Bright Career Academy Narowal

TRIGONOMETRIC FORMULAS **COMPILED BY: MUZZAMMIL SUBHAN MPHIL(MATH)**

Relations & Trigonometric Ratios

- $l = r\theta$ • $180^\circ = \pi$ radian

where θ is measure in radian

- $1\text{radian} = \dfrac{180^\circ}{\pi} \approx 57.296^\circ$

$1^\circ = \dfrac{\pi}{180}\text{radian} \approx 0.0175\text{radian}$

- $\pi \approx 3.1416$ • $e \approx 2.718$
- $\sin\theta = \dfrac{p}{h} \rightleftarrows \operatorname{cosec}\theta = \dfrac{h}{p}$
- $\cos\theta = \dfrac{b}{h} \rightleftarrows \sec\theta = \dfrac{h}{b}$
- $\tan\theta = \dfrac{p}{b} \rightleftarrows \cot\theta = \dfrac{b}{p}$
- $\sin\theta = \dfrac{1}{\csc\theta} \rightleftarrows \csc\theta = \dfrac{1}{\sin\theta}$
- $\cos\theta = \dfrac{1}{\sec\theta} \rightleftarrows \sec\theta = \dfrac{1}{\cos\theta}$
- $\tan\theta = \dfrac{1}{\cot\theta} \rightleftarrows \cot\theta = \dfrac{1}{\tan\theta}$
- $\tan\theta = \dfrac{\sin\theta}{\cos\theta} \rightleftarrows \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Trigonometric Co-ratios

- $\sin\theta \rightleftarrows \cos\theta$
- $\sec\theta \rightleftarrows \operatorname{cosec}\theta$
- $\tan\theta \rightleftarrows \cot\theta$

Pythagorean Identities

- $h^2 = p^2 + b^2$
- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Half Angle Identities

- $\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$
- $\cos\theta = \cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}$
- $2\sin^2\dfrac{\theta}{2} = 1 - \cos\theta$
- $2\cos^2\dfrac{\theta}{2} = 1 + \cos\theta$

Double Angle Identities

- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

- $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
- $2\sin^2\theta = 1 - \cos 2\theta$
- $2\cos^2\theta = 1 + \cos 2\theta$

Triple Angle Identities

- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Signs of Trigonometric Functions

- $\cos(-\theta) = \cos\theta \rightleftarrows \sec(-\theta) = \sec\theta$
- $\sin(-\theta) = -\sin\theta \rightleftarrows \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
- $\tan(-\theta) = -\tan\theta \rightleftarrows \cot(-\theta) = -\cot\theta$

Fundamental Law Of Trigonometry

- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
- $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$
- $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

Express Products as Sums or Differences

- $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$
- $2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$
- $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$
- $-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

Express Sums or Differences as Products

- $\sin\alpha + \sin\beta = 2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$
- $\sin\alpha - \sin\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$
- $\cos\alpha + \cos\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$
- $\cos\alpha - \cos\beta = -2\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

Δ = Area of Triangle

- $\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}ca\sin\beta = \dfrac{1}{2}ab\sin\gamma$
- $\Delta = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \dfrac{b^2\sin\gamma\sin\alpha}{2\sin\beta} = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$
- $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ (Hero's Formula)

where • $s = \dfrac{a+b+c}{2}$ • $\alpha+\beta+\gamma = 180^\circ$

Inverse Trigonometric Formulas

- $\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$
- $\sin^{-1}A - \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$
- $\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$
- $\cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)$
- $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\dfrac{A+B}{1-AB}\right)$
- $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\dfrac{A-B}{1+AB}\right)$
- $2\tan^{-1}A = \tan^{-1}\left(\dfrac{2A}{1-A^2}\right)$

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Half Angle Formulas

- $\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$
- $\sin\dfrac{\beta}{2} = \sqrt{\dfrac{(s-c)(s-a)}{ca}}$
- $\sin\dfrac{\gamma}{2} = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$
- $\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$
- $\cos\dfrac{\beta}{2} = \sqrt{\dfrac{s(s-b)}{ca}}$
- $\cos\dfrac{\gamma}{2} = \sqrt{\dfrac{s(s-c)}{ab}}$
- $\tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$
- $\tan\dfrac{\beta}{2} = \sqrt{\dfrac{(s-c)(s-a)}{s(s-b)}}$
- $\tan\dfrac{\gamma}{2} = \sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$

Different Radii

- $R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma}$
- $R = \dfrac{abc}{4\Delta}$
- $r = \dfrac{\Delta}{s}$ • $r_1 = \dfrac{\Delta}{s-a}$
- $r_2 = \dfrac{\Delta}{s-b}$ • $r_3 = \dfrac{\Delta}{s-c}$

The Law of Sines

- $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

The Law of Cosines

- $a^2 = b^2 + c^2 - 2bc\cos\alpha$
- $b^2 = c^2 + a^2 - 2ca\cos\beta$
- $c^2 = a^2 + b^2 - 2ab\cos\gamma$

The Law of Tangents

- $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\dfrac{\alpha-\beta}{2}\right)}{\tan\left(\dfrac{\alpha+\beta}{2}\right)}$
- $\dfrac{b-c}{b+c} = \dfrac{\tan\left(\dfrac{\beta-\gamma}{2}\right)}{\tan\left(\dfrac{\beta+\gamma}{2}\right)}$
- $\dfrac{c-a}{c+a} = \dfrac{\tan\left(\dfrac{\gamma-\alpha}{2}\right)}{\tan\left(\dfrac{\gamma+\alpha}{2}\right)}$

Types of Angles

Acute Angle: $0 < \theta < 90^\circ$

Right Angle: $\theta = 90^\circ$

Obtuse Angle: $90^\circ < \theta < 180^\circ$

Straight Angle: $\theta = 180^\circ$

Reflex Angle: $180^\circ < \theta < 360^\circ$

Rotation/Full Angle: $\theta = 360^\circ$

Period of Trigonometric Functions

Function	Period
sin x , cos x , sec x & cosec x	2π
tan x & cot x	π

Domains & Ranges of Trigonometric Functions

Function	Domian	Range
y = sin x	$-\infty < x < +\infty$	$-1 \le y \le 1$
y = cos x	$-\infty < x < +\infty$	$-1 \le y \le 1$
y = tan x	$-\infty < x < +\infty$, $x \ne (2n+1)\dfrac{\pi}{2}$	$-\infty < y < +\infty$
y = sec x	$-\infty < x < +\infty$, $x \ne (2n+1)\dfrac{\pi}{2}$	$y \le -1$ or $y \ge 1$
y = cosec x	$-\infty < x < +\infty$, $x \ne n\pi$	$y \le -1$ or $y \ge 1$
y = cot x	$-\infty < x < +\infty$, $x \ne n\pi$, $n \in Z$	$-\infty < y < +\infty$

Domains & Ranges of Inverse Trigonometric Functions

Function	Domian	Range
$y = \sin^{-1}x$	$-1 \le x \le 1$	$-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$
$y = \tan^{-1}x$	$-\infty < x < +\infty$	$-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$
$y = \operatorname{cosec}^{-1}x$	$x \le -1$ or $x \ge 1$	$-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$, $y \ne 0$
$y = \cos^{-1}x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \cot^{-1}x$	$-\infty < x < +\infty$	$0 < y < \pi$
$y = \sec^{-1}x$	$x \le -1$ or $x \ge 1$	$0 \le y \le \pi$, $y \ne \dfrac{\pi}{2}$

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