1 Evaluate the following indefinite integrals

(i) $\int (3x^2 - 2x + 1)\, dx$

Chapter 3 Ex 3.2

UCADEMY

$$\int (3x^2 - 2x + 1)\, dx$$

$$= \int 3x^2 dx - \int 2x dx + \int dx$$

$$= 3\int x^2 dx - 2\int x\, dx + \int dx$$

$$= 3\left(\frac{x^{2+1}}{2+1}\right) - 2\left(\frac{x^{1+1}}{1+1}\right) + x + C$$

$$= \cancel{3} \cdot \frac{x^3}{\cancel{3}} - \cancel{2} \cdot \frac{x^2}{\cancel{2}} + x + C$$

$$= x^3 - x^2 + x + C.$$

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

1 Evaluate the following indefinite integrals

(ii) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx \quad (x > 0)$

Chapter 3 Ex 3.2

UCADEMY

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$$

$$= \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right) dx$$

$$= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{3} x^{3/2} + 2 x^{1/2} + C. = \frac{2}{3} x\sqrt{x} + 2\sqrt{x} + C$$

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

1 Evaluate the following indefinite integrals

(iii) $\int x(\sqrt{x}+1)\,dx \quad (x>0)$

Chapter 3 Ex 3.2

UCADEMY

$$\int x(\sqrt{x}+1)\,dx$$

$$= \int (x\sqrt{x} + x)\,dx$$

$$= \int x \cdot x^{1/2}\,dx + \int x\,dx$$

$$= \int x^{3/2}\,dx + \int x\,dx$$

$$= \frac{x^{3/2+1}}{\frac{3}{2}+1} + \frac{x^2}{2} + C = \frac{x^{5/2}}{5/2} + \frac{x^2}{2} + C$$

$$= \frac{2}{5}x^{5/2} + \frac{x^2}{2} + C.$$

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

1 Evaluate the following indefinite integrals

(iv) $\int (2x+3)^{\frac{1}{2}}\, dx$

Chapter 3 Ex 3.2

UCADEMY

$$\int (2x+3)^{\frac{1}{2}}\, dx$$

$$= \frac{1}{2}\int (2x+3)^{\frac{1}{2}} \cdot 2\, dx$$

$$= \frac{1}{2} \cdot \frac{(2x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{1}{2} \cdot \frac{(2x+3)^{3/2}}{\frac{3}{2}} + C$$

$$= \frac{1}{2} \cdot \frac{2}{3}(2x+3)^{3/2} + C$$

$$= \frac{1}{3}(2x+3)^{3/2} + C.$$

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

1 Evaluate the following indefinite integrals

(ix) $\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}}\,dx \quad (\theta > 0)$

Chapter 3 Ex 3.2

UCADEMY

$$\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}}\,d\theta$$

$$= \int \frac{(\sqrt{\theta})^2 + 1 - 2\sqrt{\theta}}{\sqrt{\theta}}\,d\theta = \int \frac{\theta}{\sqrt{\theta}} + \frac{1}{\sqrt{\theta}} - \frac{2\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$$

$$= \int \left(\theta^{1-\frac{1}{2}} + \theta^{-\frac{1}{2}} - 2\right) d\theta$$

$$= \int \left(\theta^{1/2} + \theta^{-\frac{1}{2}} - 2\right) d\theta$$

$$= \int \theta^{1/2}\,d\theta + \int \theta^{-\frac{1}{2}}\,d\theta - 2\int d\theta$$

$$= \frac{\theta^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{\theta^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 2\theta + C$$

$$= \frac{\theta^{3/2}}{3/2} + \frac{\theta^{1/2}}{1/2} - 2\theta + C = \frac{2}{3}\theta^{3/2} + 2\theta^{1/2} - 2\theta + C.$$

Power Rule

$$\int f^n(x) \cdot f'(x)\,dx = \frac{f^{n+1}(x)}{n+1} + c,\ n \neq -1$$

1 Evaluate the following indefinite integrals

(v) $\int (\sqrt{x}+1)^2\, dx \quad (x>0)$

Chapter 3 Ex 3.2

UCADEMY

$$\int (\sqrt{x}+1)^2\, dx$$

$$= \int \left((\sqrt{x})^2 + 1^2 + 2\sqrt{x} \right) dx$$

$$= \int \left(x + 1 + 2x^{1/2} \right) dx$$

$$= \int x\, dx + \int dx + 2\int x^{1/2}\, dx$$

$$= \frac{x^2}{2} + x + 2\,\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C.$$

$$= \frac{x^2}{2} + x + 2 \cdot \frac{x^{3/2}}{3/2} + C$$

$$= \frac{x^2}{2} + x + 2 \cdot \frac{2}{3} x^{3/2} + C = \frac{x^2}{2} + x + \frac{4}{3} x^{3/2} + C.$$

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

1 Evaluate the following indefinite integrals

(vi) $\int (\sqrt{x} - \frac{1}{\sqrt{x}})^2 \, dx \quad (x > 0)$

Chapter 3
Ex 3.2

UCADEMY

$$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$$

$$= \int \left[(\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2\sqrt{x} \cdot \frac{1}{\sqrt{x}}\right] dx$$

$$= \int \left(x + \frac{1}{x} - 2\right) dx$$

$$= \int x \, dx + \int \frac{1}{x} \, dx - 2\int dx$$

$$= \frac{x^2}{2} + \ln|x| - 2x + C.$$

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

1 Evaluate the following indefinite integrals

(vii) $\int \frac{3x+2}{\sqrt{x}}\,dx \quad (x>0)$

Chapter 3 Ex 3.2

UCADEMY

$$\int \frac{3x+2}{\sqrt{x}}\,dx$$

$$= \int \left(\frac{3x}{\sqrt{x}} + \frac{2}{\sqrt{x}}\right)dx$$

$$= 3\int \frac{x}{\sqrt{x}}\,dx + 2\int \frac{1}{\sqrt{x}}\,dx$$

$$= 3\int x^{1-\frac{1}{2}}\,dx + 2\int x^{-\frac{1}{2}}\,dx$$

$$= 3\int x^{\frac{1}{2}}\,dx + 2\int x^{-\frac{1}{2}}\,dx$$

$$= 3\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 2\,\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = 3\cdot\frac{x^{3/2}}{3/2} + 2\,\frac{x^{1/2}}{1/2} + C$$

$$= \cancel{3}\cdot\frac{2}{\cancel{3}}x^{3/2} + 2\cdot 2\,x^{1/2} + C = 2x^{3/2} + 4x^{1/2} + C.$$

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

1 Evaluate the following indefinite integrals

(viii) $\int \frac{\sqrt{y}(y+1)}{y}\,dx \quad (y>0)$

Chapter 3 Ex 3.2

UCADEMY

$$\int \frac{\sqrt{y}\,(y+1)}{y}\,dy$$

$$= \int \frac{\sqrt{y}\,y + \sqrt{y}}{y}\,dy = \int \frac{\sqrt{y}\,y}{y}\,dy + \int \frac{\sqrt{y}}{y}\,dy$$

$$= \int y^{1/2}\,dy + \int y^{\frac{1}{2}-1}\,dy = \int y^{\frac{1}{2}}\,dy + \int y^{-\frac{1}{2}}\,dy$$

$$= \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{y^{3/2}}{3/2} + \frac{y^{1/2}}{1/2} + C$$

$$= \frac{2}{3}y^{3/2} + 2y^{1/2} + C.$$

Power Rule

$$\int f^n(x)\cdot f'(x)\,dx = \frac{f^{n+1}(x)}{n+1} + c,\ n \neq -1$$

1 Evaluate the following indefinite integrals

(x) $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}}\,dx \quad (x>0)$

Chapter 3 Ex 3.2

UCADEMY

$$\int \frac{(1-\sqrt{x})^2}{\sqrt{x}}\,dx$$

$$= \int \frac{1+(\sqrt{x})^2-2\sqrt{x}}{\sqrt{x}}\,dx = \int\left(\frac{1}{\sqrt{x}}+\frac{x}{\sqrt{x}}-\frac{2\sqrt{x}}{\sqrt{x}}\right)dx$$

$$= \int\left(x^{-1/2}+x^{1-1/2}-2\right)dx = \int\left(x^{-1/2}+x^{1/2}-2\right)dx$$

$$= \int x^{-1/2}\,dx + \int x^{1/2}\,dx - 2\int dx$$

$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 2x + C$$

$$= \frac{x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2} - 2x + C = 2x^{1/2} + \frac{2}{3}x^{3/2} - 2x + C.$$

Power Rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

1 Evaluate the following indefinite integrals

(xi) $\int \frac{e^{2x}+e^{x}}{e^{x}}\,dx$

Chapter 3 Ex 3.2

UCADEMY

$$\int \frac{e^{2x}+e^{x}}{e^{x}}\,dx$$

$$= \int \left(\frac{e^{2x}}{e^{x}} + \frac{1\,e^{x}}{e^{x}}\right) dx$$

$$= \int \left(e^{2x-x} + 1\right) dx = \int \left(e^{x} + 1\right) dx$$

$$= \int e^{x}\,dx + \int dx$$

$$= e^{x} + x + C.$$

$$\int e^{x}\,dx = e^{x} + c$$

2 Evaluate:

(i) $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} \quad \begin{pmatrix} x+a>0 \\ x+b>0 \end{pmatrix}$

UCADEMY

$$\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} \quad \begin{pmatrix} x+a>0 \\ x+b>0 \end{pmatrix}$$

$$= \int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$$

$$= \int \frac{\left(\sqrt{x+a}-\sqrt{x+b}\right)dx}{\left(\sqrt{x+a}\right)^2-\left(\sqrt{x+b}\right)^2} = \int \frac{\left(\sqrt{x+a}-\sqrt{x+b}\right)}{x+a-x-b}\,dx$$

$$= \frac{1}{a-b}\int \left((x+a)^{1/2}-(x+b)^{1/2}\right)dx = \frac{1}{a-b}\left[\int (x+a)^{1/2}dx - \int (x+b)^{1/2}dx\right]$$

$$= \frac{1}{a-b}\left[\frac{(x+a)^{3/2}}{3/2} - \frac{(x+b)^{3/2}}{3/2}\right]+C = \frac{1}{\frac{3}{2}(a-b)}\left[(x+a)^{3/2}-(x+b)^{3/2}\right]+C$$

$$= \frac{2}{3(a-b)}\left[(x+a)^{3/2}-(x+b)^{3/2}\right]+C.$$

Power Rule

$$\int f^n(x)\cdot f'(x)\,dx = \frac{f^{n+1}(x)}{n+1}+c,\ n\neq -1$$

2 Evaluate:

(ii) $\int \frac{1-x^2}{1+x^2}\,dx$

Chapter 3 Ex 3.2

UCADEMY

$$\int \frac{1-x^2}{1+x^2}\,dx$$

$$= \int \frac{-x^2+1}{x^2+1}\,dx$$

$$= \int \left(-1 + \frac{2}{x^2+1}\right) dx$$

$$= \int -dx + 2\int \frac{dx}{x^2+1}$$

$$= -x + 2\tan^{-1}x + C.$$

$$\begin{array}{r} -1 \\ x^2+1 \overline{)\,-x^2+1} \\ \mp x^2 \mp 1 \\ \hline 2 \end{array}$$

Power Rule

$$\int f^n(x)\cdot f'(x)\,dx = \frac{f^{n+1}(x)}{n+1} + c,\ n \neq -1$$

2 Evaluate:

(iii) $\int \frac{dx}{\sqrt{x+a}+\sqrt{x}} \quad (x > 0,\ a > 0)$

Chapter 3 Ex 3.2

UCADEMY

$$\int \frac{dx}{\sqrt{x+a}+\sqrt{x}} \qquad (x>0,\ a>0)$$

$$= \int \frac{dx}{\sqrt{x+a}+\sqrt{x}} \times \frac{\sqrt{x+a}-\sqrt{x}}{\sqrt{x+a}-\sqrt{x}}$$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x})}{(\sqrt{x+a})^2-(\sqrt{x})^2}\,dx = \int \frac{\sqrt{x+a}-\sqrt{x}}{x+a-x}\,dx$$

$$= \frac{1}{a}\int \left[(x+a)^{1/2} - x^{1/2}\right]dx = \frac{1}{a}\left[\int (x+a)^{1/2}dx - \int x^{1/2}dx\right]$$

$$= \frac{1}{a}\left[\frac{(x+a)^{3/2}}{3/2} - \frac{x^{3/2}}{3/2}\right] + C = \frac{1}{\frac{3}{2}a}\left[(x+a)^{3/2} - x^{3/2}\right] + C$$

$$= \frac{2}{3a}\left[(x+a)^{3/2} - x^{3/2}\right] + C.$$

Power Rule

$$\int f^n(x)\cdot f'(x)\,dx = \frac{f^{n+1}(x)}{n+1} + c,\ n \neq -1$$

2 Evaluate:

(iv) $\int (a-2x)^{\frac{3}{2}}\, dx$

Chapter 3 Ex 3.2

UCADEMY

$$\int (a-2x)^{3/2}\, dx$$

$$= \frac{1}{-2}\int (a-2x)^{3/2}(-2)\, dx$$

$$= -\frac{1}{2}\,\frac{(a-2x)^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C.$$

$$= -\frac{1}{2}\,\frac{(a-2x)^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= -\frac{1}{2}\cdot\frac{2}{5}(a-2x)^{5/2} + C$$

$$= -\frac{1}{5}(a-2x)^{5/2} + C.$$

$f(x) = a-2x$

$f'(x) = -2$

Power Rule

$$\int f^n(x)\cdot f'(x)\, dx = \frac{f^{n+1}(x)}{n+1} + c,\ n \neq -1$$

2 Evaluate:

(ix) $\int \sin^2 x \, dx$

Chapter 3
Ex 3.2

UCADEMY

$$\int \sin^2 x \, dx$$

$$= \int \frac{1-\cos 2x}{2} \, dx$$

$$= \frac{1}{2} \int (1 - \cos 2x) \, dx$$

$$= \frac{1}{2} \left[\int dx - \int \cos 2x \, dx \right]$$

$$= \frac{1}{2} \left[x - \frac{1}{2} \int \cos 2x \cdot 2 \, dx \right]$$

$$= \frac{1}{2} \left[x - \frac{1}{2} \sin 2x \right] + C.$$

$$= \frac{1}{2} x - \frac{1}{4} \sin 2x + C.$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\int \cos dx = \sin x + c$$

2 Evaluate:

(v) $\int \frac{(1+e^x)^3}{e^x}\,dx$

Chapter 3
Ex 3.2

UCADEMY

$$\int e^{f(x)} f'(x)dx = e^{f(x)} + c$$

$$\int \frac{(1+e^x)^3}{e^x}\,dx$$

$$= \int \frac{1+e^{3x}+3e^x+3e^{2x}}{e^x}\,dx$$

$$= \int \left(\frac{1}{e^x} + \frac{e^{3x}}{e^x} + \frac{3e^x}{e^x} + \frac{3e^{2x}}{e^x}\right)dx$$

$$= \int \left(e^{-x} + e^{2x} + 3 + 3e^x\right)dx$$

$$= \int e^{-x}\,dx + \int e^{2x}dx + 3\int dx + 3\int e^x dx$$

$$= -\int e^{-x}(-1)dx + \frac{1}{2}\int e^{2x}2\,dx + 3\int dx + 3\int e^x dx$$

$$= -e^{-x} + \frac{1}{2}e^{2x} + 3x + 3e^x + C.$$

2 Evaluate:

(vi) $\int \sin(a+b)x\,dx$

Chapter 3
Ex 3.2

UCADEMY

$$\int \sin(a+b)x \; dx$$

$$= \frac{1}{a+b}\int \sin(a+b)x \cdot (a+b)\; dx$$

$$= \frac{1}{a+b}\left[-\cos(a+b)x\right] + C.$$

$$= -\frac{\cos(a+b)x}{a+b} + C.$$

$$\int \sin x dx = -\cos x + c$$

2 Evaluate:

(vii) $\int \sqrt{1-\cos 2x}\, dx,\ (1-\cos 2x > 0)$

Chapter 3
Ex 3.2

UCADEMY

$$\int \sqrt{1-\cos 2x}\, dx \qquad (1-\cos 2x > 0)$$

$$= \int \sqrt{2\sin^2 x}\, dx$$

$$= \int \sqrt{2}\sqrt{\sin^2 x}\, dx$$

$$= \sqrt{2}\int \sin x\, dx$$

$$= \sqrt{2}\left(-\cos x\right) + C.$$

$$= -\sqrt{2}\cos x + C.$$

$$1-\cos 2x = 2\sin^2 x$$

$$\int \sin x dx = -\cos x + c$$

2 Evaluate:

(viii) $\int (\ln x) \times \frac{1}{x}\, dx \quad (x > 0)$

Chapter 3
Ex 3.2

UCADEMY

$$\int (\ln x) \times \frac{1}{x}\, dx \qquad (x > 0)$$

$$= \frac{(\ln x)^2}{2} + C.$$

Let $f(x) = \ln x$

$f'(x) = \frac{1}{x}$

Power Rule

$$\int f^n(x) \cdot f'(x)\, dx = \frac{f^{n+1}(x)}{n+1} + c,\ n \neq -1$$

2 Evaluate:

(x) $\int \frac{1}{1+\cos x}\,dx \quad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$

Chapter 3
Ex 3.2

UCADEMY

$$\int \frac{1}{1+\cos x}\,dx$$

$$= \int \frac{1}{2\cos^2\frac{x}{2}}\,dx$$

$$= \frac{1}{2}\int \sec^2\frac{x}{2}\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\int \sec^2\frac{x}{2}\cdot\frac{1}{2}\,dx$$

$$= \int \sec^2\frac{x}{2}\cdot\frac{1}{2}\,dx$$

$$= \tan\frac{x}{2} + C.$$

$1+\cos 2x = 2\cos^2 x$

$1+\cos x = 2\cos^2\frac{x}{2}$

$$\int \sec^2 x\,dx = \tan x + c$$

2 Evaluate:

(xi) $\int \frac{ax+b}{ax^2+2bx+c}\,dx$

Chapter 3
Ex 3.2

UCADEMY

$$\int \frac{ax+b}{ax^2+2bx+c}\,dx$$

$$= \frac{1}{2}\int \frac{2(ax+b)}{ax^2+2bx+c}\,dx$$

$$= \frac{1}{2}\int \frac{2ax+2b}{ax^2+2bx+c}\,dx$$

$$= \frac{1}{2}\ln|ax^2+2bx+c| + C_1$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

2 Evaluate:

(xii) $\int \cos 3x \sin 2x \, dx$

Chapter 3 Ex 3.2

UCADEMY

$$\int \cos 3x \sin 2x \, dx$$

$$= \frac{1}{2}\int [2 \cos 3x \sin 2x] dx$$

$$= \frac{1}{2}\int [\sin(3x+2x) - \sin(3x-2x)] dx$$

$$= \frac{1}{2}\int (\sin 5x - \sin x) dx$$

$$= \frac{1}{2}\left[\int \sin 5x \, dx - \int \sin x \, dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{5}\int \sin 5x \cdot 5 \, dx - \int \sin x \, dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{5}(-\cos 5x) - (-\cos x)\right] + C = \frac{1}{2}\left[\frac{-\cos 5x}{5} + \cos x\right] + C$$

$$2\cos\alpha \sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$

2 Evaluate:

(xiii) $\int \frac{\cos 2x - 1}{1 + \cos 2x}\, dx \ (1 - \cos 2x \neq 0)$

Chapter 3
Ex 3.2

UCADEMY

$$\int \frac{\cos 2x - 1}{1 + \cos 2x}\, dx$$

$$= -\int \frac{1 - \cos 2x}{1 + \cos 2x}\, dx$$

$$= -\int \frac{2\sin^2 x}{2\cos^2 x}\, dx = -\int \tan^2 x\, dx.$$

$$= -\int (\sec^2 x - 1)\, dx = -\int \sec^2 x\, dx + \int dx$$

$$= -\tan x + x + C.$$

$$1 + \cos 2x = 2\cos^2 x$$

$$1 - \cos 2x = 2\sin^2 x$$

2 Evaluate:

(xiv) $\int \tan^2 x \, dx$

Chapter 3
Ex 3.2

UCADEMY

$$\int \tan^2 x \, dx$$

$$= \int (\sec^2 x - 1) \, dx$$

$$= \int \sec^2 x \, dx - \int dx$$

$$= \tan x - x + c.$$

$$\tan^2 x = \sec^2 x - 1$$

$$\int \sec^2 x dx = \tan x + c$$

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