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**Govt. Ghazali Degree College, Jhang**

(Important Short Questions)

Course: Calculus and Analytic Geometry

Chapter # 01

*Functions and Limits*

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Following short questions are selected from previous 5 years papers of different boards. Solve these at your own to perform well in annual exams.

1. Define function with an example.
2. Define even and odd functions.
3. Prove that  $\cosh^2 x - \sinh^2 x = 1$ .
4. Prove the identity  $\sinh 2x = 2 \sinh x \cosh x$ .
5. Prove the identity  $\operatorname{cosech}^2 x = \coth^2 x - 1$ .
6. With the help of Vertical line test, decide whether  $x^2 + y^2 = 1$  is a function or not.
7. Express the volume  $V$  of a cube as a function of the area  $A$  of its base.
8. Find the domain and range of the function  $g(x) = |x - 3|$ .
9. Find the domain and range of  $\sqrt{x^2 - 4}$ .
10. Find the domain and range of  $\sqrt{x + 1}$ .
11. Determine whether the function  $f(x) = x^{\frac{2}{3}} + 6$  is even or odd.
12. Determine whether the function  $f(x) = \frac{3x}{x^2 + 1}$  is even or odd.
13. If  $f(x) = -2x + 8$ , find  $f^{-1}(x)$ .
14. Find  $f^{-1}(x)$ , if  $f(x) = 2 + \sqrt{x - 1}$ . Also write the domain and range of  $f^{-1}(x)$ .
15. Without finding the inverse, state the domain and range of  $f^{-1}(x)$ , if  $f(x) = x - 5^2$ .

16. Evaluate

$$\lim_{x \rightarrow 0} \left( \frac{1 - \cos x}{x} \right).$$

17. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left( 1 + \frac{3}{x} \right)^{2x}.$$

18. Prove that

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a.$$

19. Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}.$$

20. Evaluate

$$\lim_{x \rightarrow 0} \frac{x}{\tan x}.$$

21. Express in terms of  $e$ ,

$$\lim_{x \rightarrow \infty} \left(\frac{x}{1+x}\right)^x.$$

22. Evaluate

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}.$$

23. Evaluate

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n.$$

24. Evaluate

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}.$$

25. Evaluate

$$\lim_{x \rightarrow -\infty} \frac{2 - 3x}{\sqrt{3 + 4x^2}}.$$

26. Evaluate

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - x}.$$

27. Evaluate

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

28. Define a continuous function.

29. If  $f(x) = \frac{x^2-9}{x-3}$ , discuss continuity at  $x = 3$ .

30. Explain why the function  $f(x) = \frac{2x}{x-2}$  is discontinuous at  $x = 2$ .

Best of Luck