Chapter # 03   *Matrices and Determinants*

> **Following short questions are selected from previous 5 years papers of different boards. Solve these at your own to perform well in annual exams.**

1. Define the diagonal matrix with an example.

2. Define the transpose of a matrix?

3. Define the rank of a matrix.

4. Under what condition two matrices A and B are conformable for the product AB? Also what will be the order of AB? .

5. If A and B are symmetric and $AB = BA$, show that AB is symmetric.

6. If A is a symmetric matrix, show that A^2 is symmetric.

7. If A is any square matrix of arbitrary order, then show that $A - (\overline{A})^t$ is skew-hermition.

8. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$, show that $A + A^t$ is symmetric.

9. If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$.

10. Without expansion, verify that $\begin{vmatrix} \alpha & \beta + \gamma & 1 \\ \beta & \gamma + \alpha & 1 \\ \gamma & \alpha + \beta & 1 \end{vmatrix} = 0$.

11. Without expansion, verify that $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 0$.

12. Without expansion, verify that $\begin{vmatrix} 1 & a^2 & \frac{a}{bc} \\ 1 & b^2 & \frac{b}{ac} \\ 1 & c^2 & \frac{c}{ab} \end{vmatrix} = 0$.

13. Without expansion, verify that $\begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = 0$.

14. Without expansion, verify that $\begin{vmatrix} a+l & a & a \\ a & a+l & a \\ a & a & a+l \end{vmatrix} = l^2(3a + l)$.

You know my methods. Apply them. *Sherlock Holmes*

15. Find λ if $\begin{bmatrix} 4 & \lambda \\ 7 & 3 \end{bmatrix}$ is singular.

16. Find $|A|$, if $A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 4 & 2 \\ 1 & 2 & -2 \end{bmatrix}$.

17. Find the matrix A, if $A \begin{bmatrix} 5 & 2 \\ -2 & \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$.

18. Find the matrix A, if $\begin{bmatrix} 4 & 3 \\ -2 & 2 \end{bmatrix} A = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$.

19. Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$.

20. Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$.

21. Find A^{-1}, if $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$.

22. Find A^{-1} if $A = \begin{bmatrix} 2i & i \\ i & 1 \end{bmatrix}$.

23. If $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, then find $(\overline{A})^t A$.

24. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 0 & -4 \\ 2 & -1 & 3 \end{bmatrix}$, then find A_{13}, A_{22}, A_{31} and A_{32}.

25. If A and B are non singular matrices, then show that $(AB)^{-1} = B^{-1}A^{-1}$.

26. If A and B are square matrices, then explain why in general $(A+B)^2 \neq A^2 + 2AB + B^2$.

<div align="center">Best of Luck</div>