Question # 1

Verify following: (i) $\sin 60^o \cos 30^o - \cos 60^o \sin 30^o = \sin 30^o$

(ii) $\sin^2 \dfrac{\pi}{6} + \sin^2 \dfrac{\pi}{3} + \tan^2 \dfrac{\pi}{4} = 2$

(iii) $2\sin 45° + \dfrac{1}{2}\csc 45° = \dfrac{3}{\sqrt{2}}$

(iv) $\sin^2 \dfrac{\pi}{6} : \sin^2 \dfrac{\pi}{4} : \sin^2 \dfrac{\pi}{3} : \sin^2 \dfrac{\pi}{2} = 1:2:3:4$

Solution

(i) L.H.S $= \sin 60° \cos 30° - \cos 60° \sin 30°$

$= \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} \cdot \dfrac{1}{2}$ $= \dfrac{3}{4} - \dfrac{1}{4}$ $= \dfrac{3-1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$ ………… (*i*)

R.H.S $= \sin 30 = \dfrac{1}{2}$ ………… (*ii*)

From (*i*) and (*ii*)
$\qquad$ L.H.S = R.H.S

(ii) L.H.S $= \sin^2 \dfrac{\pi}{6} + \sin^2 \dfrac{\pi}{3} + \tan^2 \dfrac{\pi}{4}$

$= \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 + (1)^2$ $= \dfrac{1}{4} + \dfrac{3}{4} + 1 = \dfrac{1+3+4}{4}$ $= \dfrac{8}{4}$ $= 2$ $=$ R.H.S

(iii) L.H.S $= 2\sin 45° + \dfrac{1}{2}\csc 45° = 2\sin 45° + \dfrac{1}{2}\dfrac{1}{\sin 45°}$

$= 2\left(\dfrac{1}{\sqrt{2}}\right) + \dfrac{1}{2}\dfrac{1}{\left(\dfrac{1}{\sqrt{2}}\right)}$ $= \dfrac{2}{\sqrt{2}} + \dfrac{\sqrt{2}}{2} = \dfrac{(\sqrt{2})^2}{\sqrt{2}} + \dfrac{\sqrt{2}}{(\sqrt{2})^2}$

$= \sqrt{2} + \dfrac{1}{\sqrt{2}}$ $= \dfrac{2+1}{\sqrt{2}}$ $= \dfrac{3}{\sqrt{2}}$ $=$ R.H.S

(iv) L.H.S $= \sin^2 \dfrac{\pi}{6} : \sin^2 \dfrac{\pi}{4} : \sin^2 \dfrac{\pi}{3} : \sin^2 \dfrac{\pi}{2}$

$= \left(\dfrac{1}{2}\right)^2 : \left(\dfrac{1}{\sqrt{2}}\right)^2 : \left(\dfrac{\sqrt{3}}{2}\right)^2 : (1)^2$ $= \dfrac{1}{4} : \dfrac{1}{2} : \dfrac{3}{4} : 1$

$= 1:2:3:4$ $=$ R.H.S $\qquad\qquad$ ×ing by 4

Question # 2

Evaluate the following

(i) $\dfrac{\tan\dfrac{\pi}{3}-\tan\dfrac{\pi}{6}}{1+\tan\dfrac{\pi}{3}\tan\dfrac{\pi}{6}}$

(ii) $\dfrac{1-\tan^2\dfrac{\pi}{3}}{1+\tan^2\dfrac{\pi}{3}}$

Solution

(i) $\dfrac{\tan\dfrac{\pi}{3}-\tan\dfrac{\pi}{6}}{1+\tan\dfrac{\pi}{3}\tan\dfrac{\pi}{6}}\quad=\dfrac{\sqrt{3}-\dfrac{1}{\sqrt{3}}}{1+\sqrt{3}\cdot\dfrac{1}{\sqrt{3}}}\quad=\dfrac{\dfrac{3-1}{\sqrt{3}}}{1+1}$

$$=\dfrac{\dfrac{2}{\sqrt{3}}}{2}\quad=\dfrac{2}{2\sqrt{3}}=\dfrac{1}{\sqrt{3}}\quad\text{Answer}$$

(ii) $\dfrac{1-\tan^2\dfrac{\pi}{3}}{1+\tan^2\dfrac{\pi}{3}}\quad=\dfrac{1-\left(\sqrt{3}\right)^2}{1+\left(\sqrt{3}\right)^2}=\dfrac{1-3}{1+3}\quad=\dfrac{-2}{4}\quad=-\dfrac{1}{2}\quad\text{Answer}$

Question # 3

Verify the following when $\theta=30^o,45^o$

(i) $\sin 2\theta=2\sin\theta\cos\theta$ (ii) $\cos 2\theta=2\cos^2\theta-1$

(iii) $\cos 2\theta=2\cos^2\theta-1$ (iv) $\cos 2\theta=1-2\sin^2\theta$

(v) $\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$

Solution

(i) When $\theta=30°$

$\text{L.H.S}=\sin 2\theta=\sin 2(30)=\sin 60=\dfrac{\sqrt{3}}{2}$ ………. (i)

$\text{R.H.S}=2\sin\theta\cos\theta=2\sin 30°\cos 30°=2\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)=\dfrac{\sqrt{3}}{2}$ ……….. (ii)

From (i) and (ii)

     L.H.S = R.H.S

When $\theta=45°$

$\text{L.H.S}=\sin 2\theta=\sin 2(45)=\sin 90=1$ ………. (i)

$\text{R.H.S}=2\sin\theta\cos\theta=2\sin 45°\cos 45°=2\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{\sqrt{2}}\right)=\dfrac{2}{2}=1$ ……….. (ii)

From (i) and (ii)

     L.H.S = R.H.S

(ii) When $\theta=30°$

L.H.S $= \cos 2\theta = \cos 2(30) = \cos 60 = \dfrac{1}{2}$ ……… (i)

R.H.S $= \cos^2 \theta - \sin^2 \theta = \cos^2 30° - \sin^2 30°$

$= \left(\dfrac{\sqrt{3}}{2} \right)^2 - \left(\dfrac{1}{2} \right)^2 = \dfrac{3}{4} - \dfrac{1}{4} = \dfrac{1}{2}$ ……….. (ii)

From (i) and (ii)

      L.H.S = R.H.S

When $\quad \theta = 45°$

L.H.S $= \cos 2\theta = \cos 2(45) = \cos 90 = 0$ ……… (i)

R.H.S $= \cos^2 \theta - \sin^2 \theta = \cos^2 45° - \sin^2 45°$

$= \left(\dfrac{1}{\sqrt{2}} \right)^2 - \left(\dfrac{1}{\sqrt{2}} \right)^2 = \dfrac{1}{2} - \dfrac{1}{2} = 0$ ……….. (ii)

From (i) and (ii)

      L.H.S = R.H.S

(iii) and (iv)   *Do yourself as above*

(v)   When $\theta = 30°$

     L.H.S $= \tan 2\theta = \tan 2(30) = \tan 60 = \sqrt{3}$ ……… (i)

R.H.S $= \dfrac{2\tan \theta}{1 - \tan^2 \theta} = \dfrac{2\tan 30°}{1 - \tan^2 30°} = \dfrac{2\left(\dfrac{1}{\sqrt{3}} \right)}{1 - \left(\dfrac{1}{\sqrt{3}} \right)^2}$

$= \dfrac{\dfrac{2}{\sqrt{3}}}{1 - \dfrac{1}{3}} = \dfrac{\dfrac{2}{\sqrt{3}}}{\dfrac{2}{3}} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{3}{2} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\left(\sqrt{3}\right)^2}{2} = \sqrt{3}$ ……….. (ii)

From (i) and (ii)

      L.H.S = R.H.S

When $\quad \theta = 45°$

L.H.S $= \tan 2\theta = \tan 2(45) = \tan 90 = \infty$ ……… (i)

R.H.S $= \dfrac{2\tan \theta}{1 - \tan^2 \theta} = \dfrac{2\tan 45°}{1 - \tan^2 45°} = \dfrac{2(1)}{1 - (1)^2}$

$= \dfrac{2}{1 - 1} = \dfrac{2}{0} = \infty$ ……….. (ii)

From (i) and (ii)

      L.H.S = R.H.S

Question # 4

Find x, if $\tan^2 45^o - \cos^2 60^o = x\sin 45^o \cos 45^o \tan 60^o$

Solution

Since $\quad \tan^2 45^o - \cos^2 60^o = x \sin 45^o \cos 45^o \tan 60^o$

This gives $\quad (1)^2 - \left(\dfrac{1}{2}\right)^2 = x\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{\sqrt{2}}\right)(\sqrt{3})$

$$\Rightarrow \quad 1 - \dfrac{1}{4} = x\left(\dfrac{\sqrt{3}}{2}\right) \quad \dfrac{3}{4} = x\dfrac{\sqrt{3}}{2} \qquad \Rightarrow \quad \dfrac{6}{4\sqrt{3}} = x$$

$$\Rightarrow \quad x = \dfrac{3}{2\sqrt{3}} \qquad\qquad \Rightarrow \quad \boxed{x = \dfrac{\sqrt{3}}{2}} \quad \text{Answer}$$

Question # 5

Find the values of the trigonometric functions of the following

(i) $-\pi$ (ii) -3π (iii) $\dfrac{5}{2}\pi$ (iv) $-\dfrac{9}{2}\pi$ (v) -15π

(vi) 1530° (vii) $-2430°$ (viii) $\dfrac{235}{2}\pi$ (ix) $\dfrac{407}{2}\pi$

Solutions

(i)   Since $-\pi = -2\pi + \pi$, therefore

$\sin(-\pi) = \sin(-2\pi + \pi) = \sin\pi = 0$

$\cos(-\pi) = \cos(-2\pi + \pi) = \cos\pi = -1$

$\tan(-\pi) = \tan(-2\pi + \pi) = \tan\pi = 0$

$\csc(-\pi) = \csc(-2\pi + \pi) = \csc\pi = \infty$ (undefined)

$\sec(-\pi) = \sec(-2\pi + \pi) = \sec\pi = -1$

$\cot(-\pi) = \cot(-2\pi + \pi) = \cot\pi = \infty$ (undefined)

(ii)   Since $-3\pi = -4\pi + \pi = -2(2\pi) + \pi$, therefore

$\sin(-3\pi) = \sin(-2(2\pi) + \pi) = \sin\pi = 0$

            And now find other values yourself.

(iii)   Since $\dfrac{5}{2}\pi = 2\dfrac{1}{2}\pi = 2\pi + \dfrac{\pi}{2}$, therefore

$\sin\left(\dfrac{5}{2}\pi\right) = \sin\left(2\pi + \dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2} = 1$, $\cos\left(\dfrac{5}{2}\pi\right) = \cos\left(2\pi + \dfrac{\pi}{2}\right) = \cos\dfrac{\pi}{2} = 0$

$\tan\left(\dfrac{5}{2}\pi\right) = \tan\left(2\pi + \dfrac{\pi}{2}\right) = \tan\dfrac{\pi}{2} = \infty$, $\csc\left(\dfrac{5}{2}\pi\right) = \csc\left(2\pi + \dfrac{\pi}{2}\right) = \csc\dfrac{\pi}{2} = 1$

$\sec\left(\dfrac{5}{2}\pi\right) = \sec\left(2\pi + \dfrac{\pi}{2}\right) = \sec\dfrac{\pi}{2} = \infty$, $\cot\left(\dfrac{5}{2}\pi\right) = \cot\left(2\pi + \dfrac{\pi}{2}\right) = \cot\dfrac{\pi}{2} = 0$

(iv)   Since $-\dfrac{9}{2}\pi = -4\dfrac{1}{2}\pi = -6\pi + \dfrac{3\pi}{2}$, therefore

$\sin\left(-\dfrac{9}{2}\pi\right) = \sin\left(-6\pi + \dfrac{3\pi}{2}\right) = \sin\dfrac{3\pi}{2} = -1$

            And now find other values yourself.

(v)   Since $-15\pi = -16\pi + \pi$, therefore
$$\sin(-15\pi) = \sin(-16\pi + \pi) = \sin\pi = 0$$
And now find other values yourself.

(vi) Since $1530° = 1530 \times \dfrac{\pi}{180} = \dfrac{17}{2}\pi = 8\dfrac{1}{2}\pi = 8\pi + \dfrac{\pi}{2}$

So $\sin(1530°) = \sin\left(8\pi + \dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2} = 1$
And now find other values yourself.

(vii) Since $-2430° = -2430 \times \dfrac{\pi}{180} = -\dfrac{27}{2}\pi = -13\dfrac{1}{2}\pi = -14\pi + \dfrac{\pi}{2}$
Now do yourself

(viii) Since $\dfrac{235}{2}\pi = 116\dfrac{3}{2}\pi = 116\pi + \dfrac{3\pi}{2}$
Now do yourself

(*ix*) Since $\dfrac{407}{2}\pi = 202\dfrac{3}{2}\pi = 202\pi + \dfrac{3\pi}{2}$
Now do yourself

Question # 6
Find the values of the trigonometric functions of the following angles:
(i) $390°$ (ii) $-330°$ (iii) $765°$

(iv) $-675°$ (v) $-\dfrac{17}{3}\pi$ (vi) $\dfrac{13}{3}\pi$

(vii) $\dfrac{25}{6}\pi$ (viii) $-\dfrac{71}{6}\pi$ (ix) $-1035°$

Solutions
(i)   Since $390° = 360 + 30$

So $\sin(390°) = \sin(360 + 30) = \sin 30° = \dfrac{1}{2}$

$\cos(390°) = \cos(360 + 30) = \cos 30° = \dfrac{\sqrt{3}}{2}$

$\tan(390°) = \tan(360 + 30) = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\csc(390°) = \csc(360 + 30) = \csc 30° = 2$

$\sec(390°) = \sec(360 + 30) = \sec 30° = \dfrac{2}{\sqrt{3}}$

$\cot(390°) = \cot(360 + 30) = \cot 30° = \sqrt{3}$

(ii)   Since $-330° = -360 + 30$

So $\sin(-330°) = \sin(-360 + 30) = \sin 30° = \dfrac{1}{2}$

And now find other values yourself.

(iii)   Since $765° = 720 + 45 = 2(360) + 45$

So $\sin(760°) = \sin(2(360) + 45) = \sin 45° = \dfrac{1}{\sqrt{2}}$

And now find other values yourself.

(iv)   Since $-675° = -720 + 45 = -2(360) + 45$ *Now do yourself*

(v)   Since $-\dfrac{17}{3}\pi = -5\dfrac{2}{3}\pi = -6\pi + \dfrac{\pi}{3}$ *Now do yourself*

(vi)   Since $\dfrac{13}{3}\pi = 4\dfrac{1}{3}\pi = 4\pi + \dfrac{\pi}{3}$ *Now do yourself.*

(vii)   Since $\dfrac{25}{6}\pi = 4\dfrac{1}{6}\pi = 4\pi + \dfrac{\pi}{6}$

So $\sin\left(\dfrac{25}{6}\pi\right) = \sin\left(4\pi + \dfrac{\pi}{6}\right) = \sin\dfrac{\pi}{6} = \dfrac{1}{2}$ *Now do yourself*

(viii)   Since $-\dfrac{71}{6}\pi = -11\dfrac{5}{6}\pi = -12\pi + \dfrac{\pi}{6}$ *Now do yourself.*

(ix)   Since $-1035° = -1035 \cdot \dfrac{\pi}{180} = -\dfrac{23\pi}{4} = -5\dfrac{3\pi}{4} = -6\pi + \dfrac{\pi}{4}$

So $\sin(-1035°) = \sin\left(-6\pi + \dfrac{\pi}{4}\right) = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ *Now do yourself*
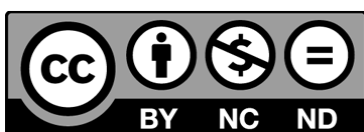

Please report us error at *www.mathcity.org/error*

Book: ***Exercise 9.3***
Text Book of Algebra and Trigonometry Class XI
Punjab Textbook Board, Lahore.

Available online at http://www.MathCity.org *in PDF Format*
(Picture format to view online).
Page setup: A4 (8.27 in × 11.02 in).
Updated: August,22,2017.

http://www.mathcity.org