F.Sc.(Part-1)
TRIGONOMETRY
CH # 9---14

IMPORTANT—FORMULAE

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Relation Between l & θ

$$l = r\theta$$

Conversion of Radian

$$1^\circ = \frac{\pi}{180} radian \quad \& \quad 1\, radian = \frac{180^\circ}{\pi}$$

Fundamental Identities

1. $sin^2\theta + cos^2\theta = 1$
2. $1 + tan^2\theta = sec^2\theta$
3. $1 + cot^2\theta = csc^2\theta$

Signs of Trigonometric Function

$\sin$ & $\csc$ $+ve$	$all + ve$
II-Quad.	I-Quad.+
III-Quad.	IV-Quad.
tan & $cot + ve$	$\cos$ & $\sec +ve$

Values of Trigonometric Functions

θ	0°	$30^\circ = \pi/6$	$45^\circ = \pi/4$	$60^\circ = \pi/3$	$90^\circ = \pi/2$
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

Fundamental Laws of Trigonometry

1. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
4. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
5. $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
6. $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double Angle Identities

1. $\sin 2\alpha = 2\sin\alpha\cos\alpha$
2. $\cos 2\alpha = \begin{cases} cos^2\alpha - sin^2\alpha \\ 2cos^2\alpha - 1 \\ 1 - 2sin^2\alpha \end{cases}$
3. $\tan 2\alpha = \frac{2\tan\alpha}{1 - tan^2\alpha}$

Triple Angle Identities

1. $\sin 3\alpha = 3\sin\alpha - 4sin^3\alpha$
2. $\cos 3\alpha = 4cos^3\alpha - 3\cos\alpha$
3. $\tan 3\alpha = \frac{3\tan\alpha - tan^3\alpha}{1 - 3tan^2\alpha}$

Sum, Difference & Product

1. $2\sin\alpha\cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
2. $2\cos\alpha\sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$
3. $2\cos\alpha\cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$
4. $-2\sin\alpha\sin\beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$
5. $\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$
6. $\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$
7. $\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$
8. $\cos P - \cos Q = -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$

Domain & Range of Trig. Functions

Functions	Domain	Range
$y = \sin x$	$-\infty < x < +\infty$	$-1 \le y \le 1$
$y = \cos x$	$-\infty < x < +\infty$	$-1 \le y \le 1$
$y = \tan x$	$-\infty < x < +\infty$ $x \ne \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \cot x$	$-\infty < x < +\infty$ $x \ne \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \sec x$	$-\infty < x < +\infty$ $x \ne n\pi$	$y \ge 1$ or $y \le -1$
$y = \csc x$	$-\infty < x < +\infty$ $x \ne n\pi$	$y \ge 1$ or $y \le -1$

where $n \in z$.

Period of Trigonometric Functions

Function	Period
$\sin a\theta$	$\frac{2\pi}{a}$
$\cos a\theta$	
$\csc a\theta$	
$\sec a\theta$	
$\tan a\theta$	$\frac{\pi}{a}$
$\cot a\theta$	

The Law of Sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

The Law of Cosine

1. $a^2 = b^2 + c^2 - 2bc\cos\alpha$
2. $b^2 = c^2 + a^2 - 2ca\cos\beta$
3. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

The Law of Tangents

1. $\frac{a-b}{a+b} = \frac{\tan(\frac{\alpha-\beta}{2})}{\tan(\frac{\alpha+\beta}{2})}$
2. $\frac{b-c}{b+c} = \frac{\tan(\frac{\beta-\gamma}{2})}{\tan(\frac{\beta+\gamma}{2})}$
3. $\frac{c-a}{c+a} = \frac{\tan(\frac{\gamma-\alpha}{2})}{\tan(\frac{\gamma+\alpha}{2})}$

Half Angle Formulas

1. $\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
2. $\sin\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$
3. $\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$
4. $\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$
5. $\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}}$
6. $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$
7. $\tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
8. $\tan\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$
9. $\tan\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

Area of Triangles (Δ)

1. $\Delta = \frac{1}{2}bc\sin\alpha = \frac{1}{2}ca\sin\beta = \frac{1}{2}ab\sin\gamma$
2. $\Delta = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \frac{b^2\sin\gamma\sin\alpha}{2\sin\beta} = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$
3. $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$where \quad s = \frac{a+b+c}{2}$

Circum – Circle (R)

1. $R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma}$
2. $R = \frac{abc}{4\Delta}$

In – Circle (r)

$$r = \frac{\Delta}{s}$$

Escribed – Circle

$$r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$$

Inverse Trigonometr Formulas

$$\sin^{-1}A + \sin^{-1}B = \sin^{-1}(A\sqrt{1-B^2} + B\sqrt{1-A^2})$$

$$\sin^{-1}A - \sin^{-1}B = \sin^{-1}(A\sqrt{1-B^2} + B\sqrt{1-A^2})$$

$$\cos^{-1}A + \cos^{-1}B = \cos^{-1}(AB - \sqrt{(1-A^2)(1-B^2)})$$

$$\cos^{-1}A - \cos^{-1}B = \cos^{-1}(AB + \sqrt{(1-A^2)(1-B^2)})$$

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}(\frac{A+B}{1-AB})$$

$$\tan^{-1}A - \tan^{-1}B = \tan^{-1}(\frac{A-B}{1+AB})$$

$$2\tan^{-1}A = \tan^{-1}(\frac{2A}{1-A^2})$$

Principal Trig. Functions

Function	Domain	Range
$y = sinx$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$-1 \le y \le 1$
$y = \sin^{-1}x$	$-1 \le y \le 1$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$
$y = cosx$	$0 \le x \le \pi$	$-1 \le y \le 1$
$y = \cos^{-1}x$	$-1 \le y \le 1$	$0 \le x \le \pi$
$y = tanx$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	R
$y = \tan^{-1}x$	R	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
$y = cotx$	$0 < x < \pi$	R
$y = \cot^{-1}x$	R	$0 < x < \pi$
$y = secx$	$[0, \pi], x \ne \frac{\pi}{2}$	$y \le -1 \text{ or } y \ge 1$
$y = \sec^{-1}x$	$x \le -1 \text{ or } x \ge 1$	$[0, \pi], y \ne \frac{\pi}{2}$
$y = \csc x$	$[-\frac{\pi}{2}, \frac{\pi}{2}], x \ne 0$	$y \le -1 \text{ or } y \ge 1$
$y = \csc^{-1}x$	$x \le -1 \text{ or } x \ge 1$	$[-\frac{\pi}{2}, \frac{\pi}{2}], y \ne 0$

Best of Luck

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