

### Question # 1

Prove that:

$$(1) \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{7}{25} = \cos^{-1} \frac{253}{325}$$

**Solution**

$$\begin{aligned} \text{L.H.S} &= \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{7}{25} \\ &\because \sin^{-1} A + \sin^{-1} B = \sin^{-1} \left( A\sqrt{1-B^2} + B\sqrt{1-A^2} \right) \\ &= \sin^{-1} \left( \frac{5}{13} \sqrt{1 - \left( \frac{7}{25} \right)^2} + \frac{7}{25} \sqrt{1 - \left( \frac{5}{13} \right)^2} \right) \\ &= \sin^{-1} \left( \frac{5}{13} \sqrt{1 - \frac{49}{625}} + \frac{7}{25} \sqrt{1 - \frac{25}{169}} \right) \\ &= \sin^{-1} \left( \frac{5}{13} \sqrt{\frac{576}{625}} + \frac{7}{25} \sqrt{\frac{144}{169}} \right) = \sin^{-1} \left( \frac{5}{13} \left( \frac{24}{25} \right) + \frac{7}{25} \left( \frac{12}{13} \right) \right) \\ &= \sin^{-1} \left( \frac{120}{325} + \frac{84}{325} \right) = \sin^{-1} \left( \frac{204}{325} \right) \\ &= \frac{\pi}{2} - \cos^{-1} \left( \frac{204}{325} \right) \quad \because \sin^{-1} \theta = \frac{\pi}{2} - \cos^{-1} \theta \\ &= \cos^{-1} (0) - \cos \left( \frac{204}{325} \right) \quad \because \frac{\pi}{2} = \cos^{-1} (0) \\ &\therefore \cos^{-1} A - \cos^{-1} B = \cos^{-1} \left( AB + \sqrt{(1-A^2)(1-B^2)} \right) \\ &= \cos^{-1} \left( (0) \left( \frac{204}{325} \right) + \sqrt{\left( 1 - (0)^2 \right) \left( 1 - \left( \frac{204}{325} \right)^2 \right)} \right) \\ &= \cos^{-1} \left( 0 + \sqrt{\left( 1 - 0 \right) \left( 1 - \frac{41616}{105625} \right)} \right) = \cos^{-1} \left( \sqrt{\left( 1 \right) \left( \frac{64009}{105625} \right)} \right) \\ &= \cos^{-1} \left( \sqrt{\frac{64009}{105625}} \right) = \cos^{-1} \frac{253}{325} = \text{R.H.S} \end{aligned}$$

### Question # 2

$$\text{Prove that: } \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} = \tan^{-1} \left( \frac{9}{19} \right)$$

**Solution**

$$\text{L.H.S} = \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5}$$

$$\begin{aligned}
 &= \tan^{-1} \left( \frac{\frac{1}{4} + \frac{1}{5}}{1 - \left( \frac{1}{4} \right) \left( \frac{1}{5} \right)} \right) \quad \because \tan^{-1} A + \tan^{-1} B = \tan^{-1} \left( \frac{A+B}{1-AB} \right) \\
 &= \tan^{-1} \left( \frac{\frac{9}{20}}{1 - \frac{1}{20}} \right) = \tan^{-1} \left( \frac{\frac{9}{20}}{\frac{19}{20}} \right) \\
 &= \tan^{-1} \left( \frac{9}{19} \right) = \text{R.H.S}
 \end{aligned}$$


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### Question # 3

Prove that:

$$2 \tan^{-1} \frac{2}{3} = \sin^{-1} \frac{12}{13}$$

**Solution** Suppose

$$\alpha = \sin^{-1} \frac{12}{13} \quad \dots \dots \dots \text{(i)}$$

$$\Rightarrow \sin \alpha = \frac{12}{13}$$

$$\text{Now } \cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \left( \frac{12}{13} \right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\text{Now } \tan \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \sqrt{\frac{1 - \frac{5}{13}}{1 + \frac{5}{13}}} = \sqrt{\frac{\frac{8}{13}}{\frac{18}{13}}} = \sqrt{\frac{8}{18}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$\Rightarrow \frac{\alpha}{2} = \tan^{-1} \left( \frac{2}{3} \right) \Rightarrow \alpha = 2 \tan^{-1} \frac{2}{3} \quad \dots \dots \dots \text{(ii)}$$

from (i) and (ii)

$$2 \tan^{-1} \frac{2}{3} = \sin^{-1} \frac{12}{13} \quad \text{proved.}$$


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### Question # 4

Prove that:

$$\tan^{-1} \frac{120}{119} = 2 \cos^{-1} \frac{12}{13}$$

**Solution** Suppose

$$\alpha = \tan^{-1} \frac{120}{119} \quad \dots \dots \dots \text{(i)}$$

$$\Rightarrow \tan \alpha = \frac{120}{119}$$

$$\text{Now } \sec \alpha = \sqrt{1 + \tan^2 \alpha}$$

$$\begin{aligned}
 &= \sqrt{1 + \left(\frac{120}{119}\right)^2} = \sqrt{1 + \frac{14400}{14161}} \\
 &= \sqrt{\frac{28561}{14161}} = \frac{169}{119}
 \end{aligned}$$

So  $\cos \alpha = \frac{1}{\sec \alpha} = \frac{1}{\cancel{169}/\cancel{119}} = \frac{119}{169}$

Now  $\cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}} = \sqrt{\frac{1 + 119/169}{2}} = \sqrt{\frac{288/169}{2}} = \sqrt{\frac{288}{2 \times 169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$$\Rightarrow \frac{\alpha}{2} = \cos^{-1} \frac{12}{13} \quad \Rightarrow \quad \alpha = 2 \cos^{-1} \frac{12}{13} \quad \dots \dots \dots \text{(ii)}$$

From (i) and (ii)

$$\tan^{-1} \frac{120}{119} = 2 \cos^{-1} \frac{12}{13} \quad \text{proved.}$$


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### Question # 5

Prove that:

$$\sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$$

**Solution** Suppose

$$\alpha = \sin^{-1} \frac{1}{\sqrt{5}} \quad \dots \dots \dots \text{(i)}$$

$$\Rightarrow \sin \alpha = \frac{1}{\sqrt{5}}$$

Now  $\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{1 - \frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

So  $\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\cancel{1}/\sqrt{5}}{\cancel{2}/\sqrt{5}} = \frac{1}{2}$

$$\Rightarrow \alpha = \tan^{-1} \frac{1}{2} \quad \dots \dots \dots \text{(ii)}$$

From (i) and (ii)

$$\sin^{-1} \frac{1}{\sqrt{5}} = \tan^{-1} \frac{1}{2}$$

Now  $\cot^{-1} 3 = \tan^{-1} \frac{1}{3} \quad \because \cot^{-1} x = \tan^{-1} \frac{1}{x}$

And L.H.S =  $\sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3$

$$\begin{aligned}
 &= \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} \\
 &= \tan^{-1} \left( \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left( \frac{1}{2} \right) \left( \frac{1}{3} \right)} \right) = \tan^{-1} \left( \frac{\frac{5}{6}}{1 - \frac{1}{6}} \right) \\
 &= \tan^{-1} \left( \frac{\frac{5}{6}}{\frac{5}{6}} \right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S} \quad \text{proved.}
 \end{aligned}$$


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### Question # 6

Prove that:

$$\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{8}{17} = \sin^{-1} \left( \frac{77}{85} \right)$$

$$\begin{aligned}
 \text{Solution} \quad \text{L.H.S} &= \sin^{-1} \frac{3}{5} + \sin^{-1} \frac{8}{17} \\
 &= \sin^{-1} \left( \frac{3}{5} \sqrt{1 - \left( \frac{8}{17} \right)^2} + \frac{8}{17} \sqrt{1 - \left( \frac{3}{5} \right)^2} \right) \\
 &= \sin^{-1} \left( \frac{3}{5} \sqrt{1 - \frac{64}{289}} + \frac{8}{17} \sqrt{1 - \frac{9}{25}} \right) = \sin^{-1} \left( \frac{3}{5} \sqrt{\frac{225}{289}} + \frac{8}{17} \sqrt{\frac{16}{25}} \right) \\
 &= \sin^{-1} \left( \frac{3}{5} \left( \frac{15}{17} \right) + \frac{8}{17} \left( \frac{4}{5} \right) \right) = \sin^{-1} \left( \frac{45}{85} + \frac{32}{85} \right) \\
 &= \sin^{-1} \left( \frac{77}{85} \right) = \text{R.H.S} \quad \text{proved.}
 \end{aligned}$$


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### Question # 7

Prove that:

$$\sin^{-1} \frac{77}{85} - \sin^{-1} \frac{3}{5} = \cos^{-1} \left( \frac{15}{17} \right)$$

$$\begin{aligned}
 \text{Solution} \quad \text{L.H.S} &= \sin^{-1} \frac{77}{85} - \sin^{-1} \frac{3}{5} \\
 &= \left( \frac{\pi}{2} - \cos^{-1} \frac{77}{85} \right) - \left( \frac{\pi}{2} - \cos^{-1} \frac{3}{5} \right) \quad \because \sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x \\
 &= \frac{\pi}{2} - \cos^{-1} \frac{77}{85} - \frac{\pi}{2} + \cos^{-1} \frac{3}{5} = \cos^{-1} \frac{3}{5} - \cos^{-1} \frac{77}{85} \\
 &= \cos^{-1} \left( \left( \frac{3}{5} \right) \left( \frac{77}{85} \right) + \sqrt{\left( 1 - \left( \frac{3}{5} \right)^2 \right) \left( 1 - \left( \frac{77}{85} \right)^2 \right)} \right) \\
 &= \cos^{-1} \left( \frac{231}{425} + \sqrt{\left( 1 - \frac{9}{25} \right) \left( 1 - \frac{5929}{7225} \right)} \right)
 \end{aligned}$$

$$\begin{aligned}
&= \cos^{-1} \left( \frac{231}{425} + \sqrt{\left(\frac{16}{25}\right)\left(\frac{1296}{7225}\right)} \right) \\
&= \cos^{-1} \left( \frac{231}{425} + \sqrt{\frac{20736}{180625}} \right) = \cos^{-1} \left( \frac{231}{425} + \frac{144}{425} \right) \\
&= \cos^{-1} \left( \frac{375}{425} \right) = \cos^{-1} \left( \frac{15}{17} \right) = \text{L.H.S}
\end{aligned}$$


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**Question # 8**

Prove that:

$$\cos^{-1} \frac{63}{65} + 2 \tan^{-1} \frac{1}{5} = \sin^{-1} \left( \frac{3}{5} \right)$$

**Solution** Suppose  $\alpha = \cos^{-1} \frac{63}{65}$  ..... (i)

$$\Rightarrow \cos \alpha = \frac{63}{65}$$

Now  $\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \left(\frac{63}{65}\right)^2} = \sqrt{1 - \frac{3969}{4225}} = \sqrt{\frac{256}{4225}} = \frac{16}{65}$

$$\Rightarrow \alpha = \sin^{-1} \left( \frac{16}{65} \right) \text{ ..... (ii)}$$

So from equation (i) and (ii)

$$\cos^{-1} \left( \frac{63}{65} \right) = \sin^{-1} \left( \frac{16}{65} \right)$$

Now suppose  $\beta = \tan^{-1} \frac{1}{5}$  ..... (iii)

$$\Rightarrow \tan \beta = \frac{1}{5}$$

So  $\sec \alpha = \sqrt{1 + \tan^2 \alpha} = \sqrt{1 + \left(\frac{1}{5}\right)^2} = \sqrt{1 + \frac{1}{25}} = \sqrt{\frac{26}{25}} = \frac{\sqrt{26}}{5}$

So  $\cos \beta = \frac{1}{\sec \beta} = \frac{1}{\sqrt{\frac{26}{5}}} = \frac{5}{\sqrt{26}}$

As  $\frac{\sin \beta}{\cos \beta} = \tan \beta \Rightarrow \sin \beta = \tan \beta \cdot \cos \beta$

$$\Rightarrow \sin \beta = \left(\frac{1}{5}\right) \left(\frac{5}{\sqrt{26}}\right) = \frac{1}{\sqrt{26}}$$

$$\Rightarrow \beta = \sin^{-1} \frac{1}{\sqrt{26}} \text{ ..... (iv)}$$

From (iii) and (iv)

$$\tan^{-1} \frac{1}{5} = \sin^{-1} \frac{1}{\sqrt{26}}$$

$$\text{Now L.H.S} = \cos^{-1} \frac{63}{65} + 2 \tan^{-1} \frac{1}{5}$$

$$= \sin^{-1} \frac{16}{65} + 2 \sin^{-1} \frac{1}{\sqrt{26}} = \sin^{-1} \frac{16}{65} + \left( \sin^{-1} \frac{1}{\sqrt{26}} + \sin^{-1} \frac{1}{\sqrt{26}} \right)$$

$$= \sin^{-1} \frac{16}{65} + \sin^{-1} \left( \frac{1}{\sqrt{26}} \sqrt{1 - \left( \frac{1}{\sqrt{26}} \right)^2} + \frac{1}{\sqrt{26}} \sqrt{1 - \left( \frac{1}{\sqrt{26}} \right)^2} \right)$$

$$= \sin^{-1} \frac{16}{65} + \sin^{-1} \left( \frac{1}{\sqrt{26}} \sqrt{1 - \frac{1}{26}} + \frac{1}{\sqrt{26}} \sqrt{1 - \frac{1}{26}} \right)$$

$$= \sin^{-1} \frac{16}{65} + \sin^{-1} \left( \frac{1}{\sqrt{26}} \sqrt{\frac{25}{26}} + \frac{1}{\sqrt{26}} \sqrt{\frac{25}{26}} \right)$$

$$= \sin^{-1} \frac{16}{65} + \sin^{-1} \left( \frac{1}{\sqrt{26}} \cdot \frac{5}{\sqrt{26}} + \frac{1}{\sqrt{26}} \cdot \frac{5}{\sqrt{26}} \right)$$

$$= \sin^{-1} \frac{16}{65} + \sin^{-1} \left( \frac{5}{26} + \frac{5}{26} \right) = \sin^{-1} \frac{16}{65} + \sin^{-1} \left( \frac{5}{13} \right)$$

$$= \sin^{-1} \left( \frac{16}{65} \sqrt{1 - \left( \frac{5}{13} \right)^2} + \frac{5}{13} \sqrt{1 - \left( \frac{16}{65} \right)^2} \right)$$

$$= \sin^{-1} \left( \frac{16}{65} \sqrt{1 - \frac{25}{169}} + \frac{5}{13} \sqrt{1 - \frac{256}{4225}} \right)$$

$$= \sin^{-1} \left( \frac{16}{65} \sqrt{1 - \frac{25}{169}} + \frac{5}{13} \sqrt{1 - \frac{256}{4225}} \right) = \sin^{-1} \left( \frac{16}{65} \sqrt{\frac{144}{169}} + \frac{5}{13} \sqrt{\frac{3969}{4225}} \right)$$

$$= \sin^{-1} \left( \frac{16}{65} \sqrt{\frac{144}{169}} + \frac{5}{13} \sqrt{\frac{3969}{4225}} \right) = \sin^{-1} \left( \frac{16}{65} \left( \frac{12}{13} \right) + \frac{5}{13} \left( \frac{63}{65} \right) \right)$$

$$= \sin^{-1} \left( \frac{192}{845} + \frac{315}{845} \right) = \sin^{-1} \left( \frac{3}{5} \right) = \text{R.H.S}$$

### Question # 9

Prove that:

$$\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5} - \tan^{-1} \frac{8}{19} = \frac{\pi}{4}$$

**Solution**      L.H.S =  $\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5} - \tan^{-1} \frac{8}{19}$

$$\begin{aligned}
&= \tan^{-1} \left( \frac{\cancel{3/4} + \cancel{3/5}}{1 - (\cancel{3/4})(\cancel{3/5})} \right) - \tan^{-1} \frac{8}{19} \\
&= \tan^{-1} \left( \frac{\cancel{27/20}}{1 - \cancel{9/20}} \right) - \tan^{-1} \frac{8}{19} = \tan^{-1} \left( \frac{\cancel{27/20}}{\cancel{11/20}} \right) - \tan^{-1} \frac{8}{19} \\
&= \tan^{-1} \left( \frac{27}{11} \right) - \tan^{-1} \left( \frac{8}{19} \right) = \tan^{-1} \left( \frac{\cancel{27/11} - \cancel{8/19}}{1 + (\cancel{27/11})(\cancel{8/19})} \right) \\
&= \tan^{-1} \left( \frac{\cancel{425/209}}{1 + \cancel{216/209}} \right) = \tan^{-1} \left( \frac{\cancel{425/209}}{1 + \cancel{216/209}} \right) \\
&= \tan^{-1} \left( \frac{\cancel{425/209}}{\cancel{425/209}} \right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S} \quad \text{proved.}
\end{aligned}$$

**Question # 10***Do Yourself***Question # 11**

Prove that:

$$\tan^{-1} \frac{1}{11} + \tan^{-1} \frac{5}{6} = \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2}$$

**Solution**      L.H.S =  $\tan^{-1} \frac{1}{11} + \tan^{-1} \frac{5}{6}$   
 $= \text{Solve this} \dots \dots \dots \text{(i)}$

R.H.S =  $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2}$   
 $= \text{Solve this} \dots \dots \dots \text{(ii)}$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

**Question # 12**

Prove that:

$$2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$$

**Solution**      L.H.S =  $2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7}$   
 $= \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7}$

$$\begin{aligned}
 &= \tan^{-1} \left( \frac{\frac{1}{3} + \frac{1}{3}}{1 - \frac{\frac{1}{3} \cdot \frac{1}{3}}{3 \cdot 3}} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left( \frac{\frac{1+1}{3}}{1 - \frac{1}{9}} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left( \frac{\frac{2}{3}}{\frac{9-1}{9}} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left( \frac{\frac{2}{3}}{\frac{8}{9}} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left( \frac{2 \times 9}{3 \times 8} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left( \frac{3}{4} \right) + \tan^{-1} \frac{1}{7} \Rightarrow \tan^{-1} \left( \frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}} \right) \\
 &= \tan^{-1} \left( \frac{\frac{21+4}{28}}{1 - \frac{3}{28}} \right) \Rightarrow \tan^{-1} \left( \frac{\frac{25}{28}}{\frac{28-3}{28}} \right) \\
 &= \tan^{-1} \left( \frac{\frac{25}{28}}{\frac{25}{28}} \right) \Rightarrow \tan^{-1}(1) \\
 &= \frac{\pi}{4} = R.H.S.
 \end{aligned}$$

### Question # 13

Show that:

$$\cos(\sin^{-1} x) = \sqrt{1 - x^2}$$

**Solution** Suppose  $y = \sin^{-1} x$

$$\Rightarrow \sin y = x$$

$$\text{Since } \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

$$\Rightarrow \cos(\sin^{-1} x) = \sqrt{1-x^2} \quad \text{Proved}$$


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**Question # 14**

Show that:

$$\sin(2\cos^{-1} x) = 2x\sqrt{1-x^2}$$

**Solution** Suppose  $y = \cos^{-1} x$   
 Then  $\cos y = x$   
 Also  $\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-x^2}$   
 Now  $\begin{aligned} \sin(2\cos^{-1} x) &= \sin(2y) \\ &= 2\sin y \cdot \cos y \\ &= 2\sqrt{1-x^2} \cdot x \\ &= 2x\sqrt{1-x^2} \end{aligned}$

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**Question # 15**

Show that:

$$\cos(2\sin^{-1} x) = 1-2x^2$$

**Solution** Suppose  $y = \sin^{-1} x \Rightarrow \sin y = x$   
 &  $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}$   
 Now  $\begin{aligned} \cos(2\sin^{-1} x) &= \cos 2y \\ &= \cos^2 y - \sin^2 y \\ &= (\sqrt{1-x^2})^2 - x^2 = 1-x^2 - x^2 \\ &= 1-2x^2 \quad \text{Proved} \end{aligned}$

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**Question # 16**

Show that:

$$\tan^{-1}(-x) = -\tan^{-1} x$$

**Solution** Suppose  $y = \tan^{-1}(-x)$  ..... (i)  
 $\Rightarrow \tan y = -x \Rightarrow -\tan y = x$   
 $\Rightarrow \tan(-y) = x \quad \because -\tan \theta = \tan(-\theta)$   
 $\Rightarrow -y = \tan^{-1} x$   
 $\Rightarrow y = -\tan^{-1} x$  ..... (ii)

From equation (i) and (ii)

$$\tan^{-1}(-x) = -\tan^{-1} x \quad \text{Proved}$$


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**Question # 17**

Do yourself as above

**Question # 18**

Show that:

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

**Solution** Suppose  $y = \pi - \cos^{-1} x$  ..... (i)

$$\Rightarrow \pi - y = \cos^{-1} x \Rightarrow \cos(\pi - y) = x$$

$$\Rightarrow \cos\pi\cos y + \sin\pi\sin y = x \Rightarrow (-1)\cos y + (0)\sin y = x$$

$$\Rightarrow -\cos y + 0 = x \Rightarrow -\cos y = x$$

$$\Rightarrow \cos y = -x \Rightarrow y = \cos^{-1}(-x)$$
 ..... (ii)

From (i) and (ii)

$$\cos^{-1}(-x) = \pi - \cos^{-1} x \quad \text{Proved}$$


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### Question # 19

Show that:

$$\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$$

**Solution** Suppose  $y = \sin^{-1} x \Rightarrow \sin y = x$

Now  $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$

&  $\tan y = \frac{\sin y}{\cos y} = \frac{x}{\sqrt{1-x^2}}$

Now  $\tan(\sin^{-1} x) = \tan y = \frac{x}{\sqrt{1-x^2}}$  proved

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### Question # 20

Given that  $x = \sin^{-1} \frac{1}{2}$ , find the values of the following trigonometric functions:

$\sin x, \cos x, \tan x, \cot x, \sec x$  and  $\csc x$ .

**Solution** Since  $x = \sin^{-1} \frac{1}{2} \Rightarrow \sin x = \frac{1}{2}$

Now  $\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}, \quad \cot x = \frac{1}{\tan x} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}, \quad \cosec x = \frac{1}{\sin x} = \frac{1}{\frac{1}{2}} = 2$$


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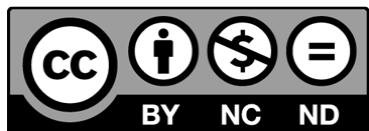
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