A \cap A' = \emptyset$
- vii. From suitable properties of Union and Intersection deduce  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- viii. Take any set, say  $A = \{1, 2, 3, 4, 5\}$  verify  $A \cup A = A$

## EXERCISE 2.4

Tick (✓) the correct answer.

1. For the propositions  $p$  and  $q$ ,  $(p \wedge q) \rightarrow p$  is:  
 (a) ✓ Tautology      (b) Absurdity      (c) contingency      (d) None of these
2. For the propositions  $p$  and  $q$ ,  $p \rightarrow (p \vee q)$  is:  
 (a) ✓ Tautology      (b) Absurdity      (c) Contingency      (d) None of these
3. The symbol which is used to denote negation of a proposition is  
 (a) ✓  $\sim$       (b)  $\rightarrow$       (c)  $\wedge$       (d)  $\vee$
4. Truth set of a tautology is  
 (a) ✓ Universal set      (b)  $\emptyset$       (c) True      (d) False
5. A statement which is always falls is called  
 (a) Tautology      (b) ✓ Absurdity      (c) Contingency      (d) Contra positive
6.  $p \rightarrow \sim p$  is  
 (a) Tautology      (b) ✓ Absurdity      (c) Contingency      (d) Contra positive
7. In a proposition if  $p \rightarrow q$  then  $q \rightarrow p$  is called  
 (a) Inverse of  $p \rightarrow q$       (b) ✓ converse of  $p \rightarrow q$       (c) contrapositive of  $p \rightarrow q$       (d) None
8. Contra positive of  $\sim p \rightarrow \sim q$  is  
 (a)  $p \rightarrow q$       (b) ✓  $q \rightarrow p$       (c)  $\sim p \rightarrow q$       (d)  $\sim q \rightarrow p$
9. The symbol " $\exists$ " is called  
 (a) Universal quantifier      (b) ✓ Existential quantifier      (c) Converse      (d) Inverse
10. The symbol " $\forall$ " is called  
 (a) ✓ Universal quantifier      (b) Existential quantifier      (c) Converse      (d) Inverse

## M. Salman Sherazi

### SHORT QUESTIONS

- i. Write converse , inverse and contra positive of  $\sim p \rightarrow q$
- ii. Construct the truth table of  $[(p \rightarrow q) \wedge p \rightarrow q]$
- iii. Show that  $\sim(p \rightarrow q) \rightarrow p$  is tautology.
- iv. Define Absurdity.

### LONG QUESTIONS

Prove that  $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

## EXERCISE 2.5

Tick (✓) the correct answer.

1. Truth set of  $p \wedge q$  is  
 (a) ✓  $P \cap Q$       (b)  $P \cup Q$       (c)  $P - Q$       (d)  $P + Q$

2.  $P = Q$  is the truth set of

- (a)  $p = q$  (b)  $p \rightarrow q$  (c)   $p \leftrightarrow q$  (d)  $p \Rightarrow q$

3. Truth set of a tautology is the

- (a) Power set (b) Subset (c)  Universal set (d) Super set

### SHORT QUESTIONS

Write logical form of

$$(A \cap B)' = A' \cup B'$$

### LONG QUESTIONS

Convert  $(A \cup B) \cup C = A \cup (B \cup C)$  into logical form and prove it by constructing the truth table.

## EXERCISE 2.6

Tick () the correct answer.

- If  $y = \sqrt{x}, x \geq 0$  is a function, then its inverse is:
 

(a) A line (b) a parabola (c) a point (d)  not a function
- An onto function is also called:
 

(a) Injective (b)  Surjective (c) Bijective (d) Inverse
- A (1 – 1) function is also called:
 

(a)  Injective (b) Surjective (c) Bijective (d) Inverse
- If set  $A$  has 2 elements and  $B$  has 4 elements, then number of elements in  $A \times B$  is:
 

(a) 6 (b)  8 (c) 16 (d) None of these
- Inverse of a line is:
 

(a)  A line (b) a parabola (c) a point (d) not defined
- The function  $f = \{(x, y), y = x\}$  is:
 

(a)  Identity function (b) Null function (c) not a function (d) similar function
- The range of  $\{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$ 

(a)  $\{2,3,4,5,6\}$  (b)   $\{1,2,3,4,5\}$  (c)  $\{2,1,3,2,4\}$  (d)  $\{1,2,3,5\}$
- The inverse of  $\{(x, y) | x^2 + y^2 = 9, |x| \leq 3, |y| \leq 3\}$  is
 

(a) Function (b)  Not function (c) (1-1) function (d) onto function

### SHORT QUESTIONS

- Find the inverse of  $\{(x, y) | y = 2x + 3, x \in R\}$ . Tell it is function or not.
- How we can find inverse of a function in "Set – builder notation".
- For  $A = \{1, 2, 3, 4\}$ , find the following relation in  $A$ . State domain and range of the relation.  $\{(x, y) | x + y < 5\}$
- Find the inverse of  $\{(x, y) | y^2 = 4ax, x \geq 0\}$ . Tell it is function or not.

## EXERCISE 2.7

Tick (✓) the correct answer.

1. Negation of a given number is example of

- (a) Binary operation      (b) ✓ Unary operation      (c) relation      (d) function

2. Cube root of a number is example of

- (a) Binary operation      (b) ✓ Unary operation      (c) relation      (d) function

3. A binary operation is denoted by

- (a) +      (b)  $\times$       (c) ✓ \*      (d)  $\div$

4.  $O + E =$

- (a) ✓  $O$       (b)  $E$       (c)  $W$       (d)  $C$

5. The set  $\{1, -1, i, -i\}$  where  $i = \sqrt{-1}$  is closed w. r. t

- (a) +      (b) ✓  $\times$       (c) \*      (d)  $\div$

6. The set  $\{1, \omega, \omega^2\}$  where  $i = \sqrt{-1}$  is closed w. r. t

- (a) +      (b) ✓  $\times$       (c) \*      (d)  $\div$

7.  $N$  is closed w. r. t

- (a) +      (b)  $\times$       (c) ✓ both (a) and (b)      (d)  $\div$

8. Inverse and identity of a set  $S$  under binary operation \* is

- (a) ✓ Unique      (b) Two      (c) Three      (d) Four

## SHORT QUESTIONS

- i. Write down two properties of Binary Operations.
- ii. Prepare a table of addition of the elements of the set of residue classes modulo 4.
- iii. Write multiplication table for the set  $\{1, -1, i, -i\}$ .

## EXERCISE 2.8

Tick (✓) the correct answer.

1. The set of natural number is not closed under binary operation

- (a) +      (b)  $\times$       (c) both (a) and (b)      (d) ✓ -

2. The set  $\{1, -1, i, -i\}$  is not closed w. r. t

- (a) ✓ +      (b)  $\times$       (c) both (a) and (b)      (d) None of these

3.  $(Z, .)$  is

- (a) Group      (b) ✓ Semi-group      (c) closed      (d) Not closed

4. Subtraction is non-commutative and non-associative on

- (a) ✓  $N$       (b)  $R$       (c)  $Z$       (d)  $Q$

5. A semi-group having an identity is called

- (a) Group (b)  monoid (c) Closed (d) Not closed
6. For every  $a, b \in G$ ,  $a * b = b * a$  then  $G$  is called  
 (a) Group (b) Monoid (c) Closed (d)  Abelian group
7. Solution of linear equation  $ax = b$  is  
 (a)  $x = ba^{-1}$  (b)   $x = a^{-1}b$  (c)  $x = ab$  (d)  $xa = b$
8. In group  $(\mathbb{Z}, +)$  inverse of 1 is  
 (a) 1 (b)  -1 (c) 0 (d) 2
9. In group  $(\mathbb{R} - \{0\}, \times)$  inverse of 3 is  
 (a)   $\frac{1}{3}$  (b) -3 (c) 0 (d) 2
10. In a group the inverse is  
 (a)  Unique (b) two (c) three (d) four

### SHORT QUESTIONS

- Prove that  $(ab)^{-1} = b^{-1}a^{-1}$
- Define Finite and Infinite group.
- Consider the set  $S = \{1, -1, i, -i\}$ . Set up its ' $\cdot$ ' table and show that the set is an abelian group under ' $\cdot$ '
- If  $G$  is a group under the operation  $*$  and  $a, b \in G$ , find the solutions of the equation  $a * x = b$  and  $x * a = b$

### LONG QUESTIONS

Show that the set  $\{1, \omega, \omega^2\}$ ,  $\omega^3 = 1$ , is an Abelian group w.r.t ordinary multiplication.

Prove that all  $2 \times 2$  non-singular matrices over the real field form a non-abelian group under multiplication.

### EXERCISE 3.1

Tick () the correct answer.

- A rectangular array of numbers enclosed by a square brackets is called:  
 (a)  Matrix (b) Row (c) Column (d) Determinant
- The horizontal lines of numbers in a matrix are called:  
 (a) Columns (b)  Rows (c) Column matrix (d) Row matrix
- The vertical lines of numbers in a matrix are called:  
 (a)  Columns (b) Rows (c) Column matrix (d) Row matrix
- If a matrix  $A$  has  $m$  rows and  $n$  columns, then order of  $A$  is :  
 (a)   $m \times n$  (b)  $n \times m$  (c)  $m + n$  (d)  $m^n$
- $[1 \ -1 \ 3 \ 4]$  is an example of  
 (a)  Row vector (b) column vector (c) Rectangular matrix (d) Square matrix
- The matrix  $A$  is said to be real if its all entries are

- (a) Rational (b)  real (c) natural (d) complex
7. If a matrix A has different number of rows and columns then A is called:  
 (a) Row vector (b)  Column vector (c) square matrix (d) Rectangular matrix
8. For the square matrix  $A = [a_{ij}]_{n \times n}$  then  $a_{11}, a_{22}, a_{33} \dots a_{nn}$  are:  
 (a)  Main diagonal (b) primary diagonal (c) preceding diagonal (d) secondary diagonal
9. For a square matrix  $A = [a_{ij}]$  if all  $a_{ij} = 0, i \neq j$  and all  $a_{ij} = k$  (non-zero) for  $i = j$  then A is called:  
 (a) Diagonal matrix (b)  Scalar Matrix (c) Unit matrix (d) Null matrix
10. A square matrix A is singular if  
 (a)   $|A| = 0$  (b)  $|A| \neq 0$  (c)  $A = 0$  (d)  $A \neq 0$
11. If order of matrix A is  $m \times n$  and order of matrix B is  $n \times p$  then order of matrix AB is  
 (a)  $m \times n$  (b)  $n \times m$  (c)  $n \times p$  (d)   $m \times p$

### SHORT QUESTIONS

- i. If  $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$ , show that  $A^4 = I_2$
- ii. Find the value of  $x$  and  $y$  if  $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$
- iii. If  $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$  and  $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , find the values of  $a$  and  $b$ .
- iv. Find the matrix X if;  $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$
- v. Find the matrix A if;  $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$

### LONG QUESTIONS

Find  $x$  and  $y$  if  $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$

### EXERCISE 3.2

Tick () the correct answer.

1. In general matrix multiplication is not  
 (a)  Commutative (b) Associative (c) Closure (d) Distributive
2.  $(A^t)^t =$   
 (a)  $A^t$  (b)  A (c)  $-A$  (d)  $(A^t)^t$
3. If  $A = [a_{ij}]_{m \times n}$  then  $A^t =$   
 (a)  $[a_{ij}]_{n \times m}$  (b)  $[a_{ji}]_{m \times n}$  (c)  $[a_{ij}]_{m \times m}$  (d)   $[a_{ji}]_{n \times m}$
4. Which of the following Sets is a field.  
 (a) R (b) Q (c) C (d)  all of these
5. Which of the following Sets is not a field.  
 (a) R (b) Q (c) C (d)  Z

### SHORT QUESTIONS

- i. Find the inverse of  $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$
- ii. If  $A$  and  $B$  are square matrices of the same order, then explain why in general  
 (a)  $(A + B)^2 \neq A^2 + 2AB + B^2$       (b)  $A^2 - B^2 \neq (A - B)(A + B)$
- iii. If  $A = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$  then find  $A^t A$

### LONG QUESTIONS

Solve the following system of linear equations:  $3x - 5y = 1$ ;  $-2x + y = -3$

Solve the following matrix equation for  $A$ :  $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A - \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix}$

### EXERCISE 3.3

Tick (✓) the correct answer.

- For any matrix  $A$ , it is always true that  
 (a)  $A = A^t$       (b)  $-A = \bar{A}$       (c) ✓  $|A| = |A^t|$       (d)  $A^{-1} = \frac{1}{A}$
- If all entries of a square matrix of order 3 is multiplied by  $k$ , then value of  $|kA|$  is equal to:  
 (a)  $k|A|$       (b)  $k^2|A|$       (c) ✓  $k^3|A|$       (d)  $|A|$
- For a non-singular matrix it is true that :  
 (a)  $(A^{-1})^{-1} = A$       (b)  $(A^t)^t = A$       (c)  $\bar{\bar{A}} = A$       (d) ✓ all of these
- For any non-singular matrices  $A$  and  $B$  it is true that:  
 (a)  $(AB)^{-1} = B^{-1}A^{-1}$       (b)  $(AB)^t = B^t A^t$       (c)  $AB \neq BA$       (d) ✓ all of these
- If a square matrix  $A$  has two identical rows or two identical columns then  
 (a)  $A = 0$       (b) ✓  $|A| = 0$       (c)  $A^t = 0$       (d)  $A = 1$
- If a matrix is in triangular form, then its determinant is product of the entries of its  
 (a) Lower triangular matrix (b) Upper triangular matrix (c) ✓ main diagonal (d) none of these
- If  $A$  is non-singular matrix then  $A^{-1} =$   
 (a) ✓  $\frac{1}{|A|} \text{adj}A$       (b)  $-\frac{1}{|A|} \text{adj}A$       (c)  $\frac{|A|}{\text{adj}A}$       (d)  $\frac{1}{|A|\text{adj}A}$
- $\begin{vmatrix} r \cos \varphi & 1 & -\sin \varphi \\ 0 & 1 & 0 \\ r \sin \varphi & 1 & \cos \varphi \end{vmatrix} =$   
 (a) 1      (b) 2      (c) ✓  $r$       (d)  $r^2$
- $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} =$   
 (a) 1      (b) 2      (c) ✓ 0      (d) -1
- $(A^{-1})^t =$   
 (a)  $A^{-1}$       (b)  $(A^{-1})^t$       (c) ✓  $(A^t)^{-1}$       (d)  $A^t$

### SHORT QUESTIONS

i. Evaluate  $\begin{vmatrix} 2a & a & a \\ b & 2b & b \\ c & c & 2c \end{vmatrix}$

ii. Without expansion show that  $\begin{vmatrix} 1 & 2 & -2 \\ -1 & 1 & 3 \\ 2 & 4 & -1 \end{vmatrix}$

iii. Without expansion verify that  $\begin{vmatrix} 1 & 2 & 3x \\ 2 & 3 & 6x \\ 3 & 5 & 9x \end{vmatrix} = 0$

iv. Find the value of  $x$  if  $\begin{vmatrix} 3 & 1 & x \\ -1 & 3 & 4 \\ x & 1 & 0 \end{vmatrix} = -30$

v. If  $A$  and  $B$  are non-singular matrices, then show that  $(A^{-1})^{-1} = A$

### LONG QUESTIONS

Show that  $\begin{vmatrix} -a & 0 & c \\ 0 & a & -b \\ b & -c & 0 \end{vmatrix} = 0$

Show that  $\begin{vmatrix} a + \lambda & b & c \\ a & b + \lambda & c \\ a & b & c + \lambda \end{vmatrix} = \lambda^2(a + b + c + \lambda)$

### EXERCISE 3.4

Tick (✓) the correct answer.

1. A square matrix  $A$  is symmetric if:

- (a) ✓  $A^t = A$                       (b)  $A^t = -A$                       (c)  $(\overline{A})^t = A$                       (d)  $(\overline{A})^t = -A$

2. A square matrix  $A$  is skew symmetric if:

- (a)  $A^t = A$                       (b) ✓  $A^t = -A$                       (c)  $(\overline{A})^t = A$                       (d)  $(\overline{A})^t = -A$

3. A square matrix  $A$  is Hermitian if:

- (a)  $A^t = A$                       (b)  $A^t = -A$                       (c) ✓  $(\overline{A})^t = A$                       (d)  $(\overline{A})^t = -A$

4. A square matrix  $A$  is skew- Hermitian if:

- (a)  $A^t = A$                       (b)  $A^t = -A$                       (c)  $(\overline{A})^t = A$                       (d) ✓  $(\overline{A})^t = -A$

5. The main diagonal elements of a skew symmetric matrix must be:

- (a) 1                      (b) 0                      (c) any non-zero number                      (d) any complex number

6. The main diagonal elements of a skew hermitian matrix must be:

- (a) 1                      (b) ✓ 0                      (c) any non-zero number                      (d) any complex number

7. In echelon form of matrix, the first non zero entry is called:

- (a) ✓ Leading entry                      (b) first entry                      (c) preceding entry                      (d) Diagonal entry

8. A square matrix  $A = [a_{ij}]$  for which  $a_{ij} = 0, i > j$  then  $A$  is called:

- (a) ✓ Upper triangular                      (b) Lower triangular                      (c) Symmetric                      (d) Hermitian

9. A square matrix  $A = [a_{ij}]$  for which  $a_{ij} = 0, i < j$  then  $A$  is called:

- (a) Upper triangular (b)  Lower triangular (c) Symmetric (d) Hermitian

10. If  $A$  is symmetric (Skew symmetric), then  $A^2$  must be

- (a) Singular solution (b) non singular (c)  symmetric (d) non trivial

### SHORT QUESTIONS

- i. If  $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$ , show that  $A + (\bar{A})^t$  is hermitian.
- ii. If  $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$ , show that  $A + A^t$  is symmetric.
- iii. If  $A$  is symmetric or skew-symmetric, show that  $A^2$  is symmetric.
- iv. If  $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$ , find  $A(\bar{A})^t$ .
- v. Define symmetric and skew-symmetric matrix.
- vi. Define lower and upper triangular matrix.

### EXERCISE 3.5

Tick () the correct answer.

1. In a homogeneous system of linear equations, the solution  $(0,0,0)$  is:  
 (a)  Trivial solution (b) non trivial solution (c) exact solution (d) anti symmetric
2. If  $AX = O$  then  $X =$   
 (a)  $I$  (b)   $O$  (c)  $A^{-1}$  (d) Not possible
3. If the system of linear equations have no solution at all, then it is called a/an  
 (a) Consistent system (b)  Inconsistent system (c) Trivial System (d) Non Trivial System
4. The value of  $\lambda$  for which the system  $x + 2y = 4; 2x + \lambda y = -3$  does not possess the unique solution  
 (a)  4 (b) -4 (c)  $\pm 4$  (d) any real number
5. If the system  $x + 2y = 0; 2x + \lambda y = 0$  has non-trivial solution, then  $\lambda$  is:  
 (a)  4 (b) -4 (c)  $\pm 4$  (d) any real number

### LONG QUESTIONS

Use matrices to solve the system

$$x_1 - 2x_2 + x_3 = -4 ; 2x_1 - 3x_2 + 2x_3 = -6 ; 2x_1 + 2x_2 + x_3 = 5$$

Solve by using Cramer's Rule

$$2x + 2y + z = 3 ; 3x - 2y - 2z = 1 ; 5x + y - 3z = 2$$

Find the value of  $\lambda$  for which the following system does not possess a unique solution. Also solve the system for the value of  $\lambda$ .

$$x_1 + 4x_2 + \lambda x_3 = 2 \quad ; \quad 2x_1 + x_2 - 2x_3 = 11 \quad ; \quad 3x_1 + 2x_2 - 2x_3 = 16$$

### EXERCISE 4.1

Tick (✓) the correct answer.

1. The equation  $ax^2 + bx + c = 0$  will be quadratic if:

- (a)  $a = 0, b \neq 0$                       (b) ✓  $a \neq 0$                       (c)  $a = b = 0$                       (d)  $b =$  any real number

2. Solution set of the equation  $x^2 - 4x + 4 = 0$  is:

- (a)  $\{2, -2\}$                       (b) ✓  $\{2\}$                       (c)  $\{-2\}$                       (d)  $\{4, -4\}$

3. The quadratic formula for solving the equation  $ax^2 + bx + c = 0; a \neq 0$  is

- (a)  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$                       (b) ✓  $x = \frac{-b \pm \sqrt{a^2 - 4ac}}{2b}$                       (c)  $x = \frac{-a \pm \sqrt{b^2 - 4ac}}{2}$                       (d) None of these

4. How many techniques to solve quadratic equations.

- (a) 1                      (b) 2                      (c) ✓ 3                      (d) 4

5. The solution of a quadratic equation are called

- (a) ✓ Roots                      (b) identity                      (c) quadratic equation                      (d) solution

### SHORT QUESTIONS

- i. Solve by factorization  $x^2 - x = 2$
- ii. Solve by completing square  $x^2 - 3x - 648$
- iii. Solve by quadratic formula  $15x^2 + 2ax - a^2 = 0$
- iv. Solve the equation  $x^2 - 7x + 10 = 0$  by factorization.
- v. Define "Quadratic equation".
- vi. Derive "Quadratic formula".

### LONG QUESTIONS

Solve by factorization  $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b ; x \neq \frac{1}{a}, \frac{1}{b}$

Solve by quadratic formula  $(a + b)x^2 + (a + 2b + c)x + b + c = 0$

Solve by quadratic formula

$$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$$

## EXERCISE 4.2

Tick (✓) the correct answer.

1. To convert  $ax^{2n} + bx^n + c = 0 (a \neq 0)$  into quadratic form, the correct substitution is:
  - (a) ✓  $y = x^n$
  - (b)  $x = y^n$
  - (c)  $y = x^{-n}$
  - (d)  $y = \frac{1}{x}$
2. The equation in which variable occurs in exponent, called:
  - (a) ✓ Exponential function
  - (b) Quadratic equation
  - (c) Reciprocal equation
  - (d) Exponential equation
3. To convert  $4^{1+x} + 4^{1-x} = 10$  into quadratic, the substitution is:
  - (a)  $y = x^{1-x}$
  - (b)  $y = 4^{1+x}$
  - (c) ✓  $y = 4^x$
  - (d)  $y = 4^{-x}$
4. The equation  $x^4 - 3x^3 + 4x^2 - 3x + 1 = 0$  is example of
  - (a) Exponential equation
  - (b) Quadratic equation
  - (c) Radical equation
  - (d) ✓ Reciprocal equation

### SHORT QUESTIONS

- i. Solve  $x^4 - 6x^2 + 8 = 0$
- ii. Solve  $x^{\frac{2}{5}} + 8 = 6x^{\frac{1}{5}}$
- iii. Define "Reciprocal equation" with an example.
- iv. Solve  $4^{1+x} + 4^{1-x} = 10$
- v. Define "Exponential equation" with an example.

### LONG QUESTIONS

Solve  $x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

Solve  $4 \cdot 2^{2x+1} - 9 \cdot 2^x + 1 = 0$

Solve  $(x-1)(x+5)(x+8)(x+2) - 880 = 0$

## EXERCISE 4.3

Tick (✓) the correct answer.

1. The equations involving radical expressions of the variable are called:
  - (a) Reciprocal equations
  - (b) ✓ Radical equations
  - (c) Quadratic functions
  - (d) exponential equations

### LONG QUESTIONS

Solve  $\sqrt{2x+8} + \sqrt{x+5} = 7$

Solve  $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Solve  $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

Solve  $(x + 4)(x + 1) = \sqrt{x^2 + 2x - 15} + 3x + 31$

### EXERCISE 4.4

Tick (✓) the correct answer.

1. The cube roots of unity are :

- (a) ✓  $1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$  (b)  $1, \frac{1+\sqrt{3}i}{2}, \frac{1+\sqrt{3}i}{2}$  (c)  $-1, \frac{-1+\sqrt{3}i}{2}, \frac{-1+\sqrt{3}i}{2}$  (d)  $-1, \frac{1+\sqrt{3}i}{2}, \frac{1+\sqrt{3}i}{2}$

2. Sum of all cube roots of 64 is :

- (a) ✓ 0 (b) 1 (c) 64 (d) -64

3. Product of cube roots of -1 is:

- (a) 0 (b) -1 (c) ✓ 1 (d) None

4.  $16\omega^8 + 16\omega^4 =$

- (a) 0 (b) ✓ -16 (c) 16 (d) -1

5. The sum of all four fourth roots of unity is:

- (a) Unity (b) ✓ 0 (c) -1 (d) None

6. The product of all four fourth roots of unity is:

- (a) Unity (b) 0 (c) ✓ -1 (d) None

7. The sum of all four fourth roots of 16 is:

- (a) 16 (b) -16 (c) ✓ 0 (d) 1

8. The complex cube roots of unity are..... each other.

- (a) ✓ Additive inverse (b) Equal to (c) Conjugate (d) None of these

9. The complex fourth roots of unity are ..... of each other.

- (a) ✓ Additive inverse (b) equal to (c) square of (d) None of these

10. The cube roots of -1 are

- (a)  $\{1, \omega, \omega^2\}$  (b)  $\{1, -\omega, \omega^2\}$  (c) ✓  $\{-1, -\omega, -\omega^2\}$  (d)  $\{-1, \omega, \omega^2\}$

### SHORT QUESTIONS

- i. Find the three cube roots of : 8, 27
- ii. Evaluate  $\omega^{28} + \omega^{29} + 1$
- iii. Show that  $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \dots 2n \text{ factors} = 1$
- iv. If  $\omega$  is cube root of  $x^2 + x + 1 = 0$ , show that its other root is  $\omega^2$  and prove that  $\omega^3 = 1$
- v. If  $\omega$  is cube root of unity, form an equation whose roots are  $2\omega$  and  $2\omega^2$ .
- vi. Find four fourth roots of 81.
- vii. Solve  $5x^5 - 5x = 0$

### LONG QUESTIONS

Show that

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$$

## EXERCISE 4.5

Tick (✓) the correct answer.

1. The expression  $x^2 + \frac{1}{x} - 3$  is polynomial of degree:
 

|       |       |       |                        |
|-------|-------|-------|------------------------|
| (a) 2 | (b) 3 | (c) 1 | (d) ✓ not a polynomial |
|-------|-------|-------|------------------------|
  2. If  $f(x)$  is divided by  $-a$ , then dividend = (Divisor)(.....)+ Remainder.
 

|             |              |                |            |
|-------------|--------------|----------------|------------|
| (a) Divisor | (b) Dividend | (c) ✓ Quotient | (d) $f(a)$ |
|-------------|--------------|----------------|------------|
  3. If  $f(x)$  is divided by  $x - a$  by remainder theorem then remainder is:
 

|              |             |                |                 |
|--------------|-------------|----------------|-----------------|
| (a) ✓ $f(a)$ | (b) $f(-a)$ | (c) $f(a) + R$ | (d) $x - a = R$ |
|--------------|-------------|----------------|-----------------|
  4. The polynomial  $(x - a)$  is a factor of  $f(x)$  if and only if
 

|                  |                |                    |              |
|------------------|----------------|--------------------|--------------|
| (a) ✓ $f(a) = 0$ | (b) $f(a) = R$ | (c) Quotient = $R$ | (d) $x = -a$ |
|------------------|----------------|--------------------|--------------|
  5.  $x - 2$  is a factor of  $x^2 - kx + 4$ , if  $k$  is:
 

|       |         |       |        |
|-------|---------|-------|--------|
| (a) 2 | (b) ✓ 4 | (c) 8 | (d) -4 |
|-------|---------|-------|--------|
  6. If  $x = -2$  is the root of  $kx^4 - 13x^2 + 36 = 0$ , then  $k =$ 

|       |        |       |          |
|-------|--------|-------|----------|
| (a) 2 | (b) -2 | (c) 1 | (d) ✓ -1 |
|-------|--------|-------|----------|
- 
7.  $x + a$  is a factor of  $x^n + a^n$  when  $n$  is
 

|                 |                          |                       |                     |
|-----------------|--------------------------|-----------------------|---------------------|
| (a) Any integer | (b) any positive integer | (c) ✓ any odd integer | (d) any real number |
|-----------------|--------------------------|-----------------------|---------------------|
  8.  $x - a$  is a factor of  $x^n - a^n$  when  $n$  is
 

|                   |                          |                     |                     |
|-------------------|--------------------------|---------------------|---------------------|
| (a) ✓ Any integer | (b) any positive integer | (c) any odd integer | (d) any real number |
|-------------------|--------------------------|---------------------|---------------------|

### SHORT QUESTIONS

- i. Find the numerical value of  $k$  if the polynomial  $x^3 + kx^2 - 7x + 6$  has a remainder of  $-4$ , when divided by  $x + 2$ .
- ii. Show that  $(x - 2)$  is a factor of  $x^4 - 13x^2 + 36$ .
- iii. When the polynomial  $x^3 + 2x^2 + kx + 4$  is divided by  $x - 2$ , the remainder is 14. Find the value of  $k$ .
- iv. Use synthetic division to find the quotient and the remainder when the polynomial  $x^4 - 10x^2 - 2x + 4$  is divided by  $x + 3$ .
- v. Use factor theorem to determine if  $x + a$  is a factor of  $x^n + a^n$ , where  $n$  is odd integer.

### LONG QUESTIONS

Use synthetic division to find the values of  $p$  and  $q$  if  $x + 1$  and  $x - 2$  are factors of the polynomial  $x^3 + px^2 + qx + 6$ . Find the values of  $a$  and  $b$  if  $-2$  and  $2$  are the roots of the polynomial  $x^3 - 4x^2 + ax + b$ .

## EXERCISE 4.6

Tick (✓) the correct answer.

1. Sum of roots of  $ax^2 - bx - c = 0$  is ( $a \neq 0$ )  
 (a)  $\frac{b}{a}$  (b)  $-\frac{b}{a}$  (c)  $\frac{c}{a}$  (d) ✓  $-\frac{c}{a}$
2. Product of roots of  $ax^2 - bx - c = 0$  is ( $a \neq 0$ )  
 (a) ✓  $\frac{b}{a}$  (b)  $-\frac{b}{a}$  (c)  $\frac{c}{a}$  (d)  $-\frac{c}{a}$
3. If 2 and -5 are roots of a quadratic equation, then equation is:  
 (a)  $x^2 - 3x - 10 = 0$  (b)  $x^2 - 3x + 10 = 0$  (c) ✓  $x^2 + 3x - 10 = 0$  (d)  $x^2 + 3x + 10 = 0$
4. If  $\alpha$  and  $\beta$  are the roots of  $3x^2 - 2x + 4 = 0$ , then the value of  $\alpha + \beta$  is:  
 (a) ✓  $\frac{2}{3}$  (b)  $-\frac{2}{3}$  (c)  $\frac{4}{3}$  (d)  $-\frac{4}{3}$
5. The equation whose roots are given is  
 (a)  $x^2 + Sx + P = 0$  (b)  $x^2 - Sx - P = 0$  (c)  $x^2 + Sx - P = 0$  (d) ✓  $x^2 - Sx + P = 0$

### SHORT QUESTIONS

- i. If  $\alpha, \beta$  are the roots of  $3x^2 - 2x + 4 = 0$ , find the values of  
 (a)  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  (b)  $\alpha^2 - \beta^2$
- ii. If  $\alpha, \beta$  are the roots of  $x^2 - px - p - c = 0$ , prove that  $(1 + \alpha)(1 + \beta) = 1 - c$
- iii. Find the condition that one root of  $x^2 + px + q = 0$  is additive inverse of the other.
- iv. If  $\alpha, \beta$  are the roots of the equation  $ax^2 + bx + c = 0$ , form the equations whose roots are  $\alpha^3, \beta^3$ .
- v. If the roots of  $px^2 + qx + q = 0$  are  $\alpha$  and  $\beta$  then prove that  $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{p}{q}} = 0$

### LONG QUESTIONS

If  $\alpha$  and  $\beta$  are the roots of  $x^2 - 3x + 5 = 0$ , form the equation whose roots are  $\frac{1-\alpha}{1-\beta}$  and  $\frac{1-\beta}{1+\alpha}$ .

Find the condition that  $\frac{a}{x-a} + \frac{b}{x-b} = 5$  may have roots equal in magnitude but opposite in signs.

## EXERCISE 4.7

Tick (✓) the correct answer.

1. If roots of  $ax^2 + bx + c = 0$ , ( $a \neq 0$ ) are real, then  
 (a) ✓  $\text{Disc} \geq 0$  (b)  $\text{Disc} < 0$  (c)  $\text{Disc} \neq 0$  (d)  $\text{Disc} \leq 0$
2. If roots of  $ax^2 + bx + c = 0$ , ( $a \neq 0$ ) are complex, then  
 (a)  $\text{Disc} \geq 0$  (b) ✓  $\text{Disc} < 0$  (c)  $\text{Disc} \neq 0$  (d)  $\text{Disc} \leq 0$
3. If roots of  $ax^2 + bx + c = 0$ , ( $a \neq 0$ ) are equal, then  
 (a) ✓  $\text{Disc} = 0$  (b)  $\text{Disc} < 0$  (c)  $\text{Disc} \neq 0$  (d) None of these

4. The expression  $b^2 - 4ac$  is called:

- (a) ✓ Discriminant (b) Quadratic equation (c) Linear equation (d) roots

5. Disc of  $x^2 + 2x + 3 = 0$  is

- (a) 16 (b) -16 (c) ✓ -8 (d) -16

### SHORT QUESTIONS

- i. Discuss the nature of  $2x^2 - 5x + 1 = 0$
- ii. For what values of  $m$  will the equation  $(m + 1)x^2 + 2(m + 3)x + 2m + 3 = 0$  have equal root?
- iii. Show that the roots of the equation  $px^2 - (p - q)x - q = 0$  will be rational.

### LONG QUESTIONS

Show that the roots of  $x^2 + (mx + c)^2 = a^2$  will be equal if  $c^2 = a^2(1 + m^2)$

Show that the roots of the equation  $bcx^2 + 2(b^2 - ca)x + c^2 - ab = 0$  will be equal, if either  $a^3 + b^3 + c^3 = 3abc$  or  $b = 0$ .

### EXERCISE 4.8

### LONG QUESTIONS

Solve the following systems of equations.

$$x + y = a + b \quad ; \quad \frac{a}{x} + \frac{b}{y} = 2$$

Solve the following systems of equations.

$$x + y = 5 \quad ; \quad x^2 + 2y^2 = 17$$

Solve the following systems of equations.

$$(x + 3)^2 + (y - 1)^2 = 5 \quad ; \quad x^2 + y^2 + 2x = 9$$

### EXERCISE 4.9

### LONG QUESTIONS

Solve the following systems of equations.

$$2x^2 - 8 = 5y^2 \quad ; \quad x^2 - 13 = -2y^2$$

Solve the following systems of equations.

$$x^2 - y^2 = 16 \quad ; \quad xy = 15$$

Solve the following systems of equations.

$$x^2 + xy = 9 \quad ; \quad x^2 - y^2 = 2$$

### EXERCISE 4.10

#### LONG QUESTIONS

The sum of a positive number and its reciprocal is  $\frac{26}{5}$ . Find the number.

Find two consecutive numbers, whose product is 132.

The sum of a positive number and its square is 380. Find the number.

### EXERCISE 5.1

Tick (✓) the correct answer.

- An open sentence formed by using sign of “ = ” is called a/an
  - ✓ Equation
  - Formula
  - Rational fraction
  - Theorem
- If an equation is true for all values of the variable, then it is called:
  - a conditional equation
  - ✓ an identity
  - proper rational fraction
  - All of these
- $(x + 3)(x + 4) = x^2 + 7x + 12$  is a/an:
  - Conditional equation
  - ✓ an identity
  - proper rational fraction
  - a formula
- The quotient of two polynomials  $\frac{P(x)}{Q(x)}$ ,  $Q(x) \neq 0$  is called :
  - ✓ Rational fraction
  - Irrational fraction
  - Partial fraction
  - Proper fraction
- A fraction  $\frac{P(x)}{Q(x)}$ ,  $Q(x) \neq 0$  is called proper fraction if :
  - ✓ Degree of  $P(x) <$  Degree of  $Q(x)$
  - Degree of  $P(x) =$  Degree of  $Q(x)$
  - Degree of  $P(x) >$  Degree of  $Q(x)$
  - Degree of  $P(x) \geq$  Degree of  $Q(x)$
- A fraction  $\frac{P(x)}{Q(x)}$ ,  $Q(x) \neq 0$  is called proper fraction if :
  - Degree of  $P(x) <$  Degree of  $Q(x)$
  - Degree of  $P(x) =$  Degree of  $Q(x)$
  - Degree of  $P(x) >$  Degree of  $Q(x)$
  - ✓ Degree of  $P(x) \geq$  Degree of  $Q(x)$
- A mixed form of fraction is :
  - An integer+ improper fraction
  - a polynomial+improper fraction
  - ✓ a polynomial+proper fraction
  - a polynomial+rational fraction
- When a rational fraction is separated into partial fractions, then result is always :
  - A conditional equations
  - ✓ an identity
  - a partial fraction
  - an improper fraction

#### SHORT QUESTIONS

- Resolve  $\frac{7x+5}{(x+3)(x+4)}$  into Partial fraction.
- Resolve  $\frac{1}{x^2-1}$  into Partial fraction.

### LONG QUESTIONS

Resolve  $\frac{1}{(1-ax)(1-bx)(1-cx)}$  into Partial fraction.

Resolve  $\frac{x}{(x-a)(x-b)(x-c)}$  into Partial fraction.

### EXERCISE 5.2

Tick (✓) the correct answer.

- The number of Partial fraction of  $\frac{x^3}{x(x+1)(x^2-1)}$  are:  
 (a) 2 (b) 3 (c) ✓ 4 (d) None of these
- The number of Partial fraction of  $\frac{x^5}{x(x+1)(x^2-4)}$  are:  
 (a) 2 (b) 3 (c) 4 (d) ✓ 6
- $\frac{9x^2}{x^3-1}$  is an  
 (a) Improper fraction (b) ✓ Proper fraction (c) Polynomial (d) equation

### LONG QUESTIONS

Resolve  $\frac{1}{(x+1)^2(x^2-1)}$  into Partial fraction.

Resolve  $\frac{x^2}{(x-2)(x-1)^2}$  into Partial fraction.

Resolve  $\frac{1}{(x-3)^2(x+1)}$  into Partial fraction.

### EXERCISE 5.3

Tick (✓) the correct answer.

- Which is a reducible factor:  
 (a)  $x^3 - 6x^2 + 8x$  (b)  $x^2 + 16x$  (c)  $x^2 + 5x - 6$  (d) ✓ all of these
- A quadratic factor which cannot written as a product of linear factors with real coefficients is called:  
 (a) ✓ An irreducible factor (b) reducible factor (c) an irrational factor (d) an improper factor

### LONG QUESTIONS

Resolve  $\frac{1}{(x^2+1)(x+1)}$  into Partial fraction.

Resolve  $\frac{x^2+1}{x^3+1}$  into Partial fraction.

Resolve  $\frac{3x+7}{(x^2+4)(x+3)}$  into Partial fraction.

### EXERCISE 5.4

#### LONG QUESTIONS

Resolve  $\frac{x^2}{(x^2+1)^2(x-1)}$  into Partial fraction.

Resolve  $\frac{4x^2}{(x^2+1)^2(x-1)}$  into Partial fraction.

Resolve  $\frac{8x^2}{(x^2+1)^2(1-x^2)}$  into Partial fraction.

### EXERCISE 6.1

Tick (✓) the correct answer.

1. An arrangement of numbers according to some definite rule is called:

- (a) ✓ Sequence (b) Combination (c) Series (d) Permutation

2. A sequence is also known as:

- (a) Real sequence (b) ✓ Progression (c) Arrangement (d) Complex sequence

3. A sequence is function whose domain is

- (a)  $Z$  (b) ✓  $N$  (c)  $Q$  (d)  $R$

4. As sequence whose range is  $R$  i. e., set of real numbers is called:

- (a) ✓ Real sequence (b) Imaginary sequence (c) Natural sequence (d) Complex sequence

5. If  $a_n = \{n + (-1)^n\}$ , then  $a_{10} =$

- (a) 10 (b) 11 (c) 12 (d) 13

6. The last term of an infinite sequence is called :

- (a)  $n$ th term (b)  $a_n$  (c) last term (d) does not exist

7. The next term of the sequence  $-1, 2, 12, 40, \dots$  is

- (a) ✓ 112 (b) 120 (c) 124 (d) None of these

8. For  $a_n = (-1)^{n+1}$ ,  $a_{26} =$

- (a) 1 (b) ✓ -1 (c) 0 (d) 2

9. The next two terms of the sequence  $1, -3, 5, -7, 9, -11, \dots$  are

- (a) 13, 15 (b)  $-13, -15$  (c) ✓  $13, -15$  (d)  $-13, 15$

10. For  $a_n = \frac{1}{2^n}$ ,  $a_1 =$

- (a) 2 (b) ✓  $\frac{1}{2}$  (c) 4 (d) 8

#### SHORT QUESTIONS

i. Write first two, 21<sup>st</sup> and 26<sup>th</sup> terms of the sequence whose general term is  $(-1)^{n+1}$ .

- |      |                               |   |
|------|-------------------------------|---|
| ii.  | Write the first four terms of | $a_n = (-1)^n n^2$                                    |
| iii. | Write the first four terms of | $a_n = (n + 1)a_{n-1}, a_1 = 1$                       |
| iv.  | Find the indicated term of    | $1, 3, 12, 60, \dots a_6$                             |
| v.   | Find the indicated term of    | $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \dots a_7$ |
| vi.  | Find the next two terms of    | $1, 3, 7, 15, 31 \dots$                               |
| vii. | Find the next two terms of    | $-1, 2, 12, 40, \dots$                                |

## EXERCISE 6.2

Tick (✓) the correct answer.

- A sequence  $\{a_n\}$  in which  $a_n - a_{n-1}$  is the same number for all  $n \in N, n > 1$  is called:
 

|           |         |         |                   |
|-----------|---------|---------|-------------------|
| (a) ✓ A.P | (b) G.P | (c) H.P | (d) None of these |
|-----------|---------|---------|-------------------|
- $n$ th term of an A.P is  $3n - 1$  then 10<sup>th</sup> term is :
 

|       |          |        |                       |
|-------|----------|--------|-----------------------|
| (a) 9 | (b) ✓ 29 | (c) 12 | (d) cannot determined |
|-------|----------|--------|-----------------------|
- For  $a_n - a_{n-1} = d$ 

|             |             |               |             |
|-------------|-------------|---------------|-------------|
| (a) $n = 0$ | (b) $n = 1$ | (c) ✓ $n > 1$ | (d) $n < 1$ |
|-------------|-------------|---------------|-------------|

### SHORT QUESTIONS

- Define "Arithmetic Progression".
- Prove that  $a_n = a_1 + (n - 1)d$
- If  $a_{n-2} = 3n - 11$ , find the  $n$ th term of the sequence.
- Find the 13th term of the sequence  $x, 1, 2 - x, 3 - 2x, \dots$
- If  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  are in A.P., show that  $b = \frac{2ac}{a+c}$ .
- Find the  $n$ th term of the sequence  $, \left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \dots$
- Which term of the A.P.  $5, 2, -1, \dots$  is  $-85$ ?

### LONG QUESTIONS

If  $l, m, n$  are the  $p$ th,  $q$ th and  $r$ th terms of an A.P., show that

$$l(q - r) + m(r - p) + n(p - q) = 0$$

If 5th term of an A.P., is 16 and the 20th term is 46,

what is its 12th term?

## EXERCISE 6.3

Tick (✓) the correct answer.

- If  $a_{n-1}, a_n, a_{n+1}$  are in A.P, then  $a_n$  is
 

|           |         |         |               |
|-----------|---------|---------|---------------|
| (a) ✓ A.M | (b) G.M | (c) H.M | (d) Mid point |
|-----------|---------|---------|---------------|

2. Arithmetic mean between  $c$  and  $d$  is:

- (a)   $\frac{c+d}{2}$  (b)  $\frac{c+d}{2cd}$  (c)  $\frac{2cd}{c+d}$  (d)  $\frac{2}{c+d}$

3. The arithmetic mean between  $\sqrt{2}$  and  $3\sqrt{2}$  is:

- (a)  $4\sqrt{2}$  (b)  $\frac{4}{\sqrt{2}}$  (c)   $2\sqrt{2}$  (d) none of these

4.  $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$  may be the A.M between  $a$  and  $b$  if

- (a)   $n = 1$  (b)  $n = 0$  (c)  $n > 1$  (d)  $n < 1$

### SHORT QUESTIONS

- Find A.M. between  $1 - x + x^2$  and  $1 + x + x^2$
- If 5, 8 are two A.Ms between  $a$  and  $b$ , find  $a$  and  $b$ .
- Define "Arithmetic mean".

### LONG QUESTIONS

Find 6 A.Ms. between 2 and 5.

Find  $n$  so that  $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$  may be the A.M. between  $a$  and  $b$ .

Show that the sum of  $n$  A.Ms between  $a$  and  $b$  is equal to  $n$  times their A.M.

### EXERCISE 6.4

Tick (✓) the correct answer.

1. The sum of terms of a sequence is called:

- (a) Partial sum (b)  Series (c) Finite sum (d) none of these

2. Forth partial sum of the sequence  $\{n^2\}$  is called:

- (a) 16 (b)   $1+4+9+16$  (c) 8 (d)  $1+2+3+4$

3. Sum of  $n$  - term of an Arithmetic series  $S_n$  is equal to:

- (a)   $\frac{n}{2}[2a + (n - 1)d]$  (b)  $\frac{n}{2}[a + (n - 1)d]$  (c)  $\frac{n}{2}[2a + (n + 1)d]$  (d)  $\frac{n}{2}[2a + l]$

### SHORT QUESTIONS

- How many terms of the series  $-9 - 6 - 3 + 0 + \dots$  amount to 66?
- Sum the series  $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + \dots + a_{13}$
- Sum the series  $(x - a) + (x + a) + (x + 3a) + \dots$  to  $n$  terms.
- How many terms of the series  $-7 + (-5) + (-3) + \dots$  amount to 65?
- Find the sum of 20 terms of the series whose  $r$ th term is  $3r + 1$ .

### LONG QUESTIONS

If  $S_n = n(2n - 1)$ , then find the series.

The ratio of the sums of  $n$  terms of two series in A.P. is  $3n + 2 : n + 1$ . Find the ratio of the 8th terms.

If  $S_2, S_3, S_5$  are the sums of  $2n, 3n, 5n$  terms of an A.P., show that  $S_5 = 5(S_3 - S_2)$

Find four numbers in A.P. whose sum is 32 and the sum of whose squares is 276.

If  $a^2, b^2$  and  $c^2$  are in A.P., show that  $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$  are in A.P.

## EXERCISE 6.5

### LONG QUESTIONS

A clock strikes once when its hour hand is at one, twice when it is at two and so on. How many times does the clock strike in twelve hours.

The sum of interior angles of polygons having sides 3,4,5,... etc form an A.P. Find the sum of interior angles for a 16 sided polygon.

## EXERCISE 6.6

Tick (✓) the correct answer.

1. For any G.P., the common ratio  $r$  is equal to:

- (a) ✓  $\frac{a_n}{a_{n+1}}$                       (b)  $\frac{a_{n-1}}{a_n}$                       (c)  $\frac{a_n}{a_{n-1}}$                       (d)  $a_{n+1} - a_n, n \in N, n > 1$

2. No term of a G.P., is:

- (a) ✓ 0                                      (b) 1                                      (c) negative                                      (d) imaginary number

3. The general term of a G.P., is :

- (a) ✓  $a_n = ar^{n-1}$                       (b)  $a_n = ar^n$                       (c)  $a_n = ar^{n+1}$                       (d) None of these

### SHORT QUESTIONS

- i. Find the 5th term of G.P., 3,6,12,...
- ii. Find the 11th term of the sequence,  $1 + i, 2, \frac{4}{1+i}, \dots$
- iii. Which term of the sequence:  $x^2 - y^2, x + y, \frac{x+y}{x-y}, \dots$  is  $\frac{x+y}{(x-y)^9}$ ?
- iv. If  $a, b, c, d$  are in G.P, prove that  $a - b, b - c, c - d$  are in G.P.
- v. If  $\frac{1}{a}, \frac{1}{b}$  and  $\frac{1}{c}$  are in G.P. show that the common ratio is  $\pm \sqrt{\frac{a}{c}}$ .

## LONG QUESTIONS

Find three, consecutive numbers in G.P whose sum is 26 and their product is 216.

If three consecutive numbers in A.P. are increased by 1, 4, 15 respectively, the resulting numbers are in G.P. Find the original numbers if their sum is 6.

### EXERCISE 6.7

Tick (✓) the correct answer.

1. Geometric mean between 4 and 16 is

- (a)  $\pm 2$                                       (b)  $\pm 4$                                       (c)  $\pm 6$                                       (d) ✓  $\pm 8$

2. For what value of  $n$ ,  $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$  is the positive geometric mean between  $a$  and  $b$ ?

- (a) 1    (b) 2    (c) ✓  $\frac{1}{2}$                                       (d)  $\frac{3}{2}$

### SHORT QUESTIONS

- i. If  $a, b, c$  and  $d$  are in G.P. show that  $a + b, b + c, c + d$  are in G.P.
- ii. Find G.M. between  $-2i$  and  $8i$
- iii. The A.M. between two numbers is 5 and their (positive) G.M is 4. Find the numbers.
- iv. The A.M. of two positive integral numbers exceeds their (positive) G.M by 2 and their sum is 20, find the numbers.
- v. Insert four real geometric means between 3 and 96.
- vi. If both  $x$  and  $y$  are positive distinct real numbers, show that the geometric mean between  $x$  and  $y$  is less than their arithmetic mean.

## LONG QUESTIONS

For what value  $n$ ,  $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$  is the positive geometric mean between  $a$  and  $b$ ?

The A.M. between two numbers is 5 and their (Positive) G.M. is 4. Find the numbers.

### EXERCISE 6.8

Tick (✓) the correct answer.

1. The sum of infinite geometric series is valid if

- (a)  $|r| > 1$                                       (b)  $|r| = 1$                                       (c)  $|r| \geq 1$                                       (d) ✓  $|r| < 1$

2. For the series  $1 + 5 + 25 + 125 + \dots + \infty$ , the sum is

- (a) -4    (b) 4    (c)  $\frac{1-5^n}{-4}$                                       (d) ✓ not defined

3. An infinite geometric series is convergent if

- (a)  $|r| > 1$                                       (b)  $|r| = 1$                                       (c)  $|r| \geq 1$                                       (d) ✓  $|r| < 1$

4. An infinite geometric series is divergent if

- (a)  $|r| < 1$                                       (b)  $|r| \neq 1$                                       (c)  $r = 0$                                       (d) ✓  $|r| > 1$

5. If sum of series is defined then it is called:

- (a) ✓ Convergent series      (b) Divergent series      (c) finite series      (d) Geometric series

6. If sum of series is not defined then it is called:

- (a) Convergent series (b) ✓ Divergent series (c) finite series (d) Geometric series

7. The interval in which series  $1 + 2x + 4x^2 + 8x^3 + \dots$  is convergent if :

- (a)  $-2 < x < 2$  (b) ✓  $-\frac{1}{2} < x < \frac{1}{2}$  (c)  $|2x| > 1$  (d)  $|x| < 1$

### SHORT QUESTIONS

- i. Sum to  $n$  terms of the series :  $r + (1 + k)r^2 + (1 + k + k^2)r^3 + \dots$
- ii. Sum the series  $2 + (1 - i) + \left(\frac{1}{i}\right) + \dots$  to 8 terms.
- iii. Find the sums of the infinite geometric series :  $0.1 + 0.05 + 0.025 + \dots$
- iv. Find vulgar fractions equivalent to the recurring decimal :  $0.\dot{7}$
- v. If  $a = 1 - x + x^2 - x^3 + \dots$   $|x| < 1$  and  $b = 1 + x + x^2 + x^3 + \dots$   $|x| < 1$  show that  $2ab = a + b$

### LONG QUESTIONS

If  $y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \dots$  and if  $0 < x < \frac{3}{2}$ , then

show that  $x = \frac{3y}{2(1+y)}$

If  $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots$

- i) Show that  $x = 2\left(\frac{y-1}{y}\right)$
- ii) Find the interval in which the series is convergent.

### EXERCISE 6.9

### LONG QUESTIONS

The population of a certain village is 62500. What will be its population after 3 years if it increases geometrically at the rate of 4% annually.

A singular cholera bacteria produces two complete bacteria in  $\frac{1}{2}$  hours. If we start with a colony of A bacteria, how many bacteria will have in  $n$  hours?

## EXERCISE 6.10

Tick (✓) the correct answer.

1. If the reciprocal of the terms a sequence form an *A.P.*, then it is called:
  - (a) ✓ *H.P*
  - (b) *G.P*
  - (c) *A.P*
  - (d) sequence
2. The  $n$ th term of  $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \dots$  is
  - (a) ✓  $\frac{1}{3n-1}$
  - (b)  $3n - 1$
  - (c)  $2n + 1$
  - (d)  $\frac{1}{3n+1}$
3. Harmonic mean between 2 and 8 is:
  - (a) ✓ 5
  - (b)  $\frac{16}{5}$
  - (c)  $\pm 4$
  - (d)  $\frac{5}{16}$
4. If *A, G* and *H* are Arithmetic, Geometric and Harmonic means between two positive numbers then
  - (a)  $G^2 = AH$
  - (b) ✓ *A, G, H* are in *G.P*
  - (c)  $A > G > H$
  - (d) all of these
5. If *A, G* and *H* are Arithmetic, Geometric and Harmonic means between two negative numbers then
  - (a)  $G^2 = AH$
  - (b) *A, G, H* are in *G.P*
  - (c)  $A < G < H$
  - (d) ✓ all of these
6. If *a* and *b* are two positive number then
  - (a)  $A < G < H$
  - (b) ✓  $A > G > H$
  - (c)  $A = G = H$
  - (d)  $A \geq G \geq H$
7. If *a* and *b* are two negative number then
  - (a) ✓  $A < G < H$
  - (b)  $A > G > H$
  - (c)  $A = G = H$
  - (d)  $A \geq G \geq H$
8. If  $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$  is H.M between *a* and *b* then *n* is equal to:
  - (a) ✓ 0
  - (b) -1
  - (c) 1
  - (d)  $\frac{1}{2}$

### SHORT QUESTIONS

- i. Find the  $n$ th and 8th term of  $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \dots$
- ii. If the numbers  $\frac{1}{k}, \frac{1}{2k+1}$  and  $\frac{1}{4k-1}$  are in harmonic sequence, find *k*.
- iii. If  $a^2, b^2$  and  $c^2$  are in *A.P.* show that  $a + b, c + a$  and  $b + c$  are in *H.P.*
- iv. Find *A, G, H* and show that  $G^2 = AH$  if  $a = -2$  and  $b = -6$
- v. Verify that  $A < G < H$  ( $G < 0$ ), if  $a = -2, b = -8$

### LONG QUESTIONS

Find *n* so that  $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$  may be H.M between *a* and *b*.

If the (positive) G.M. and H.M. between two numbers are 4 and  $\frac{16}{5}$ , find the numbers.

## EXERCISE 6.11

Tick (✓) the correct answer.

1. If  $S_n = (n + 1)^2$ , then  $S_{2n}$  is equal to:

- (a)  $2n + 1$                       (b) ✓  $4n^2 + 4n + 1$       (c)  $(2n - 1)^2$       (d) cannot be determined

2.  $\sum_{k=1}^n k^3 =$

- (a)  $\frac{n(n+1)}{2}$                       (b)  $\frac{n(n+1)(n+2)}{6}$                       (c) ✓  $\frac{n^2(n+1)^2}{4}$                       (d)  $\frac{n(n+1)^2}{2}$

3.  $\sum_{k=1}^n 1 =$

- (a) 1                      (b) 0                      (c)  $k$                       (d) ✓  $n$

### LONG QUESTIONS

Given  $n$ th terms of the series, find the sum to  $2n$  terms:  $3n^2 + 2n + 1$

Sum the series upto  $n$  terms:  $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$

Sum the series upto  $n$  terms:  $3 \times 2^2 + 5 \times 3^2 + 7 \times 4^2 + \dots$

Sum the series:  $1^2 - 2^2 + 3^2 - 4^2 + \dots + (2n - 1)^2 - (2n)^2$

Sum the series:  $\frac{1^2}{1} + \frac{1^2+2^2}{2} + \frac{1^2+2^2+3^2}{3} + \dots$  to  $n$  terms.

## EXERCISE 7.1

Tick (✓) the correct answer.

1.  $\frac{8!}{7!} =$

- (a) ✓ 8                      (b) 7                      (c) 56                      (d)  $\frac{8}{7}$

2.  $0! =$

- (a) 0                      (b) ✓ 1                      (c) 2                      (d) cannot be defined

3.  $n! =$

- (a)  $n(n - 1)$                       (b)  $(n - 1)!$                       (c)  $(n - 2)!$                       (d) ✓  $n(n - 1)!$

4.  $\frac{9!}{6!3!} =$

- (a) 80                      (b) ✓ 84                      (c) 90                      (d) 94

5. Factorial form of  $\frac{8.7.6}{3.2.1}$  is

- (a)  $\frac{8!}{3!4!}$                       (b)  $\frac{8!}{3!3!}$                       (c) ✓  $\frac{8!}{3!5!}$                       (d)  $\frac{8!}{3!6!}$

### SHORT QUESTIONS

- i. Evaluate  $\frac{8!}{7!}$  and  $\frac{11!}{2!4!5!}$
- ii. Evaluate  $\frac{9!}{2!(9-2)!}$  and  $4!0!1!$
- iii. Write in factorial form: **20. 19. 18. 17**
- iv. Write in factorial form:  $n(n+1)(n+2)$
- v. Write in factorial form:  $\frac{(n+1)(n)(n-1)}{3.2.1}$
- vi. Prove that  $0! = 1$
- vii. Write in factorial form:  $n(n-1)(n-2) \dots (n-r+1)$

## EXERCISE 7.2

Tick (✓) the correct answer.

1.  ${}^{20}P_3 =$   
 (a) 6890 (b) 6810 (c) ✓ 6840 (d) 6880
2. If  ${}^{n}P_2 = 30$  then  $n =$   
 (a) 4 (b) 5 (c) 6 (d) 10
3.  ${}^{n}P_n =$   
 (a)  $n$  (b)  $p!$  (c) ✓  $n!$  (d)  $(n-1)!$
4.  ${}^{n}P_r =$   
 (a)  $n!$  (b)  $\frac{n!}{r!}$  (c) ✓  $\frac{n!}{(n-r)!}$  (d)  $r!$
5. \_\_\_\_\_ of  $n$  different objects is called permutation.  
 (a) Combination (b) ✓ Permutation (c) Probability (d) Arrangements
6. In how many ways the letters of the "WORD" can be write?  
 (a)  $2!$  ways (b)  $3!$  ways (c) ✓  $4!$  ways (d)  $5!$  ways
7. In how many ways three books can be arranged?  
 (a)  $2!$  ways (b) ✓  $3!$  ways (c)  $4!$  ways (d)  $5!$  ways

### SHORT QUESTIONS

- i. Define "PERMUTATION".
- ii. Prove that  ${}^{n}P_r = \frac{n!}{(n-r)!}$
- iii. Find the value of  ${}^{11}P_n = 11.10.9$
- iv. Evaluate  ${}^{10}P_7$
- v. Prove from the first principle :  ${}^{n}P_r = n \cdot n - 1 P_{r-1}$
- vi. How many words can be formed from the letters of "FASTING" using all letters when no letter is to be repeated.

### LONG QUESTIONS

Prove that  ${}^{n}P_r = n - 1 P_r + r \cdot n - 1 P_{r-1}$

Find the numbers greater than 23000 that can be formed from the digits 1,2,3,5,6, without repeating any digit.

**How many 5-digit multiples of 5 can be formed from the digits 2,3,5,7,9, when no digit is repeated.**

### EXERCISE 7.3

Tick (✓) the correct answer.

- How many arrangement of the word "MATHEMATICS" can be made  
 (a)  $11!$  (b)  $\binom{11}{3,2,1,1,1,1,1}$  (c) ✓  $\binom{11}{2,2,2,1,1,1,1}$  (d)  $10!$
- The permutation of things which can be represented by the points on a circle are called  
 (a) Combinations (b) ✓ Circular permutation (c) Probability (d) factorial
- In how many ways can a necklace of 8 beads of different colours be made?  
 (a)  $\frac{1}{2} \times 8!$  (b) ✓  $\frac{1}{2} \times 7!$  (c)  $\frac{1}{2} \times 6!$  (d)  $\frac{1}{2} \times 5!$

### SHORT QUESTIONS

- How many arrangements of the letters of "PAKISTAN" taken all together, can be made.
- In how many ways can a necklace of 8 beads of different colours be made?
- In how many ways can 4 keys be arranged on a circular key ring?
- The Governor of the Punjab calls a meeting of 12 officers. In how many ways can they be seated at a round table?
- How many necklace can be made from 6 beads of different colors?

### LONG QUESTIONS

How many numbers greater than 1000,000 can be formed from the digits 0, 2, 2, 3, 4, 4. How many 6-digit can be formed from the digits 2, 2, 3, 3, 4, 4? How many of them will lie between 400,000 and 430,000?

### EXERCISE 7.4

Tick (✓) the correct answer.

- $n - {}_1C_r + n - {}_1C_{r-1}$  equals  
 (a) ✓  $n_{C_r}$  (b)  $n_{C_{r-1}}$  (c)  $n - {}_1C_r$  (d)  $n + {}_1C_r$
- How many signals can be given by 5 flags of different colors, using 3 at a time  
 (a) 120 (b) 60 (c) 24 (d) 15
- $n_{C_n} =$   
 (a)  $n!$  (b)  $0!$  (c) ✓ 1 (d) 0
- $n_{C_r} \times r! =$   
 (a)  $n_{C_r}$  (b) ✓  $n_{P_r}$  (c)  $n_{C_n}$  (d)  $r!$
- $n_{C_0} =$

- (a) 0 (b)  1 (c) 2 (d)  $n!$
6. For complementary combination  $n_{C_r} =$
- (a)  $n_{C_n}$  (b)   $n_{C_{n-r}}$  (c)  $n_{C_r}$  (d) None of these
7. If  $n_{C_8} = n_{C_{12}}$  then  $n =$
- (a) 10 (b)  20 (c) 30 (d) 40
8. In a permutation  $n_{P_r}$  or  $P(n, r)$ , it is always true that
- (a)   $n \geq r$  (b)  $n < r$  (c)  $n \leq r$  (d)  $n < 0, r < 0$

### SHORT QUESTIONS

- i. Show that :  $16_{C_{11}} + 16_{C_{10}} = 17_{C_{11}}$
- ii. Find the values of  $n$  and  $r$ , when  $n_{C_r} = 35$  and  $n_{P_r} = 210$
- iii. How many (a) diagonals and (b) triangles can be formed by joining the vertices of the polygon having 8 sides.
- iv. Find the value of  $n$ , when  $n_{C_{10}} = \frac{12 \times 11}{2!}$
- v. Find the values of  $n$  and  $r$ , when  $n - 1_{C_{r-1}} : n_{C_r} : n + 1_{C_{r+1}} = 3 : 6 : 11$
- vi.

### LONG QUESTIONS

Prove that  $n - 1_{C_r} + n - 1_{C_{r-1}} = n_{C_r}$

Prove that  $n_{C_r} + n_{C_{r-1}} = n + 1_{C_r}$

### EXERCISE 7.5

Tick () the correct answer.

1. Probability of non-occurrence of an event E is equal to :
- (a)   $1 - P(E)$  (b)  $P(E) + \frac{n(S)}{n(E)}$  (c)  $\frac{n(S)}{n(E)}$  (d)  $1 + P(E)$
2. Non occurrence of an event E is denoted by:
- (a)  $\sim E$  (b)   $\bar{E}$  (c)  $E^c$  (d) All of these
3. A card is drawn from a deck of 52 playing cards. The probability of card that it is an ace card is:
- (a)  $\frac{2}{13}$  (b)  $\frac{4}{13}$  (c)   $\frac{1}{13}$  (d)  $\frac{17}{13}$
4. Four persons wants to sit in a circular sofa, the total ways are:
- (a)  24 (b) 6 (c) 4 (d) None of these
5. Let  $S = \{1, 2, 3, \dots, 10\}$  the probability that a number is divided by 4 is :
- (a)  $\frac{2}{5}$  (b)   $\frac{1}{5}$  (c)  $\frac{1}{10}$  (d)  $\frac{1}{2}$
6. A die is rolled , the probability of getting 3 or 5 is:
- (a)  $\frac{2}{3}$  (b)   $\frac{15}{36}$  (c)  $\frac{15}{36}$  (d)  $\frac{1}{36}$
7. If E is a certain event , then
- (a)  $P(E) = 0$  (b)   $P(E) = 1$  (c)  $0 < P(E) < 1$  (d)  $P(E) > 1$
8. If E is an impossible event ,then
- (a)   $P(E) = 0$  (b)  $P(E) = 1$  (c)  $P(E) \neq 0$  (d)  $0 < P(E) < 1$
9. Sample space for tossing a coin is:

- (a)  $\{H\}$                       (b)  $\{T\}$                       (c)  $\{H, H\}$                       (d)   $\{H, T\}$

### SHORT QUESTIONS

- i. What is the probability that a slip of numbers divisible by 4 are picked from the slips bearing numbers 1,2,3,....., 10?
- ii. A die is rolled . What is the probability that the dots on the top are greater than 4?
- iii. Define "*Certain event and Impossible event*".
- iv. Define "*Equally likely events and Mutually exclusive events*".
- v. What is "*Sample space and event*"?

### EXERCISE 7.6

Tick () the correct answer.

1. If  $P(E) = \frac{7}{12}$ ,  $n(S) = 8400$ ,  $n(E) =$   
 (a) 108                      (b)  4900                      (c) 144                      (d) 14400
2.  $P(E) =$   
 (a)   $\frac{n(E)}{n(S)}$                       (b)  $\frac{n(S)}{n(E)}$                       (c)  $\frac{P(S)}{n(E)}$                       (d)  $\frac{n(E)}{P(S)}$

### EXERCISE 7.7

Tick () the correct answer.

1. For independent events  $P(A \cup B) =$   
 (a)  $P(A) + P(B)$                       (b)   $P(A) + P(B) - P(A \cap B)$                       (c)  $P(A) \cdot P(B)$                       (d)  $\frac{P(A)}{P(B)}$
2. If  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{1}{2}$  and  $P(A \cap B) = \frac{1}{3}$  then  $P(A \cup B) =$   
 (a)  $\frac{1}{2}$                       (b)   $\frac{2}{3}$                       (c)  $\frac{1}{3}$                       (d)  $\frac{1}{4}$

### SHORT QUESTIONS

- i. If sample space  $S = \{1, 2, 3, \dots, 9\}$ , Event  $A = \{2, 4, 6, 8\}$ , Event  $B = \{1, 3, 5\}$ , find  $P(A \cup B)$ .
- ii. A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen numbers is a multiple of 3 or of 5?
- iii. Two dice are thrown. What is the probability that the sum of number of dots appearing on them is 4 or 6?

## EXERCISE 7.8

Tick (✓) the correct answer.

1. For independent events  $P(A \cap B) =$   
 (a)  $P(A) + P(B)$       (b)  $P(A) - P(B)$       (c) ✓  $P(A) \cdot P(B)$       (d)  $\frac{P(A)}{P(B)}$
2. If an event  $A$  can occur in  $p$  ways and  $B$  can occur  $q$  ways, then number of ways that both events occur is:  
 (a)  $p + q$       (b) ✓  $p \cdot q$       (c)  $(pq)!$       (d)  $(p + q)!$
3. If  $P(A) = 0.8$  and  $P(B) = 0.75$  then  $P(A \cap B) =$   
 (a) 0.5      (b) ✓ 0.6      (c) 0.7      (d) 0.9

### SHORT QUESTIONS

- i. Find the probability that the sum of dots appearing in two successive throws of two dice in every time 7.
- ii. Define "*Independent events*".

## EXERCISE 8.1

Tick (✓) the correct answer.

1. The statement  $4^n + 3^n + 4$  is true when :  
 (a)  $n = 0$       (b)  $n = 1$       (c) ✓  $n \geq 2$       (d)  $n$  is any +iv integer
2. The method of induction was given by Francesco who lived from:  
 (a) ✓ 1494-1575      (b) 1500-1575      (c) 1498-1575      (d) 1494-1570
3. The statement  $3^n < n!$  is true, when  
 (a)  $n = 2$       (b)  $n = 4$       (c)  $n = 6$       (d) ✓  $n > 6$

### LONG QUESTIONS

Prove by mathematical induction that all positive integral values of  $n$

- i)  $5^n - 2^n$  is divisible by 3
- ii)  $x - y$  is a factor  $x^n - y^n$ ; ( $x \neq y$ )

Use the principle of mathematical induction prove that  $\ln x^n = n \ln x$  for any integer  $n \geq 0$  if  $x$  is a positive number.

Use the principle of extended mathematical induction to prove that

$$1 + nx \leq (1 + x)^n \text{ for } n \geq 2 \text{ and } x > -1$$

Prove by mathematical induction that all positive integral values of  $n$

$$1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^{n-1}} = 2 \left[ 1 - \frac{1}{2^n} \right]$$

Prove by mathematical induction that all positive integral values of  $n$   $r +$

$$r^2 + r^3 + \dots + r^n = \frac{r(1-r^n)}{1-r}, r \neq 1$$

## EXERCISE 8.2

Tick (✓) the correct answer.

4. General term in the expansion of  $(a + b)^n$  is:

- (a)  $\binom{n+1}{r} a^{n-r} x^r$  (b) ✓  $\binom{n}{r-1} a^{n-r} x^r$  (c)  $\binom{n}{r+1} a^{n-r} x^r$  (d)  $\binom{n}{r} a^{n-r} x^r$

5. The number of terms in the expansion of  $(a + b)^n$  are:

- (a)  $n$  (b) ✓  $n + 1$  (c)  $2^n$  (d)  $2^{n-1}$

6. Middle term/s in the expansion of  $(a - 3x)^{14}$  is/are :

- (a)  $T_7$  (b) ✓  $T_8$  (c)  $T_6$  &  $T_7$  (d)  $T_7$  &  $T_8$

7. The coefficient of the last term in the expansion of  $(2 - x)^7$  is :

- (a) 1 (b) ✓  $-1$  (c) 7 (d)  $-7$

8.  $\binom{2n}{0} + \binom{2n}{1} + \binom{2n}{2} + \dots + \binom{2n}{2n}$  is equal to:

- (a)  $2^n$  (b) ✓  $2^{2n}$  (c)  $2^{2n-1}$  (d)  $2^{2n+1}$

9.  $1 + x + x^2 + x^3 + \dots$

- (a)  $(1 + x)^{-1}$  (b) ✓  $(1 - x)^{-1}$  (c)  $(1 + x)^{-2}$  (d)  $(1 - x)^{-2}$

10. The middle term in the expansion of  $(a + b)^n$  is  $\binom{n}{\frac{n}{2} + 1}$ ; then  $n$  is

- (a) Odd (b) ✓ even (c) prime (d) none of these

11. The number of terms in the expansion of  $(a + b)^{20}$  is:

- (a) 18 (b) ✓ 20 (c) 21 (d) 19

### SHORT QUESTIONS

- i. Using binomial theorem expand  $(a + 2b)^5$
- ii. Calculate  $(2.02)^4$
- iii. Expand and simplify  $(2 + i)^5 - (2 - i)^5$
- iv. Find the term involving  $x^{-2}$  in the expansion of  $\left(x - \frac{2}{x^3}\right)^{13}$
- v. Determine the middle term in  $\left(2x - \frac{1}{2x}\right)^{2m+1}$

### LONG QUESTIONS

Show that the middle term of  $(1 + x)^{2n}$  is  $\frac{1.3.5 \dots (2n-1)}{n!} 2^n x^n$

Show that :  $\binom{n}{0} + \frac{1}{2} \binom{n}{1} + \frac{1}{3} \binom{n}{2} + \frac{1}{4} \binom{n}{3} + \dots + \frac{1}{n+1} \binom{n}{n} = \frac{2^{n+1} - 1}{n+1}$

## EXERCISE 8.3

Tick (✓) the correct answer.

1. The expansion  $(1 - 4x)^{-2}$  is valid if:

- (a) ✓  $|x| < \frac{1}{4}$                       (b)  $|x| > \frac{1}{4}$                       (c)  $-1 < x < 1$                       (d)  $|x| < -1$

### SHORT QUESTIONS

- i. Expand  $(1 + 2x)^{-1}$  upto 4 terms, taking the values of  $x$  such that the expansion is valid.
- ii. Expand  $(2 - 3x)^{-2}$  upto 4 terms, taking the values of  $x$  such that the expansion is valid.
- iii. Use Binomial theorem find the value of  $(.98)^{\frac{1}{2}}$
- iv. Use Binomial theorem find the value of  $\sqrt[5]{31}$
- v. Find the coefficient of  $x^n$  in the expansion of  $\frac{1+x^3}{1-x^2}$
- vi. Find The coefficient of  $x^n$  in the expansion of  $\frac{1+x^2}{(1+x)^2}$

### LONG QUESTIONS

If  $x$  is so small that its square and higher powers can be neglected , then show

that 
$$\frac{\sqrt{4+x}}{(1-x)^3} \approx 2 + \frac{25}{4}x$$

If  $x$  is so small that its cube and higher powers can be neglected , then show that

$$\sqrt{1 - x - 2x^2} \approx 1 - \frac{1}{2}x - \frac{9}{2}x^2$$

Show that  $\left[ \frac{n}{2(n+N)} \right]^{\frac{1}{2}} \approx \frac{8n}{9n-N} - \frac{n+N}{4n}$  where  $n$  and  $N$  are nearly equal.

If  $x$  is very nearly equal 1, then prove that  $px^p - qx^q \approx (p - q)x^{p+q}$

If  $y = \frac{2}{5} + \frac{1.3}{2!} \left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!} \left(\frac{2}{5}\right)^3 + \dots$ , then prove that  $y^2 + 2y - 4 = 0$

## EXERCISE 9.1

Tick (✓) the correct answer.

1. Two rays with a common starting point form:

- (a) Triangle                      (b) ✓ Angle                      (c) Radian                      (d) Minute

2. The common starting point of two rays is called:

- (a) Origin                      (b) Initial Point                      (c) ✓ Vertex                      (d) All of these

3. If the rotation of the angle is counter clock wise, then angle is:

- (a) Negative (b)  Positive (c) Non-Negative (d) None of these

4. One right angle is equal to

- (a)   $\frac{\pi}{2}$  radian (b)  $90^\circ$  (c)  $\frac{1}{4}$  rotation (d) All of these

5.  $1^\circ$  is equal to

- (a) 30 minutes (b)  60 minutes (c)  $\frac{1}{60}$  minutes (d)  $\frac{1}{2}$  minutes

6.  $1^\circ$  is equal to

- (a)   $60'$  (b)  $3600''$  (c)  $(\frac{1}{360})'$  (d)  $60''$

7.  $60^{\text{th}}$  part of  $1^\circ$  is equal to

- (a) One second (b)  One minute (c) 1 Radian (d)  $\pi$  radian

8. 3 radian is:

- (a)   $171.888^\circ$  (b)  $120^\circ$  (c)  $300^\circ$  (d)  $270^\circ$

9. Area of sector of circle of radius  $r$  is:

- (a)  $\frac{1}{2}r^2\theta$  (b)   $\frac{1}{2}r\theta^2$  (c)  $\frac{1}{2}(r\theta)^2$  (d)  $\frac{1}{2r^2\theta}$

10. Circular measure of angle between the hands of a watch at 4' O clock is

- (a)  $\frac{\pi}{6}$  (b)   $\frac{2\pi}{3}$  (c)  $\frac{3\pi}{2}$  (d)  $\frac{\pi}{3}$

### SHORT QUESTIONS

i. Define "Radian".

ii. Express  $45^\circ$  and  $35^\circ 20'$  into radians.

iii. Convert  $\frac{25\pi}{36}$  and  $\frac{\pi}{6}$  into sexagesimal system.

iv. Find  $r$ , when  $l = 5 \text{ cm}$ ,  $\theta = \frac{1}{2}$  radian

v. What is circular measure of the angle between the hands of a watch at 4 O'clock?

vi. Find the radius of the circle, in which the arms of a central angle of measure 1 radian cut off an arc of length 35cm.

vii. Show that the area of a sector of a circular region of radius  $r$  is  $\frac{1}{2}r^2\theta$ , where  $\theta$  is the circular measure of the central angle of the sector.

## EXERCISE 9.2

Tick () the correct answer.

1. Angles with same initial and terminal sides are called:

- (a) Acute angles (b) Allied Angles (c)  Coterminal angles (d) Quadrantal angles

2. If angle  $\theta$  is in radian, then the angle coterminal with  $\theta$  is:

- (a)   $\theta + 2\pi k, k \in Z$  (b)  $\theta + \pi k, k \in Z$  (c)  $\theta + \frac{\pi}{2}k, k \in Z$  (d)  $\theta + \frac{\pi}{3}k, k \in Z$

3. An angle is in standard position, if its vertex is

- (a) At origin (b)  at  $x$  - axis (c) at  $y$  - axis (d) in 1<sup>st</sup> Quad Only

4. If initial and the terminal side of an angle falls on  $x$  - axis or  $y$  - axis then it is called:

- (a) Coterminal angle (b)  Quadrantal angle (c) Allied angle (d) None of these

5.  $0^\circ, 90^\circ, 180^\circ, 270^\circ$  and  $360^\circ$  are called

- (a) Coterminal angle (b)  Quadrantal angle (c) Allied angle (d) None of these

6.  $\sin^2\theta + \cos^2\theta$  is equal to:

- (a) 0 (b) -1 (c) 2 (d)  1

7.  $1 + \tan^2\theta$  is equal to:

- (a)  $\csc^2\theta$  (b)  $\sin^2\theta$  (c)   $\sec^2\theta$  (d)  $\tan^2\theta$

8.  $\csc^2\theta - \cot^2\theta$  is equal to:

- (a) 0 (b)  1 (c) -1 (d) 2  
 (a) I (b) II (c) III (d) IV

9. If  $\tan\theta < 0$  and  $\operatorname{cosec}\theta > 0$  then the terminal arm of angle lies in \_\_\_\_\_ Quad.

- (a) I (b)  II (c) III (d) IV

10. If  $\sec\theta < 0$  and  $\sin\theta < 0$  then the terminal arm of angle lies in \_\_\_\_\_ Quad.

- (a) I (b) II (c)  III (d) IV

### SHORT QUESTIONS

- If  $\sin\theta = \frac{12}{13}$  and the terminal arm of the angle is in quadrant Ist. Find the remaining trigonometric functions.
- For  $\cot\theta > 0$ , and  $\sin\theta < 0$  In when Quadrant  $\theta$  lies ?
- Find  $\cos\theta$  and  $\tan\theta$  if  $\sin\theta = \frac{12}{13}$  and the terminal arm of the angle is in quad 1<sup>st</sup>.

### LONG QUESTIONS

If  $\csc\theta = \frac{m^2+1}{2m}$  and  $m > 0$  ( $0 < \theta < \frac{\pi}{2}$ ), find the values of the remaining trigonometric ratios.

If  $\cot\theta = \frac{5}{2}$  and the terminal arm of the angle is in I quad., find the value of  $\frac{3\sin\theta+4\cos\theta}{\cos\theta-\sin\theta}$ .

### EXERCISE 9.3

Q1. Tick () the correct answer. (5)

- In right angle triangle, the measure of the side opposite to  $30^\circ$  is:
 

(a)  Half of Hypotenuse (b) Half of Base (c) Double of base (d) None of these
- The point  $(0, 1)$  lies on the terminal side of angle:
 

(a)  $0^\circ$  (b)   $90^\circ$  (c)  $180^\circ$  (d)  $270^\circ$
- The point  $(-1, 0)$  lies on the terminal side of angle:
 

(a)  $0^\circ$  (b)  $90^\circ$  (c)   $180^\circ$  (d)  $270^\circ$
- The point  $(0, -1)$  lies on the terminal side of angle:
 

(a)  $0^\circ$  (b)  $90^\circ$  (c)  $180^\circ$  (d)   $270^\circ$
- $2\sin 45^\circ + \frac{1}{2}\operatorname{Cosec} 45^\circ =$ 

(a)  $\sqrt{\frac{2}{3}}$  (b)   $\frac{3}{\sqrt{2}}$  (c) -1 (d) 1

### SHORT QUESTIONS

- Verify  $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1 : 2 : 3 : 4$
- Verify  $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$  when  $\theta = 45^\circ$
- Find  $x$ , if  $\tan^2 45^\circ - \cos^2 60^\circ = x \sin^2 45^\circ \cos 45^\circ \tan 45^\circ$

- iv. Find the values of the trigonometric functions of  $\frac{5}{2}\pi$   
 v. Find the values of the trigonometric functions of  $765^\circ$   
 vi. Define "Quadrantal angles".  
 vii. Evaluate  $\frac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$

## EXERCISE 9.4

Tick (✓) the correct answer.

1. Domain of  $\sin\theta$  is:  
 (a) ✓  $R$  (b)  $\theta \in R$  but  $\theta \neq n\pi, n \in Z$  (c)  $\theta \in R$  but  $\theta \neq \frac{(2n+1)\pi}{2}, n \in Z$  (d) None of these
2. Domain of  $\cos\theta$  is:  
 (a) ✓  $R$  (b)  $\theta \in R$  but  $\theta \neq n\pi, n \in Z$  (c)  $\theta \in R$  but  $\theta \neq \frac{(2n+1)\pi}{2}, n \in Z$  (d) None of these
3. Domain of  $\tan\theta$  is:  
 (a)  $R$  (b)  $\theta \in R$  but  $\theta \neq n\pi, n \in Z$  (c)  $\theta \in R$  but  $\theta \neq \frac{(2n+1)\pi}{2}, n \in Z$  (d) None of these
4. Domain of  $\sec\theta$  is:  
 (a)  $R$  (b)  $\theta \in R$  but  $\theta \neq n\pi, n \in Z$  (c) ✓  $\theta \in R$  but  $\theta \neq \frac{(2n+1)\pi}{2}, n \in Z$  (d) None of these
5.  $\operatorname{cosec}\theta \sec\theta \sin\theta \cos\theta =$   
 (a) ✓ 1 (b) 0 (c)  $\sin\theta$  (d)  $\cos\theta$
6.  $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) =$   
 (a) ✓ 1 (b) 0 (c)  $\sec\theta$  (d)  $\tan\theta$
7.  $\frac{1-\sin\theta}{\cos\theta} =$   
 (a)  $\frac{\cos\theta}{1-\sin\theta}$  (b) ✓  $\frac{\cos\theta}{1+\sin\theta}$  (c)  $\frac{\sin\theta}{1-\cos\theta}$  (d)  $\frac{\sin\theta}{1+\cos\theta}$

### SHORT QUESTIONS

- i. Show that  $\cot^4\theta + \cot^2\theta = \operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta$ , where  $\theta$  is not an integral multiple of  $\frac{\pi}{2}$ .
- ii. Prove that  $\cos\theta + \tan\theta \sin\theta = \sec\theta$
- iii. Prove that  $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$
- iv. Prove that  $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\cot\theta - 1}{\cot\theta + 1}$
- v. Prove that  $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
- vi. Prove that  $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$
- vii. Prove that  $(\sin^3\theta - \cos^3\theta) = (\sin\theta - \cos\theta)(1 - \sin^2\theta \cos^2\theta)$

### LONG QUESTIONS

- Prove that  $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta \cos^2\theta$   
 Prove that  $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

## EXERCISE 10.1

Tick (✓) the correct answer.

1. Fundamental law of trigonometry is ,  $\cos(\alpha - \beta)$ 
  - (a) ✓  $\cos\alpha\cos\beta + \sin\alpha\sin\beta$
  - (b)  $\cos\alpha\cos\beta - \sin\alpha\sin\beta$
  - (c)  $\sin\alpha\cos\beta + \cos\alpha\sin\beta$
  - (d)  $\sin\alpha\cos\beta - \cos\alpha\sin\beta$
2.  $\sin(\alpha + \beta)$  is equal to:
  - (a)  $\cos\alpha\cos\beta + \sin\alpha\sin\beta$
  - (b)  $\cos\alpha\cos\beta - \sin\alpha\sin\beta$
  - (c) ✓  $\sin\alpha\cos\beta + \cos\alpha\sin\beta$
  - (d)  $\sin\alpha\cos\beta - \cos\alpha\sin\beta$
3.  $\cos\left(\frac{\pi}{2} - \beta\right) =$ 
  - (a)  $\cos\beta$
  - (b)  $-\cos\beta$
  - (c) ✓  $\sin\beta$
  - (d)  $-\sin\beta$
4.  $\sin(2\pi - \theta) =$ 
  - (a)  $\cos\theta$
  - (b)  $-\cos\theta$
  - (c) ✓  $\sin\theta$
  - (d)  $-\sin\theta$
5.  $\tan(\alpha - \beta) =$ 
  - (a) ✓  $\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$
  - (b)  $\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
  - (c)  $\frac{\tan\alpha - \tan\beta}{1 - \tan\alpha\tan\beta}$
  - (d)  $\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
6. Angles associated with basic angles of measure  $\theta$  to a right angle or its multiple are called:
  - (a) Coterminal angle
  - (b) angle in standard position
  - (c) ✓ Allied angle
  - (d) obtuse angle
7.  $\sin\left(\frac{3\pi}{2} + \theta\right) =$ 
  - (a)  $\sin\theta$
  - (b)  $\cos\theta$
  - (c)  $-\sin\theta$
  - (d) ✓  $-\cos\theta$
8.  $\cos 315^\circ$  is equal to:
  - (a) 1
  - (b) 0
  - (c) ✓  $\frac{1}{\sqrt{2}}$
  - (d)  $\frac{\sqrt{3}}{2}$
9.  $\sin(180^\circ + \alpha)\sin(90^\circ - \alpha) =$ 
  - (a) ✓  $\sin\alpha\cos\alpha$
  - (b)  $-\sin\alpha\cos\alpha$
  - (c)  $\cos\alpha$
  - (d)  $-\cos\alpha$
10. If  $\alpha, \beta$  and  $\gamma$  are the angles of a triangle ABC then  $\cos\left(\frac{\alpha+\beta}{2}\right) =$ 
  - (a) ✓  $\sin\frac{\gamma}{2}$
  - (b)  $-\sin\frac{\gamma}{2}$
  - (c)  $\cos\frac{\gamma}{2}$
  - (d)  $-\cos\frac{\gamma}{2}$
11. Which is the allied angle
  - (a) ✓  $90^\circ + \theta$
  - (b)  $60^\circ + \theta$
  - (c)  $45^\circ + \theta$
  - (d)  $30^\circ + \theta$

### SHORT QUESTIONS

- i. Without using Calculator. Find the value of  $\sec(-960^\circ)$
- ii. Prove that  $\sin(180^\circ + \alpha)\sin(90^\circ - \alpha) = -\sin\alpha\cos\alpha$
- iii. If  $\alpha, \beta, \gamma$  are angles of triangle ABC, then prove that  $\cos\left(\frac{\alpha+\beta}{2}\right) = \sin\frac{\gamma}{2}$
- iv. Prove that  $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ = -1$
- v. State "Distance formula".

### LONG QUESTIONS

Prove that 
$$\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\operatorname{cosec}(2\pi-\theta)} = \cos\theta$$

Prove that  $\frac{\cos(90^\circ + \theta) \sec(-\theta) \tan(180^\circ - \theta)}{\sec(360^\circ - \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)} = -1$

## EXERCISE 10.2

Tick (✓) the correct answer.

1.  $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} =$

- (a) ✓  $\tan 56^\circ$       (b)  $\tan 34^\circ$       (c)  $\cot 56^\circ$       (d)  $\cot 34^\circ$

2. If  $\sin(\alpha + \beta)$  is *-ive* and  $\cos(\alpha + \beta)$  is *+ive* then terminal arm of  $(\alpha + \beta)$  lies in

- (a) I Quad      (b) II Quad      (c) III Quad      (d) ✓ IV Quad

3.  $(\sin \alpha + \sin \beta)(\sin \alpha - \sin \beta) =$

- (a) ✓  $\sin^2 \alpha - \sin^2 \beta$     (b)  $\sin^2 \alpha - \cos^2 \beta$     (c)  $\cos^2 \alpha - \sin^2 \beta$     (d) None of these

### SHORT QUESTIONS

- i. Find the value of  $\cos 15^\circ$  and  $\tan 105^\circ$
- ii. Prove that  $\cos(360^\circ - \theta) = \cos \theta$
- iii. Prove that  $\tan(45^\circ + A) \tan(45^\circ - A) = 1$
- iv. Show that  $\frac{1 - \tan \theta \tan \varphi}{1 + \tan \theta \tan \varphi} = \frac{\cos(\theta + \varphi)}{\cos(\theta - \varphi)}$
- v. Prove that  $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$
- vi. If  $\alpha, \beta, \gamma$  are angles of a triangle  $ABC$ , Show that  $\cot \frac{\alpha}{2} + \cot \frac{\beta}{2} + \cot \frac{\gamma}{2} = \cot \frac{\alpha}{2} \cot \frac{\beta}{2} \cot \frac{\gamma}{2}$
- vii. Express  $5 \sin \theta - 4 \cos \theta$  in the form  $r \sin(\theta + \varphi)$ , where the terminal sides of the angles of measures  $\theta$  and  $\varphi$  are in the first quadrant.

### LONG QUESTIONS

If  $\cos \alpha = -\frac{24}{25}$ ,  $\tan \beta = \frac{9}{40}$ , then terminal side of the angle of measure of  $\alpha$  in the II quadrant and that of  $\beta$  is in the III quadrant, find the value of  $\cos(\alpha + \beta)$ .

Show that  $\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta = \cos^2 \beta - \sin^2 \alpha$

## EXERCISE 10.3

Tick (✓) the correct answer.

1.  $\sin 2\alpha$  is equal to:
 

|                                     |                        |                                   |                                   |
|-------------------------------------|------------------------|-----------------------------------|-----------------------------------|
| (a) $\cos^2 \alpha - \sin^2 \alpha$ | (b) $1 + \cos 2\alpha$ | (c) ✓ $2 \sin \alpha \cos \alpha$ | (d) $2 \sin 2\alpha \cos 2\alpha$ |
|-------------------------------------|------------------------|-----------------------------------|-----------------------------------|
2.  $\cos 2\alpha =$ 

|                                       |                           |                           |                  |
|---------------------------------------|---------------------------|---------------------------|------------------|
| (a) ✓ $\cos^2 \alpha - \sin^2 \alpha$ | (b) $1 - 2 \sin^2 \alpha$ | (c) $2 \cos^2 \alpha - 1$ | (d) All of these |
|---------------------------------------|---------------------------|---------------------------|------------------|
3.  $\tan 2\alpha =$ 

|   |   |   |   |
|---|---|---|---|
| (a) $\frac{2 \tan \alpha}{1 + \tan^2 \alpha}$ | (b) ✓ $\frac{2 \tan \alpha}{1 - \tan^2 \alpha}$ | (c) $\frac{2 \tan^2 \alpha}{1 - \tan^2 \alpha}$ | (d) $\frac{\tan^2 \alpha}{1 - \tan^2 \alpha}$ |
|---|---|---|---|
4.  $\sin 3\alpha =$ 

|                                       |                                       |   |                                       |
|---------------------------------------|---------------------------------------|---|---------------------------------------|
| (a) $3 \sin \alpha - 2 \sin^3 \alpha$ | (b) $3 \sin \alpha + 2 \sin^3 \alpha$ | (c) ✓ $3 \sin \alpha - 4 \sin^3 \alpha$ | (d) $3 \cos \alpha - 2 \sin^3 \alpha$ |
|---------------------------------------|---------------------------------------|---|---------------------------------------|

### SHORT QUESTIONS

- i. Find the value of  $\cos 2\alpha$ , when  $\sin \alpha = \frac{12}{13}$  where  $0 < \alpha < \frac{\pi}{2}$
- ii. Prove that  $\sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}} = \frac{\sin \alpha + \cos \alpha}{\sin \alpha - \cos \alpha}$
- iii. Prove that  $1 + \tan \alpha \tan 2\alpha = \sec 2\alpha$
- iv. Prove that  $\frac{\sin 3\theta}{\cos \theta} + \frac{\cos 3\theta}{\sin \theta} = 2 \cot 2\theta$
- v. Prove that  $\frac{\tan \frac{\theta}{2} + \cot \frac{\theta}{2}}{\cot \frac{\theta}{2} - \tan \frac{\theta}{2}} = \sec \theta$

### LONG QUESTIONS

Reduce  $\cos^4 \theta$  to an expression involving only function of multiples of raised to the first power. Prove

that 
$$\frac{\operatorname{cosec} \theta + 2 \operatorname{cosec} 2\theta}{\sec \theta} = \cot \frac{\theta}{2}$$

## EXERCISE 10.4

Tick (✓) the correct answer.

1.  $\sin \alpha + \sin \beta$  is equal to:
 

|   |   |
|---|---|
| (a) ✓ $2 \sin \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)$ | (b) $2 \cos \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right)$ |
| (c) $-2 \sin \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right)$  | (d) $2 \cos \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)$ |
2.  $\sin \alpha - \sin \beta$  is equal to:
 

|  |   |
|--|---|
| (a) $2 \sin \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)$  | (b) ✓ $2 \cos \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right)$ |
| (c) $-2 \sin \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right)$ | (d) $2 \cos \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)$   |
3.  $\cos \alpha + \cos \beta$  is equal to:

- (c)  $2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)$   
 $-2 \sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)$
- (b)  $2 \cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)$  (c)  
 (d) ✓  $2 \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)$
4.  $\cos\alpha - \cos\beta$  is equal to:  
 (d)  $2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)$  (b)  $2 \cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)$  (c)  
 ✓  $-2 \sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)$  (d)  $2 \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)$
5.  $2 \sin 7\theta \cos 3\theta =$   
 (a) ✓  $\sin 10\theta + \sin 4\theta$  (b)  $\sin 5\theta - \sin 2\theta$  (c)  $\cos 10\theta + \cos 4\theta$  (d)  $\cos 5\theta - \cos 2\theta$
6.  $2 \cos 5\theta \sin 3\theta =$   
 (a) ✓  $\sin 8\theta - \sin 2\theta$  (b)  $\sin 8\theta + \sin 2\theta$  (c)  $\cos 8\theta + \cos 2\theta$  (d)  $\cos 8\theta - \cos 2\theta$

### SHORT QUESTIONS

- i. Express  $\sin 5x + \sin 7x$  as product.  
 ii. Express  $2 \sin 3\theta \cos \theta$  as sum or difference.  
 iii. Prove that  $\frac{\sin 8x + \cos 3x}{\cos 5x + \cos 2x} = \tan 5x$   
 iv. Express  $\sin(x + 45^\circ) \sin(x - 45^\circ)$  as sum or difference.  
 v. Express  $\sin(x + 30^\circ) + \sin(x - 30^\circ)$  as product.  
 vi. Prove the identity:  $\frac{\sin \alpha - \sin \beta}{\sin \alpha + \sin \beta} = \tan \frac{\alpha - \beta}{2} \cot \frac{\alpha + \beta}{2}$   
 vii. Prove that  $\sin\left(\frac{\pi}{4} - \theta\right) \sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2} \cos 2\theta$

### LONG QUESTIONS

Prove that  $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

Prove that  $\sin \frac{\pi}{9} \sin \frac{2\pi}{9} \sin \frac{\pi}{3} \sin \frac{4\pi}{9} = \frac{3}{16}$

### EXERCISE 11.1

Tick (✓) the correct answer.

- Range of  $y = \sec x$  is  
 (a)  $R$  (b) ✓  $y \geq 1$  or  $y \leq -1$  (c)  $-1 \leq y \leq 1$  (d)  $R - [-1, 1]$
- Range of  $y = \operatorname{cosec} x$  is  
 (a)  $R$  (b) ✓  $y \geq 1$  or  $y \leq -1$  (c)  $-1 \leq y \leq 1$  (d)  $R - [-1, 1]$
- Smallest +ive number which when added to the original circular measure of the angle gives the same value of the function is called:  
 (a) Domain (b) Range (c) Co domain (d) ✓ Period
- Domain of  $y = \cos x$  is  
 (a) ✓  $-\infty < x < \infty$  (b)  $-1 \leq x \leq 1$  (c)  $-\infty < x < \infty, x \neq n\pi, n \in Z$  (d)  $x \geq 1, x \leq -1$
- Domain of  $y = \tan x$  is  
 (a)  $-\infty < x < \infty$  (b)  $-1 \leq x \leq 1$  (c) ✓  $-\infty < x < \infty, x \neq \frac{2n+1}{2}\pi, n \in Z$  (d)  $x \geq 1, x \leq -1$
- Period of  $\cos \theta$  is  
 (a)  $\pi$  (b) ✓  $2\pi$  (c)  $-2\pi$  (d)  $\frac{\pi}{2}$

7. Period of  $\tan 4x$  is

- (a)  $\pi$  (b)  $2\pi$  (c)  $-2\pi$  (d)   $\frac{\pi}{4}$

8. Period of  $\cot 3x$  is

- (a)  $\pi$  (b)   $\frac{\pi}{3}$  (c)  $-2\pi$  (d)  $\frac{\pi}{4}$

9. Period of  $3\cos \frac{x}{5}$  is

- (a)  $2\pi$  (b)  $\frac{\pi}{2}$  (c)  $\pi$  (d)   $10\pi$

10. The graph of trigonometric functions have:

- (a) Break segments (b) Sharp corners (c) Straight line segments (d) smooth curves

11. Curves of the trigonometric functions repeat after fixed intervals because trigonometric functions are

- (a) Simple (b) linear (c) quadratic (d)  periodic

12. The graph of  $y = \cos x$  lies between the horizontal line  $y = -1$  and

- (a)   $+1$  (b) 0 (c) 2 (d)  $-2$

### SHORT QUESTIONS

- Define the *period of trigonometric function*.
- Find the period of (a)  $\operatorname{Cosec} \frac{x}{4}$  (b)  $\tan \frac{x}{7}$  (c)  $3\cos \frac{x}{5}$
- Find the domain and range of  $y = \sin x$  and  $y = \cos x$
- Find the period of  $3\sin x$
- Find the period of  $\cot 8x$

### EXERCISE 12.1

Tick () the correct answer.

1. A "Triangle" has :

- (a) Two elements (b) 3 elements (c) 4 elements (d)  6 elements

2.  $\sin 38^\circ 24' =$

- (a) 0.2611 (b) 0.2622 (c) 0.6211 (d) 0.5211

3. When  $\theta$  increases from  $0^\circ$  to  $90^\circ$  then  $\sin \theta$ ,  $\sec \theta$  and  $\tan \theta$  go on

- (a)  Increasing (b) Decreasing (c) Constant (d) None of these

4. When  $\theta$  increases from  $0^\circ$  to  $90^\circ$  then  $\cos \theta$ ,  $\operatorname{cosec} \theta$  and  $\cot \theta$  go on

- (a) Increasing (b)  Decreasing (c) Constant (d) None of these

5. If  $\sin x = 0.5100$  then  $x =$

- (a)   $30^\circ 40'$  (b)  $35^\circ 40'$  (c)  $40^\circ 40'$  (d)  $44^\circ 44'$

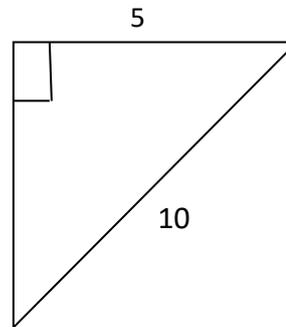
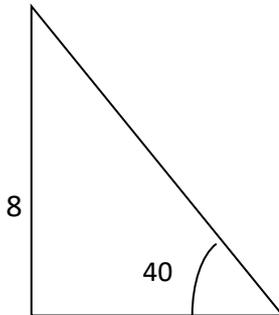
### SHORT QUESTIONS

- Find the values of  $\cos 36^\circ 20'$  and  $\cot 89^\circ 9'$
- Find  $\theta$ , if  $\sin \theta = 0.5791$  and  $\tan \theta = 1.705$

## EXERCISE 12.2

### SHORT QUESTIONS

- i. Find the unknown angles and sides of the given triangles.



- ii. Solve the right triangle  $ABC$ , in which  $\gamma = 90^\circ$ ,  $a = 3.28$ ,  $b = 5.74$   
 iii. Solve the right triangle  $ABC$ , in which  $\gamma = 90^\circ$ ,  $\beta = 50^\circ 10'$ ,  $c = 0.832$

## EXERCISE 12.3

Tick (✓) the correct answer.

1. When we look an object above the horizontal ray, the angle formed is called angle of:
 

|               |                |               |              |
|---------------|----------------|---------------|--------------|
| (a) Elevation | (b) depression | (c) incidence | (d) reflects |
|---------------|----------------|---------------|--------------|
2. When we look an object below the horizontal ray, the angle formed is called angle of:
 

|               |                |               |              |
|---------------|----------------|---------------|--------------|
| (a) Elevation | (b) depression | (c) incidence | (d) reflects |
|---------------|----------------|---------------|--------------|

### LONG QUESTIONS

A vertical pole is  $8m$  high and length of its shadow is  $6m$ . What is the angle of elevation of the sun at that moment?

What the angle between the ground and the sun is  $30^\circ$ , flag pole casts a shadow of  $40m$  long. Find the height of the top of the flag.

## EXERCISE 12.4

Tick (✓) the correct answer.

1. A triangle which is not right is called:

- |                        |                        |                      |                              |
|------------------------|------------------------|----------------------|------------------------------|
| (a) ✓ Oblique triangle | (b) Isosceles triangle | (c) Scalene triangle | (d) Right isosceles triangle |
|------------------------|------------------------|----------------------|------------------------------|

2. To solve an oblique triangle we use:

- |                 |                   |                     |                    |
|-----------------|-------------------|---------------------|--------------------|
| (a) Law of Sine | (b) Law of Cosine | (c) Law of Tangents | (d) ✓ All of these |
|-----------------|-------------------|---------------------|--------------------|

3. In any triangle  $ABC$ ,  $\frac{b^2+c^2-a^2}{2bc} =$   
 (a)  $\cos\alpha$  (b)  $\sin\alpha$  (c)  $\cos\beta$  (d)  $\cos\gamma$
4. Which can be reduced to Pythagoras theorem,  
 (a) Law of sine (b)  law of cosine (c) law of tangents (d) Half angle formulas
5. In any triangle  $ABC$ , if  $\beta = 90^\circ$ , then  $b^2 = a^2 + c^2 - 2accos\beta$  becomes:  
 (a) Law of sine (b) Law of tangents (c) Law of cosine (d)  Pythagoras theorem
6. In any triangle  $ABC$ , law of tangent is :  
 (a)  $\frac{a-b}{a+b} = \frac{\tan(\alpha-\beta)}{\tan(\alpha+\beta)}$  (b)  $\frac{a+b}{a-b} = \frac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$  (c)   $\frac{a-b}{a+b} = \frac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}$  (d)  $\frac{a-b}{a+b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$
7. In any triangle  $ABC$ ,  $\sqrt{\frac{(s-a)(s-b)}{ab}} =$   
 (a)  $\sin\frac{\alpha}{2}$  (b)  $\sin\frac{\beta}{2}$  (c)   $\sin\frac{\gamma}{2}$  (d)  $\cos\frac{\alpha}{2}$
8. In any triangle  $ABC$ ,  $\sqrt{\frac{(s-b)(s-c)}{bc}} =$   
 (a)   $\sin\frac{\alpha}{2}$  (b)  $\sin\frac{\beta}{2}$  (c)  $\sin\frac{\gamma}{2}$  (d)  $\cos\frac{\alpha}{2}$
9. In any triangle  $ABC$ ,  $\sqrt{\frac{(s-a)(s-c)}{ac}} =$   
 (a)  $\sin\frac{\alpha}{2}$  (b)   $\sin\frac{\beta}{2}$  (c)  $\sin\frac{\gamma}{2}$  (d)  $\cos\frac{\alpha}{2}$
10. In any triangle  $ABC$ ,  $\cos\frac{\alpha}{2} =$   
 (a)  $\sqrt{\frac{s(s-a)}{ab}}$  (b)  $\sqrt{\frac{s(s-b)}{ac}}$  (c)   $\sqrt{\frac{s(s-a)}{bc}}$  (d)  $\sqrt{\frac{s(s-c)}{ab}}$
11. In any triangle  $ABC$ ,  $\cos\frac{\beta}{2} =$   
 (a)  $\sqrt{\frac{s(s-a)}{ab}}$  (b)   $\sqrt{\frac{s(s-b)}{ac}}$  (c)  $\sqrt{\frac{s(s-a)}{bc}}$  (d)  $\sqrt{\frac{s(s-c)}{ab}}$
12. In any triangle  $ABC$ ,  $\cos\frac{\gamma}{2} =$   
 (a)  $\sqrt{\frac{s(s-a)}{ab}}$  (b)  $\sqrt{\frac{s(s-b)}{ac}}$  (c)  $\sqrt{\frac{s(s-a)}{bc}}$  (d)   $\sqrt{\frac{s(s-c)}{ab}}$
13. In any triangle  $ABC$ , with usual notations,  $s$  is equal to  
 (a)  $a + b + c$  (b)  $\frac{a+b+c}{3}$  (c)   $\frac{a+b+c}{2}$  (d)  $\frac{abc}{2}$
14. In any triangle  $ABC$ ,  $\sqrt{\frac{s(s-c)}{(s-a)(s-b)}} =$   
 (a)  $\sin\frac{\gamma}{2}$  (b)  $\cos\frac{\gamma}{2}$  (c)  $\tan\frac{\gamma}{2}$  (d)  $\cot\frac{\gamma}{2}$
15. In any triangle  $ABC$ ,  $\sqrt{\frac{(s-a)(s-b)}{s(s-c)}} =$   
 (a)  $\sin\frac{\gamma}{2}$  (b)  $\cos\frac{\gamma}{2}$  (c)   $\tan\frac{\gamma}{2}$  (d)  $\cot\frac{\gamma}{2}$

### SHORT QUESTIONS

- i. Solve the triangle  $ABC$ , if  $c = 16.1$ ,  $\alpha = 42^\circ 45'$ ,  $\gamma = 74^\circ 32'$   
 ii. Solve the triangle  $ABC$ , if  $a = 53$ ,  $\beta = 88^\circ 36'$ ,  $\gamma = 31^\circ 54'$

### LONG QUESTIONS

State and Prove "Law of Sine".

## EXERCISE 12.5

### LONG QUESTIONS

Solve the triangle  $ABC$  in which :

$$b = 3, \quad c = 6 \quad \text{and} \quad \beta = 36^\circ 20'$$

Solve the triangle  $ABC$  in which :

$$a = \sqrt{3} - 1, \quad b = \sqrt{3} + 1 \quad \text{and} \quad \gamma = 60^\circ$$

Solve the triangle using first law of tangents and then law of sines:

$$\begin{array}{lll} \text{(a) } b = 14.8 & , & c = 16.1 \quad \text{and } \alpha = 42^\circ 45' \\ \text{(b) } b = 61 & , & a = 32 \quad \text{and } \alpha = 59^\circ 30' \end{array}$$

## EXERCISE 12.6

Tick (✓) the correct answer.

1. The smallest angle of  $\triangle ABC$ , when  $a = 37.34$  ,  $b = 3.24$  ,  $c = 35.06$  is

- (a)  $\alpha$                       (b) ✓  $\beta$                       (c)  $\gamma$                       (d) cannot be determined

### SHORT QUESTIONS

- i. Solve the triangle , in which  
 $a = 7$  ,  $b = 7$  ,  $c = 9$
- ii. Find the smallest angle  $ABC$ , when  $a = 37.34$  ,  $b = 3.24$  ,  $c = 35.06$
- iii. Find the measure of the greatest angle , if sides of the angle are 16, 20, 33.
- iv. The sides of triangle are  $x^2 + x + 1$ ,  $2x + 1$  and  $x^2 - 1$ . Prove that the greatest angle of the triangle is  $120^\circ$ .

## EXERCISE 12.7

Tick (✓) the correct answer.

1. To solve an oblique triangles when measure of three sides are given , we can use:
 

|                      |                   |                 |                     |
|----------------------|-------------------|-----------------|---------------------|
| (a) ✓ Hero's formula | (b) Law of cosine | (c) Law of sine | (d) Law of tangents |
|----------------------|-------------------|-----------------|---------------------|
2. In any triangle  $ABC$  Area if triangle is :
 

|                      |                                 |                                |                                 |
|----------------------|---------------------------------|--------------------------------|---------------------------------|
| (a) $bc \sin \alpha$ | (b) $\frac{1}{2}ca \sin \alpha$ | (c) $\frac{1}{2}ab \sin \beta$ | (d) ✓ $\frac{1}{2}absin \gamma$ |
|----------------------|---------------------------------|--------------------------------|---------------------------------|

### SHORT QUESTIONS

- i. Find the area of the triangle  $ABC$ , in which  $b = 21.6$  ,  $c = 30.2$  and  $\alpha = 52^\circ 40'$
- ii. Find the area of the triangle  $ABC$ , given one side and two angles:  
 $b = 25.4$  ,  $\gamma = 36^\circ 41'$  and  $\alpha = 45^\circ 17'$

iii. Find the area of the triangle  $ABC$ , given three sides :

$$a = 32.65, \quad b = 42.81, \quad c = 64.92$$

### LONG QUESTIONS

The area of the triangle is 2437. If  $a = 79$  and  $c = 97$ , then find angle  $\beta$ .

### EXERCISE 12.8

Tick (✓) the correct answer.

1. The circle passing through the three vertices of a triangle is called:

- (a) ✓ Circum circle      (b) in-circle      (c) ex-centre      (d) escribed circle

2. The point of intersection of the right bisectors of the sides of the triangle is :

- (a) ✓ Circum centre      (b) In-centre      (c) Escribed center      (d) Diameter

3. In any triangle  $ABC$ , with usual notations,  $\frac{a}{2\sin\alpha} =$

- (a)  $r$       (b)  $r_1$       (c) ✓  $R$       (d)  $\Delta$

4. In any triangle  $ABC$ , with usual notations,  $\frac{a}{\sin\beta} =$

- (a)  $2r$       (b)  $2r_1$       (c) ✓  $2R$       (d)  $2\Delta$

5. In any triangle  $ABC$ , with usual notations,  $\sin\gamma =$

- (a)  $R$       (b) ✓  $\frac{c}{2R}$       (c)  $\frac{2R}{c}$       (d)  $\frac{R}{2}$

6. In any triangle  $ABC$ , with usual notations,  $abc =$

- (a)  $R$       (b)  $Rs$       (c) ✓  $4R\Delta$       (d)  $\frac{\Delta}{s}$

7. In any triangle  $ABC$ , with usual notations,  $\frac{\Delta}{s-a} =$

- (a)  $r$       (b)  $R$       (c) ✓  $r_1$       (d)  $r_2$

8. In any triangle  $ABC$ , with usual notations,  $\frac{\Delta}{s-b} =$

- (a)  $r$       (b)  $R$       (c)  $r_1$       (d) ✓  $r_2$

9. In any triangle  $ABC$ , with usual notations,  $\frac{\Delta}{s-c} =$

- (a) ✓  $r_3$       (b)  $R$       (c)  $r_1$       (d)  $r_2$

10. In any triangle  $ABC$ , with usual notation,  $r:R:r_1 =$

- (a) 3:2:1      (b) 1:2:2      (c) ✓ 1:2:3      (d) 1:1:1

11. In any triangle  $ABC$ , with usual notation,  $r:R:r_1:r_2:r_3 =$

- (a) 3:3:3:2:1      (b) 1:2:2:3:3      (c) ✓ 1:2:3:3:3      (d) 1:1:1:1:1

12. In a triangle  $ABC$ , if  $\beta = 60^\circ$ ,  $\gamma = 15^\circ$  then  $\alpha =$

- (a)  $90^\circ$       (b)  $180^\circ$       (c)  $150^\circ$       (d) ✓  $105^\circ$

### SHORT QUESTIONS

i. Prove that  $R = \frac{abc}{4\Delta}$

ii. Define "Incircle".

iii. Show that  $r_2 = s \tan \frac{\beta}{2}$

- iv. Prove that  $rr_1r_2r_3 = \Delta^2$   
 v. Prove that  $r_1r_2r_3 = rs^2$   
 vi. Show that  $\frac{1}{2rR} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$

### LONG QUESTIONS

Prove that  $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

Prove that  $r_1 + r_2 + r_3 - r = 4R$

Prove that in an equilateral triangle,  $r:R:r_1:r_2:r_3 = 1:2:3:3:3$

Prove that  $r = s \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2}$

### EXERCISE 13.1

Tick (✓) the correct answer.

1. Inverse of a function exist only if it is:

- (a) Trigonometric function (b) ✓  $(1-1)$  function (c) onto function (d) an into function

2.  $\sin^{-1}x =$

- (a) ✓  $\frac{\pi}{2} - \cos^{-1}x$  (b)  $\frac{\pi}{2} - \sin^{-1}x$  (c)  $\frac{\pi}{2} + \cos^{-1}x$  (d)  $\frac{\pi}{2} - \operatorname{cosec}^{-1}x$

3.  $\cos^{-1}x =$

- (a)  $\frac{\pi}{2} - \cos^{-1}x$  (b) ✓  $\frac{\pi}{2} - \sin^{-1}x$  (c)  $\frac{\pi}{2} + \cos^{-1}x$  (d)  $\frac{\pi}{2} - \operatorname{cosec}^{-1}x$

4.  $\sec^{-1}x =$

- (a)  $\frac{\pi}{2} - \sec^{-1}x$  (b)  $\frac{\pi}{2} - \sin^{-1}x$  (c)  $\frac{\pi}{2} + \sec^{-1}x$  (d) ✓  $\frac{\pi}{2} - \operatorname{cosec}^{-1}x$

5.  $\tan^{-1}x =$

- (a)  $\frac{\pi}{2} - \sec^{-1}x$  (b)  $\frac{\pi}{2} - \sin^{-1}x$  (c) ✓  $\frac{\pi}{2} - \cot^{-1}x$  (d)  $\frac{\pi}{2} - \operatorname{cosec}^{-1}x$

6.  $\cot^{-1}x =$

- (a)  $\frac{\pi}{2} - \sec^{-1}x$  (b) ✓  $\frac{\pi}{2} - \tan^{-1}x$  (c)  $\frac{\pi}{2} + \sec^{-1}x$  (d)  $\frac{\pi}{2} - \operatorname{cosec}^{-1}x$

7.  $\sin(\cos^{-1}\frac{\sqrt{3}}{2}) =$

- (a)  $\frac{\pi}{6}$  (b) ✓  $\frac{1}{2}$  (c)  $-\frac{1}{2}$  (d)  $\frac{\sqrt{3}}{2}$

8.  $\tan^{-1}(\sqrt{3}) =$

- (a)  $\frac{\pi}{6}$  (b)  $-\frac{\pi}{6}$  (c)  $-\frac{\pi}{3}$  (d) ✓  $\frac{\pi}{3}$

9.  $\sin(\sin^{-1}\frac{1}{2}) =$

- (a) ✓  $\frac{1}{2}$  (b)  $\frac{2}{3}$  (c) 2 (d)  $\frac{1}{3}$

### SHORT QUESTIONS

- i. Find the value of  $\sin^{-1}\frac{\sqrt{3}}{2}$   
 ii. Prove that  $\operatorname{csc}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$   
 iii. Evaluate  $\cos^{-1}(\frac{1}{2})$   
 iv. Find the value of  $\tan(\sin^{-1}(-\frac{1}{2}))$

- v. Find the value of  $\sin(\tan^{-1}(-1))$   
 vi. Prove that  $2\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{24}{25}$

## EXERCISE 13.2

Tick (✓) the correct answer.

1.  $\sin^{-1}A - \sin^{-1}B =$

- (a) ✓  $\sin^{-1}(A\sqrt{1-B^2} + B\sqrt{1-A^2})$       (b)  $\sin^{-1}(A\sqrt{1-A^2} - B\sqrt{1-B^2})$   
 (c)  $\sin^{-1}(B\sqrt{1-A^2} + A\sqrt{1-B^2})$       (d)  $\sin^{-1}(AB\sqrt{(1-A^2)(1-B^2)})$

2.  $\cos^{-1}A + \cos^{-1}B =$

- (a)  $\cos^{-1}(AB - \sqrt{(1-A^2)(1-B^2)})$       (b) ✓  $\cos^{-1}(AB + \sqrt{(1-A^2)(1-B^2)})$   
 (c)  $\cos^{-1}(AB - \sqrt{(1+A^2)(1+B^2)})$       (d)  $\cos^{-1}(AB + \sqrt{(1+A^2)(1+B^2)})$

4.  $\tan^{-1}A + \tan^{-1}B =$

- (a) ✓  $\tan^{-1}\left(\frac{A+B}{1-AB}\right)$       (b)  $\tan^{-1}\left(\frac{A+B}{1+AB}\right)$       (c)  $\tan^{-1}\left(\frac{A-B}{1-AB}\right)$       (d)  $\tan^{-1}\left(\frac{A+B}{1+AB}\right)$

5.  $\sin^{-1}(-x) =$

- (a) ✓  $-\sin^{-1}x$       (b)  $\sin^{-1}x$       (c)  $\pi - \sin^{-1}x$       (d)  $\pi - \sin x$

6.  $\cos^{-1}(-x) =$

- (a)  $-\cos^{-1}x$       (b)  $\cos^{-1}x$       (c) ✓  $\pi - \cos^{-1}x$       (d)  $\pi - \cos x$

7.  $\tan^{-1}(-x) =$

- (a) ✓  $-\tan^{-1}x$       (b)  $\tan^{-1}x$       (c)  $\pi - \tan^{-1}x$       (d)  $\pi - \tan x$

8.  $\operatorname{cosec}^{-1}(-x) =$

- (a) ✓  $-\operatorname{cosec}^{-1}x$       (b)  $\operatorname{cosec}^{-1}x$       (c)  $\pi - \operatorname{cosec}^{-1}x$       (d)  $\pi - \operatorname{cosec} x$

9.  $\sec^{-1}(-x) =$

- (a)  $-\sec^{-1}x$       (b)  $\sec^{-1}x$       (c) ✓  $\pi - \sec^{-1}x$       (d)  $\pi - \sec x$

10.  $\cot^{-1}(-x) =$

- (a)  $-\cot^{-1}x$       (b)  $\cot^{-1}x$       (c) ✓  $\pi - \cot^{-1}x$       (d)  $\pi - \cot x$

### SHORT QUESTIONS

- i. Show that  $\cos(\sin^{-1}x) = \sqrt{1-x^2}$   
 ii. Show that  $\tan^{-1}(-x) = -\tan^{-1}x$   
 iii. Show that  $\cos^{-1}(-x) = \pi - \cos^{-1}x$   
 iv. Show that  $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$

### LONG QUESTIONS

Prove that  $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} = \tan^{-1} \frac{9}{19}$

Prove that  $\sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$

Prove that  $\tan^{-1} \frac{120}{119} = 2\cos^{-1} \frac{12}{13}$

Prove that  $\sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{65} = \frac{\pi}{2}$

Prove that  $2\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$

## EXERCISE 14

1. General solution of  $\tan x = 1$  is:

- (a)   $\{\frac{\pi}{4} + n\pi, \frac{5\pi}{4} + n\pi\}$  (b)  $\{\frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi\}$  (c)  $\{\frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi\}$  (d)  $\{\frac{\pi}{4} + 2n\pi, \frac{3\pi}{4} + 2n\pi\}$

2. If  $\tan 2x = -1$ , then solution in the interval  $[0, \pi]$  is:

- (a)   $\frac{\pi}{8}$  (b)  $\frac{\pi}{4}$  (c)  $\frac{3\pi}{8}$  (d)  $\frac{3\pi}{4}$

3. If  $\sin x + \cos x = 0$  then value of  $x \in [0, 2\pi]$

- (a)  $\{\frac{\pi}{4}, \frac{3\pi}{4}\}$  (b)  $\{\frac{\pi}{4}, \frac{7\pi}{4}\}$  (c)   $\{\frac{3\pi}{4}, \frac{7\pi}{4}\}$  (d)  $\{\frac{\pi}{4}, \frac{-\pi}{4}\}$

4. General solution of  $4\sin x - 8 = 0$  is:

- (a)  $\{\pi + 2n\pi\}$  (b)  $\{\pi + n\pi\}$  (c)  $\{-\pi + n\pi\}$  (d)  not possible

5. General solution of  $1 + \cos x = 0$  is:

- (a)   $\{\pi + 2n\pi\}$  (b)  $\{\pi + n\pi\}$  (c)  $\{-\pi + n\pi\}$  (d) not possible

6. For the general solution, we first find the solution in the interval whose length is equal to its:

- (a) Range (b) domain (c) co-domain (d)  period

7. All trigonometric functions are ..... functions.

- (a)  Periodic (b) continues (c) injective (d) bijective

8. General solution of every trigonometric equation consists of :

- (a) One solution only (b) two solutions (c)  infinitely many solutions (d) no real solution

9. Solution of the equation  $2\sin x + \sqrt{3} = 0$  in the 4<sup>th</sup> quadrant is:

- (a)  $\frac{\pi}{2}$  (b)   $\frac{-\pi}{3}$  (c)  $\frac{-\pi}{6}$  (d)  $\frac{11\pi}{6}$

10. If  $\sin x = \cos x$ , then general solution is:

- (a)  $\{\frac{\pi}{4} + n\pi, n \in Z\}$  (b)  $\{\frac{\pi}{4} + 2n\pi, n \in Z\}$   
 (c)   $\{\frac{\pi}{4} + n\pi, \frac{5\pi}{4} + n\pi\}$  (d)  $\{\frac{\pi}{4} + n\pi, \frac{5\pi}{4} + n\pi\}$

11. In which quadrant is the solution of the equation  $\sin x + 1 = 0$

- (a) 1<sup>st</sup> and 2<sup>nd</sup> (b) 2<sup>nd</sup> and 3<sup>rd</sup> (c)  3<sup>rd</sup> and 4<sup>th</sup> (d) Only 1<sup>st</sup>

12. If  $\sin x = 0$  then  $x =$

- (a)   $n\pi, n \in Z$  (b)  $\frac{n\pi}{2}, n \in Z$  (c) 0 (d)  $\frac{\pi}{2}$

## SHORT QUESTIONS

- i. Solve  $\sin^2 x = \frac{3}{4}$  in  $[0, 2\pi]$
- ii. Solve  $1 + \cos x = 0$

- iii. Define trigonometric equation.
- iv. Solve  $\tan x = \frac{1}{\sqrt{3}}$
- v. Find the values of  $\theta$  satisfying the equation  $2\sin^2\theta - \sin\theta = 0$

**M.SALMAN SHERAZI**

03337727666/03067856232

*This document contains all the definitions of Mathematics HSSC-I (FSc Part 1) from the Textbook of Algebra and Trigonometry for Class XI. It has been done to help the students and teachers at no cost. This work (pdf) is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0.*

*Merging man and math*

*by*

**M. Salman Sherazi**