1 Find δy and dy in the following cases:

(i) $y = x^2 - 1$ when x changes from 3 to 3.02

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INTEGRATION

Ucademy

Ex 3.1

$y = x^2 - 1$, $\boxed{x = 3,}$

$$\boxed{\begin{aligned} x + \delta x &= 3.02 \\ \delta x &= 3.02 - x \\ \delta x &= 3.02 - 3 = 0.02 \end{aligned}}$$

$$\boxed{d(f+g) = df + dg}$$

$y + \delta y = (x+\delta x)^2 - 1$

$\delta y = (x+\delta x)^2 - 1 - y$

$\delta y = (x+\delta x)^2 - \cancel{1} - x^2 + \cancel{1} = (x+\delta x)^2 - x^2$

$\delta y = (3.02)^2 - 3^2 = 9.1204 - 9 = 0.1204$

Again $y = x^2 - 1$

$d(y) = d(x^2 - 1)$

$dy = d(x^2) - d(1) = 2x\,dx - 0 = 2x\,\delta x$

$dy = 2(3)(0.02) = 6(0.02) = 0.12$

1 Find δy and dy in the following cases:

(ii) $y = x^2 + 2x$ when x changes from 2 to 1.8

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Ex 3.1

$x = 2,$

$$x + \delta x = 1.8$$
$$\delta x = 1.8 - x$$
$$\delta x = 1.8 - 2$$
$$\delta x = -0.2$$

$$d(f+g) = df + dg$$

$$y = x^2 + 2x$$
$$y + \delta y = (x+\delta x)^2 + 2(x+\delta x)$$
$$\delta y = (x+\delta x)^2 + 2(x+\delta x) - y$$
$$\delta y = (x+\delta x)^2 + \cancel{2x} + 2\delta x - x^2 - \cancel{2x}$$
$$\delta y = (1.8)^2 + 2(-0.2) - 2^2$$
$$= 3.24 - 0.4 - 4 = -1.16$$

Again

$$y = x^2 + 2x$$
$$dy = d(x^2 + 2x) = d(x^2) + d(2x)$$
$$dy = 2x\,dx + 2\,dx = 2x\delta x + 2\delta x$$
$$dy = 2(2)(-0.2) + 2(-0.2) = -0.8 - 0.4 = -1.2.$$

1 Find δy and dy in the following cases:

(iii) $y = \sqrt{x}$ when x changes from 4 to 4.41

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INTEGRATION

Ucademy

Ex 3.1

$$y = \sqrt{x}$$

$$\boxed{x = 4 ,}$$

$$\boxed{\begin{aligned} x + \delta x &= 4.41 \\ \delta x &= 4.41 - x \\ \delta x &= 4.41 - 4 \\ \delta x &= 0.41 \end{aligned}}$$

$$y + \delta y = \sqrt{x + \delta x}$$

$$\delta y = \sqrt{x + \delta x} - y = \sqrt{x + \delta x} - \sqrt{x}$$

$$\delta y = \sqrt{4.41} - \sqrt{4}$$

$$= 2.1 - 2 = 0.1$$

$$\boxed{d(f+g) = df + dg}$$

Again

$$y = \sqrt{x}$$

$$dy = d(x^{1/2}) = \frac{1}{2} x^{\frac{1}{2} - 1} dx = \frac{1}{2} \cdot x^{-1/2} dx$$

$$dy = \frac{1}{2\sqrt{x}} \delta x = \frac{1}{2\sqrt{4}} (0.41) = \frac{1}{2(2)} (0.41)$$

$$= \frac{0.41}{4} = 0.1025$$

2 Using differentials, find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following equations:
(i) $xy + x = 4$

$$d(xy + x) = d(4)$$
$$d(xy) + d(x) = 0$$
$$x\,dy + y\,dx + dx = 0$$
$$x\,dy = -y\,dx - dx$$
$$x\,dy = -(y+1)\,dx$$
$$\frac{dy}{dx} = -\frac{(y+1)}{x}$$
$$\frac{dx}{dy} = -\frac{x}{y+1}.$$

$$d(f \cdot g) = f \cdot d(g) + g \cdot d(f)$$

2

Using differentials, find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following equations:

(ii) $x^2 + 2y^2 = 16$

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INTEGRATION

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Ex 3.1

$$d(x^2 + 2y^2) = d(16)$$

$$d(x^2) + d(2y^2) = 0$$

$$2x\,dx + 2(2y\,dy) = 0$$

$$2x\,dx + 4y\,dy = 0$$

$$4y\,dy = -2x\,dx$$

$$\frac{dy}{dx} = \frac{-\cancel{2}x}{{}_2\cancel{4}y} = -\frac{x}{2y}$$

$$\frac{dx}{dy} = -\frac{2y}{x}.$$

$$d[f(x)] = f'(x) \cdot dx$$

Using differentials, find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following equations:

(iii) $x^4 + y^2 = xy^2$

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INTEGRATION

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Ex 3.1

$$d(x^4 + y^2) = d(xy^2)$$

$$d(x^4) + d(y^2) = x\,d(y^2) + y^2\,d(x)$$

$$4x^3\,dx + 2y\,dy = x(2y\,dy) + y^2\,dx$$

$$2y\,dy - 2xy\,dy = y^2\,dx - 4x^3\,dx$$

$$(2y - 2xy)\,dy = (y^2 - 4x^3)\,dx$$

$$\frac{dy}{dx} = \frac{y^2 - 4x^3}{2y - 2xy} = \frac{y^2 - 4x^3}{2y(1-x)}$$

$$\frac{dx}{dy} = \frac{2y(1-x)}{y^2 - 4x^3}.$$

$$d(f \cdot g) = f \cdot d(g) + g \cdot d(f)$$

$$d[f(x)] = f'(x) \cdot dx$$

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Ex 3.1

2 Using differentials, find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following equations:

(iv) $xy - \ln x = c$

$$d(xy - \ln x) = d(c)$$

$$d(xy) - d(\ln x) = 0$$

$$x\,dy + y\,dx - \frac{1}{x}\,dx = 0$$

$$x\,dy = \frac{1}{x}\,dx - y\,dx$$

$$x\,dy = \left(\frac{1}{x} - y\right)dx$$

$$x\,dy = \left(\frac{1-xy}{x}\right)dx$$

$$dy/dx = \frac{1-xy}{x^2}$$

$$dx/dy = \frac{x^2}{1-xy}$$ Ans.

$$d(f \cdot g) = f \cdot d(g) + g \cdot d(f)$$

$$d[f(x)] = f'(x) \cdot dx$$

3 Use differentials to appoximate the values of: (i) $\sqrt[4]{17}$

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INTEGRATION

Ex 3.1

$\sqrt[4]{17} = (17)^{1/4}$

Let $y = x^{1/4}$

$\boxed{x = 16}$, $\boxed{\begin{aligned} x + \delta x &= 17 \\ \delta x &= 17 - x \\ \delta x &= 17 - 16 \\ \delta x &= 1 \end{aligned}}$

$y = x^{1/4}$

$dy = d(x^{1/4})$

$dy = \frac{1}{4} x^{\frac{1}{4}-1} dx$

$dy = \frac{1}{4} x^{-3/4} dx$

$dy = \frac{1}{4x^{3/4}} \delta x$ — ①

Again $y = x^{1/4}$

$y + \delta y = (x + \delta x)^{1/4}$

$(x+\delta x)^{1/4} = y + \delta y$

$(x+\delta x)^{1/4} \approx y + dy$

$(x+\delta x)^{1/4} \approx x^{1/4} + \frac{1}{4x^{3/4}} \delta x$

$(17)^{1/4} \approx (16)^{1/4} + \frac{1}{4(16)^{3/4}}(1)$

$(17)^{1/4} \approx (2^4)^{1/4} + \frac{1}{4(2^4)^{3/4}}$

$(17)^{1/4} \approx 2 + \frac{1}{32} = \frac{64+1}{32} = 2.03125$

$$d[f(x)] = f'(x) \cdot dx$$

3 Use differentials to appoximate the values of: (ii) $(31)^{\frac{1}{5}}$

$(31)^{1/5}$

Let $y = x^{1/5}$

$\boxed{x = 32}$ $\boxed{\begin{aligned} x+\delta x &= 31 \\ \delta x &= 31 - x \\ \delta x &= 32 - 32 \\ \delta x &= -1 \end{aligned}}$

$y = x^{1/5}$

$dy = d(x^{1/5})$

$dy = \frac{1}{5} x^{\frac{1}{5}-1} dx = \frac{1}{5} x^{-4/5} dx$

$dy = \frac{1}{5x^{4/5}} \delta x$ — ①

Again $y = x^{1/5}$

$y + \delta y = (x+\delta x)^{1/5}$

$(x+\delta x)^{1/5} = y + \delta y$

$(x+\delta x)^{1/5} \approx y + dy$

$(x+\delta x)^{1/5} \approx x^{1/5} + \frac{\delta x}{5x^{4/5}}$

$(31)^{1/5} \approx (32)^{1/5} + \frac{-1}{5(32)^{4/5}}$

$(31)^{1/5} \approx (2^5)^{1/5} + \frac{-1}{5(2^5)^{4/5}}$

$(31)^{1/5} \approx 2 - \frac{1}{80} = \frac{160-1}{80}$

$(31)^{1/5} \approx 1.9875.$

$$d[f(x)] = f'(x) \cdot dx$$

3 Use differentials to appoximate the values of: (iii) $\cos 29^\circ$

$\cos 29^\circ$

Let $y = \cos x$

$\boxed{x = 30^\circ}$, $\boxed{\begin{aligned} x + \delta x &= 29^\circ \\ \delta x &= 29^\circ - x \\ \delta x &= 29^\circ - 30^\circ \\ \delta x &= -1^\circ \end{aligned}}$

$y = \cos x$

$dy = d(\cos x)$

$dy = -\sin x \cdot dx$

$dy = -\sin x . \delta x$ —(1)

Again

$y = \cos x$

$y + \delta y = \cos(x + \delta x)$

$\cos(x+\delta x) = y + \delta y \approx y + dy$

$\cos(x + \delta x) \approx \cos x + (-\sin x)\delta x$

$\cos(29^\circ) \approx \cos 30^\circ - \sin 30^\circ \cdot (-0.01746)$

$\cos 29^\circ \approx 0.866 - 0.5(-0.01746)$

$\cos 29^\circ \approx 0.866 + 0.0087$

$\cos 29^\circ \approx 0.8747.$

$$d[f(x)] = f'(x) \cdot dx$$

$$1^\circ = \frac{\pi}{180} \text{ rad} \approx 0.01746 \text{ rad}$$

3 Use differentials to appoximate the values of: (iv) $\sin 61^\circ$

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INTEGRATION

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Ex 3.1

$\sin 61^\circ = \sin(90^\circ - 29^\circ) = \cos 29^\circ$

Let $y = \cos x$

$\boxed{x = 30^\circ}$, $\boxed{\begin{aligned} x + \delta x &= 29^\circ \\ \delta x &= 29^\circ - x \\ \delta x &= 29^\circ - 30^\circ \\ \delta x &= -1^\circ \end{aligned}}$

$y = \cos x$

$dy = d(\cos x)$

$dy = -\sin x \, dx$

$dy = -\sin x \, \delta x \quad — (1)$

Again $y = \cos x$

$y + \delta y = \cos(x + \delta x)$

$\cos(x + \delta x) = y + \delta y \approx y + dy$

$\cos(x + \delta x) \approx \cos x + (-\sin x)\delta x$

$\cos 29^\circ \approx \cos 30^\circ - \sin 30^\circ(-0.01746)$

$\cos 29^\circ \approx 0.866 - 0.5(-0.01746)$

$\cos 29^\circ \approx 0.866 + 0.0087$

$\sin 61^\circ \approx 0.8747$

$$d[f(x)] = f'(x) \cdot dx$$

$$1^\circ = \frac{\pi}{180} \text{ rad} \approx 0.01746 \text{ rad}$$

4

Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02.

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Ex 3.1

Let x be the length of each edge,

then

$$V = x^3$$

$$dV = d(x^3)$$

$$dV = 3x^2 dx$$

$$dV = 3x^2 \delta x$$

$$dV = 3(5)^2(0.02)$$

$$= 3(\cancel{25})\frac{\cancel{2}}{\cancel{100}\ \cancel{4}\ 2} = \frac{3}{2} = 1.5 \text{ cubic units.}$$

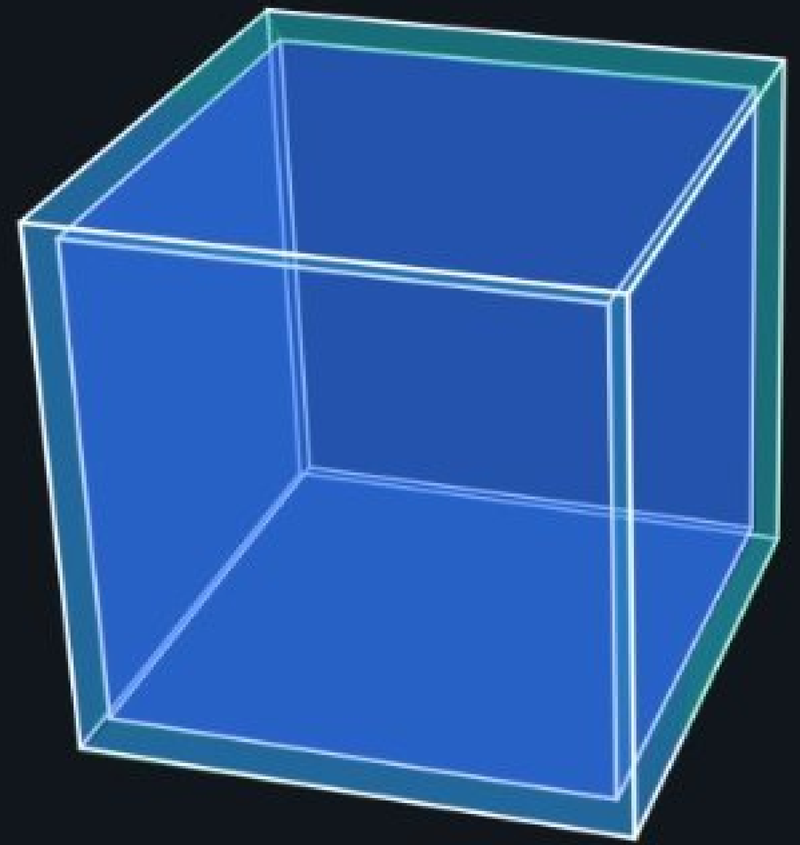
$x = 5$

$x + \delta x = 5.02$

$\delta x = 5.02 - x$

$\delta x = 5.02 - 5$

$\delta x = 0.02$



$$d[f(x)] = f'(x) \cdot dx$$

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