EXERCISE 2.6

1. For $A = \{1, 2, 3, 4\}$ find the following relations in A. State the domain and range of each relation. Also draw the graph of each.

(i) $\{(x, y) \mid y = x\}$

$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Dom $R_1 = \{1, 2, 3, 4\} = A$

Range $R_1 = \{1, 2, 3, 4\} = A$

(ii) $R_2 = \{(x, y) \mid y + x = 5\}$

$R_2 = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

Dom $R_2 = \{1, 2, 3, 4\} = A$

Range $R_2 = \{1, 2, 3, 4\} = A$

(iii) $R_3 = \{(x, y) \mid x + y < 5\}$

$R_3 = \{(1, 2), (1, 3), (2, 1), (3, 1), (2, 2), (1, 1)\}$

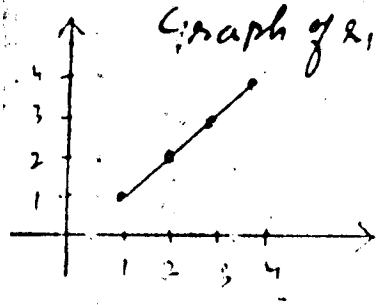
Dom $(R_3) = \{1, 2, 3\}$ Range $R_3 = \{1, 2, 3\}$

(iv) $R_4 = \{(x, y) \mid x + y > 5\}$

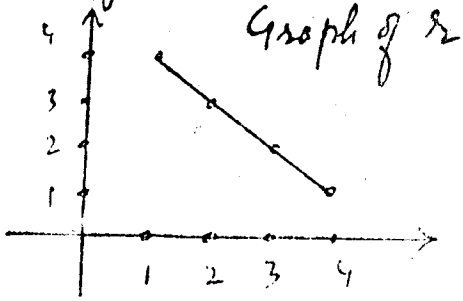
$R_4 = \{(2, 4), (3, 3), (4, 3), (3, 4), (4, 2), (4, 4)\}$

Dom $R_4 = \{2, 3, 4\}$ Range $R_4 = \{2, 3, 4\}$

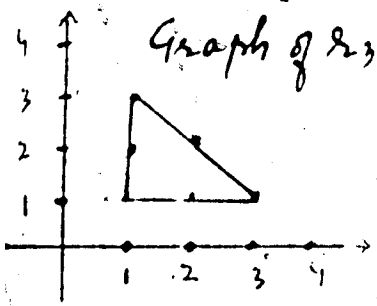
Graph of R_1



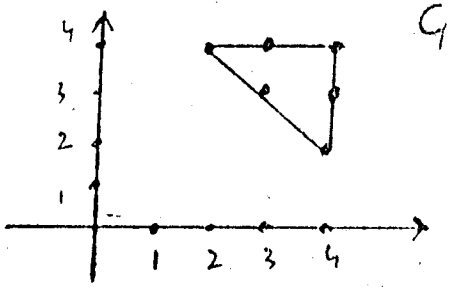
Graph of R_2



Graph of R_3



Graph of R_4



2. Repeat Q.1 when $A = \mathbb{R}$ Set of real Numbers which of the real lines are functions.

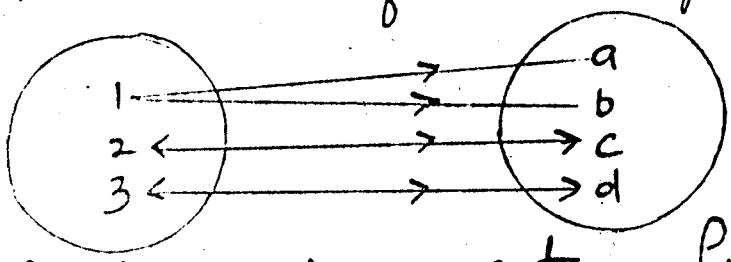
$R_1 = \{(x, y) \mid y = x\}$ is a function

$R_2 = \{(x, y) \mid x + y = 5\}$ is a function

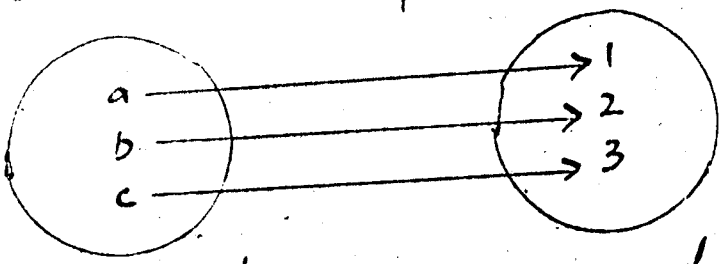
$R_3 = \{(x, y) \mid x + y < 5\}$ is not a function because Domain is repeated

$R_4 = \{(x, y) \mid x+y > 5\}$ is not a function because Domain is repeated.

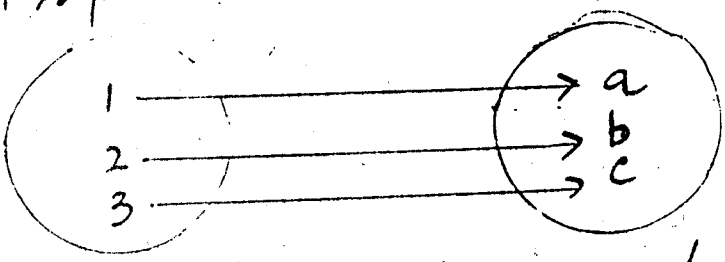
3. Which of the following diagrams represe functions and of which type.



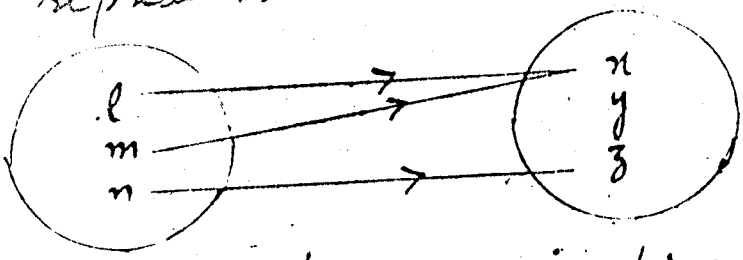
It doesn't not represents a function



It represents one-to-one and onto function



It represents one-to-one and onto function



It represents an injective (Into) functi

4. Find the inverse of each of the following relations. Tell whether each relation and inverse is a function or not.

(i) $R = \{(2,1), (3,2), (4,3), (5,4), (6$

'R' is a function $Dom(R) = \{2, 3, 4, 5, 6\}$

$R^{-1} = \{(1,2), (2,3), (3,4), (4,5), (5,6)$

R^{-1} is also a function with

$Dom(R^{-1}) = \{1, 2, 3, 4, 5\}$

(ii) $R = \{(1,3), (2,5), (3,7), (4,9), (5,11)\}$

'R' is a function $\quad Dom(R) = \{1, 2, 3, 4, 5\}$

$R^{-1} = \{(3,1), (5,2), (7,3), (9,4), (11,5)\}$

R^{-1} is also a function

As domain is not repeated.

(iii) $R = \{(x, y) \mid y = 2x + 3, \; x \in R\}$

put $x = 0, 1, 2, \ldots\ldots$

$R = \{(0,3), (1,5), (2,7), \ldots\ldots\}$

R is a function

$R^{-1} = \{(x, y) \mid y = \frac{x-3}{2}, \; x \in R\}$

put $x = 0, 1, 2, \ldots\ldots\ldots$

$R^{-1} = \{(0, -\frac{3}{2}), (1, -\frac{1}{2}), \ldots\ldots\ldots\}$

R^{-1} is a function

(iv) $R = \{(x, y) \mid y^2 = 4ax, \; x \geq 0\}$

put $x = 0, 1, 2, 3, \ldots$

$R = \{(0,0), (1, 2\sqrt{a}), (2, \sqrt{8a}), \ldots\}$

R is a function

$R^{-1} = \{(x, y) \mid y = \frac{1}{4a} x^2, \; x \geq 0\}$

R^{-1} is a function

(v) $R = \{(x, y) \mid x^2 + y^2 = 9, \; |x, y| \leq 3\}$

$R^{-1} = \{(x, y) \mid y^2 + x^2 = 9, \; |x, y| \leq 3\}$

R, and R^{-1} are not functions

put $x = 0, \pm 1, \pm 2, \ldots$

$R = \{(0,0)($

As Domain is repeated.