1 Evaluate the following integrals by parts.

(i) $\int x \sin x \, dx$

$$\int \underline{x}\ \underline{\sin x}\, dx$$

Let $\overset{\checkmark}{u} = x$, $dv = \sin x\, dx$

$du = dx$, $v = -\cos x$

Formula $\int u\,dv = uv - \int v\,du$

$$\int x \sin x\, dx = x(-\cos x) - \int (-\cos x)\, dx$$

$$= -x\cos x + \int \cos x\, dx$$

$$= -x\cos x + \sin x + C.$$

$$\int u\, dv = uv - \int v\, du$$

I Inverse
L Logarithmic
A **Algebraic**
T Trigonometric
E Exponential

1 Evaluate the following integrals by parts.

(ii) $\int \ln x \, dx$

$$\int \underline{\ln x} \, dx$$

Formula

$$\int u\,dv = uv - \int v\,du$$

Let $u = \overset{\checkmark}{\ln x}$, $dv = dx$

$du = \frac{1}{x} dx$, $v = x$

$$\int \ln x \, dx = (\ln x)(x) - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx$$

$$= x \ln x - \int dx$$

$$= x \ln x - x + C.$$

$$\int u\,dv = uv - \int v\,du$$

- **I** Inverse
- **L** **Logarithmic**
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

1 Evaluate the following integrals by parts.

(iii) $\int x \ln x \, dx$

$$\int \underline{x}\, \underline{\ln x}\, dx$$

$u = \ln x$, $dv = x\,dx$

$du = \frac{1}{x}\,dx$, $v = \frac{x^2}{2}$

Formula

$$\int u\,dv = uv - \int v\,du$$

$$\int x \ln x\,dx = (\ln x)\cdot\left(\frac{x^2}{2}\right) - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$$\int u\,dv = uv - \int v\,du$$

- **I** Inverse
- **L** **Logarithmic**
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

1

Evaluate the following integrals by parts.

(iv) $\int x^2 \ln x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int \underline{x^2}\, \underline{\ln x}\, dx$$

$u = \ln x$, $dv = x^2 dx$

$du = \frac{1}{x} dx$, $v = \frac{x^3}{3}$

Formula

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \ln x\,dx = (\ln x)\left(\frac{x^3}{3}\right) - \int \left(\frac{\overset{2}{\cancel{x^3}}}{3}\right) \cdot \frac{1}{\cancel{x}}\,dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \cdot \left(\frac{x^3}{3}\right) + C$$

$$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + C.$$

$$\int u\, dv = uv - \int v\, du$$

I Inverse

L Logarithmic

A Algebraic

T Trigonometric

E Exponential

1 Evaluate the following integrals by parts.

(v) $\int x^3 \ln x \, dx$

$$\int \underline{x^3}\, \underline{\ln x}\, dx$$

$$u = \ln x, \qquad dv = x^3 dx$$
$$du = \frac{1}{x} dx, \qquad v = \frac{x^4}{4}$$

Formula

$$\int u\,dv = uv - \int v\,du$$

$$\int x^3 \ln x \, dx = (\ln x)\left(\frac{x^4}{4}\right) - \int\left(\frac{x^{\overset{3}{\cancel{4}}}}{4}\right)\cdot\frac{1}{\cancel{x}}\,dx$$

$$= \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 dx$$

$$= \frac{x^4}{4}\ln x - \frac{1}{4}\left(\frac{x^4}{4}\right) + C.$$

$$= \frac{x^4}{4}\ln x - \frac{x^4}{16} + C.$$

$$\int u\,dv = uv - \int v\,du$$

I Inverse

L Logarithmic

A Algebraic

T Trigonometric

E Exponential

1 Evaluate the following integrals by parts.

(vi) $\int x^4 \ln x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int x^4 \ln x \, dx$$

$u = \ln x$, $dv = x^4 dx$

$du = \frac{1}{x} dx$, $v = \frac{x^5}{5}$

Formula

$$\int u dv = uv - \int v du$$

$$\int x^4 \ln x dx = (\ln x)\left(\frac{x^5}{5}\right) - \int \left(\frac{x^{\overset{4}{\cancel{5}}}}{5}\right) \frac{1}{\cancel{x}} dx$$

$$= \frac{x^5}{5} \ln x - \frac{1}{5} \int x^4 dx$$

$$= \frac{x^5}{5} \ln x - \frac{1}{5}\left(\frac{x^5}{5}\right) + C.$$

$$= \frac{x^5}{5} \ln x - \frac{x^5}{25} + C.$$

$$\int u \, dv = uv - \int v \, du$$

I Inverse

L Logarithmic

A Algebraic

T Trigonometric

E Exponential

1 Evaluate the following integrals by parts.

(vii) $\int \mathrm{Tan}^{-1} x \, dx$

$$\int \mathrm{Tan}^{-1} x \, dx$$

$u = \mathrm{Tan}^{-1} x$, $dv = dx$

$du = \frac{1}{1+x^2} dx$, $v = x$

Formula

$$\int u dv = uv - \int v du$$

$$\int \mathrm{Tan}^{-1} x \, dx = (\mathrm{Tan}^{-1} x)(x) - \int x \cdot \frac{1}{1+x^2} dx$$

$$= x \, \mathrm{Tan}^{-1} x - \frac{1}{2} \int \frac{2x^{\checkmark}}{1+x^2 \, \checkmark} dx$$

$$= x \, \mathrm{Tan}^{-1} x - \frac{1}{2} \ln|1+x^2| + c.$$

$$= x \, \mathrm{Tan}^{-1} x - \frac{1}{2} \ln(1+x^2) + C.$$

$$\int u \, dv = uv - \int v \, du$$

I Inverse
L Logarithmic
A Algebraic
T Trigonometric
E Exponential

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

1 Evaluate the following integrals by parts.

(viii) $\int x^2 \sin x \, dx$

$$\int \underline{x^2} \, \underline{\sin x} \, dx$$

Formula

$$\int u dv = uv - \int v du$$

$u = x^2$, $dv = \sin x dx$

$du = 2x dx$, $v = -\cos x$

$$\int x^2 \sin x dx = (x^2)(-\cos x) - \int (-\cos x) 2x dx$$

$$= -x^2 \cos x + 2 \underline{\int \underline{x} \, \underline{\cos x} \, dx}$$

$u = x$, $dv = \cos x dx$

$du = dx$, $v = \sin x$

$$= -x^2 \cos x + 2\left[x \sin x - \int \sin x dx\right]$$

$$= -x^2 \cos x + 2x \sin x - 2\int \sin x dx$$

$$= -x^2 \cos x + 2x \sin x - 2(-\cos x) + c.$$

$$= -x^2 \cos x + 2x \sin x + 2\cos x + c.$$

$$\int u \, dv = uv - \int v \, du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

1 Evaluate the following integrals by parts.

(ix) $\int x^2 \mathrm{Tan}^{-1} x \, dx$

$$\int \underline{x^2} \, \underline{\mathrm{Tan}^{-1} x} \, dx$$

$u = \mathrm{Tan}^{-1} x$, $dv = x^2 dx$

$du = \frac{1}{1+x^2} dx$, $v = \frac{x^3}{3}$

$$\int u \, dv = uv - \int v \, du$$

Formula

$$\int u dv = uv - \int v du$$

$$\int x^2 \mathrm{Tan}^{-1} x \, dx = (\mathrm{Tan}^{-1} x)\left(\frac{x^3}{3}\right) - \int \left(\frac{x^3}{3}\right) \cdot \frac{1}{1+x^2} dx$$

$$= \frac{x^3}{3} \mathrm{Tan}^{-1} x - \frac{1}{3} \int \frac{x^3}{x^2+1} dx$$

$$\begin{array}{r} x \\ x^2+1 \overline{) \; \cancel{x^3} \quad} \\ \underline{\pm \cancel{x^3} \pm x} \\ -x \end{array}$$

$$= \frac{x^3}{3} \mathrm{Tan}^{-1} x - \frac{1}{3} \int \left(x - \frac{x}{x^2+1} \right) dx$$

$$= \frac{x^3}{3} \mathrm{Tan}^{-1} x - \frac{1}{3} \int x dx + \frac{1}{3} \cdot \frac{1}{2} \int \frac{2x}{x^2+1} dx \quad ✓$$

$$= \frac{x^3}{3} \mathrm{Tan}^{-1} x - \frac{1}{3}\left(\frac{x^2}{2}\right) + \frac{1}{6} \ln(x^2+1) + C = \frac{x^3}{3} \mathrm{Tan}^{-1} x - \frac{x^2}{6} + \frac{1}{6} \ln(x^2+1) + C.$$

I Inverse
L Logarithmic
A Algebraic
T Trigonometric
E Exponential

1 Evaluate the following integrals by parts.

(x) $\int x\text{Tan}^{-1}x\,dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int \underline{x}\,\underline{\text{Tan}^{-1}x}\,dx$$

Formula

$$\int u\,dv = uv - \int v\,du$$

$u = \text{Tan}^{-1}x$, $dv = x\,dx$

$du = \frac{1}{1+x^2}dx$, $v = \frac{x^2}{2}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\,\text{Tan}^{-1}x\,dx = (\text{Tan}^{-1}x)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\cdot\frac{1}{x^2+1}dx$$

$$= \frac{x^2}{2}\text{Tan}^{-1}x - \frac{1}{2}\int \frac{x^2}{x^2+1}dx$$

$$= \frac{x^2}{2}\text{Tan}^{-1}x - \frac{1}{2}\int\left(1 - \frac{1}{x^2+1}\right)dx$$

$$= \frac{x^2}{2}\text{Tan}^{-1}x - \frac{1}{2}\int dx + \frac{1}{2}\int\frac{1}{x^2+1}dx$$

$$= \frac{x^2}{2}\text{Tan}^{-1}x - \frac{x}{2} + \frac{1}{2}\text{Tan}^{-1}x + C.$$

$$\begin{array}{r|l} & 1 \\ \hline x^2+1 & \cancel{x^2} \\ & \pm\cancel{x^2}\pm 1 \\ \hline & -1 \end{array}$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

1

Evaluate the following integrals by parts.

(xi) $\int x^3 \mathrm{Tan}^{-1} x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

$\int \underline{x^3}\ \underline{\mathrm{Tan}^{-1}x}\,dx$

$\int u\,dv = uv - \int v\,du$

$u = \mathrm{Tan}^{-1}x, \quad dv = x^3 dx$

$du = \frac{1}{1+x^2}dx, \quad v = \frac{x^4}{4}$

$\int x^3 \mathrm{Tan}^{-1}x\,dx = (\mathrm{Tan}^{-1}x)\left(\frac{x^4}{4}\right) - \int \frac{x^4}{4}\cdot\frac{1}{1+x^2}dx$

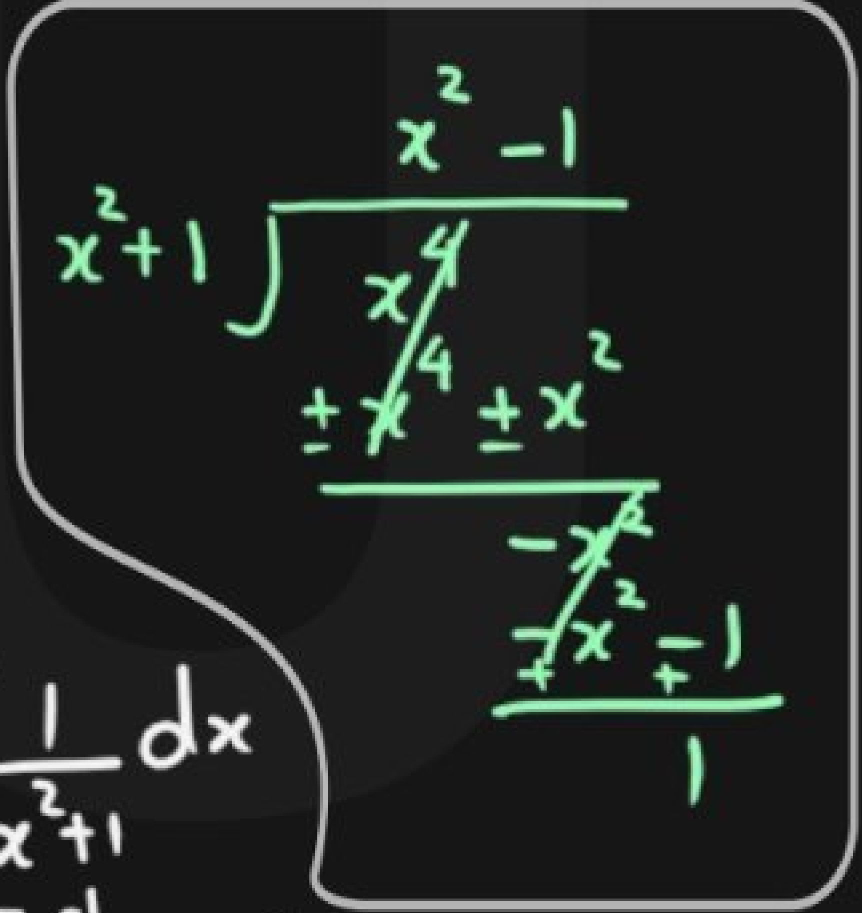
$= \frac{x^4}{4}\mathrm{Tan}^{-1}x - \frac{1}{4}\int\frac{x^4}{x^2+1}dx$

$= \frac{x^4}{4}\mathrm{Tan}^{-1}x - \frac{1}{4}\int\left(x^2-1+\frac{1}{x^2+1}\right)dx$

$= \frac{x^4}{4}\mathrm{Tan}^{-1}x - \frac{1}{4}\int x^2dx + \frac{1}{4}\int dx - \frac{1}{4}\int\frac{1}{x^2+1}dx$

$= \frac{x^4}{4}\mathrm{Tan}^{-1}x - \frac{1}{4}\left(\frac{x^3}{3}\right) + \frac{1}{4}x - \frac{1}{4}\mathrm{Tan}^{-1}x + C$

$= \frac{x^4}{4}\mathrm{Tan}^{-1}x - \frac{x^3}{12} + \frac{x}{4} - \frac{1}{4}\mathrm{Tan}^{-1}x + C.$



$$\int u\,dv = uv - \int v\,du$$

I Inverse
L Logarithmic
A Algebraic
T Trigonometric
E Exponential

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

1 Evaluate the following integrals by parts.

(xii) $\int x^2 \sin x \, dx$

$$\int \underline{x^3} \, \underline{\cos x} \, dx$$

$$= (x^3)(\sin x) - \int \sin x \cdot 3x^2 dx$$

$$= x^3 \sin x - 3 \int \underline{x^2} \, \underline{\sin x} \, dx$$

$$= x^3 \sin x - 3 \left[x^2(-\cos x) - \int (-\cos x) 2x \, dx \right]$$

$$= x^3 \sin x + 3x^2 \cos x - 3(2) \int \underline{x} \, \underline{\cos x} \, dx$$

$$= x^3 \sin x + 3x^2 \cos x - 6 \left[x \sin x - \int \sin x \, dx \right]$$

$$= x^3 \sin x + 3x^2 \cos x - 6x \sin x + 6 \int \sin x \, dx$$

$$= x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C.$$

$u = x^3$, $dv = \cos x \, dx$
$du = 3x^2 dx$, $v = \sin x$

$u = x^2$, $dv = \sin x \, dx$
$du = 2x \, dx$, $v = -\cos x$

$u = x$, $dv = \cos x \, dx$
$du = dx$, $v = \sin x$

$$\int u \, dv = uv - \int v \, du$$

- I Inverse
- L Logarithmic
- **A Algebraic**
- T Trigonometric
- E Exponential

1 Evaluate the following integrals by parts.

(xiii) $\int \mathrm{Sin}^{-1}x\,dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int \mathrm{Sin}^{-1}x\,dx$$

Formula

$$\int u\,dv = uv - \int v\,du$$

$u = \mathrm{Sin}^{-1}x$, $dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx$, $v = x$

$$\int \mathrm{Sin}^{-1}x\,dx = (\mathrm{Sin}^{-1}x)x - \int x\cdot\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= x\,\mathrm{Sin}^{-1}x + \frac{1}{2}\int (1-x^2)^{-1/2}\cdot(-2)x\,dx$$

$$= x\,\mathrm{Sin}^{-1}x + \frac{1}{2}\,\frac{(1-x^2)^{-1/2+1}}{-1/2+1} + C$$

$$= x\,\mathrm{Sin}^{-1}x + \frac{1}{\cancel{2}}\,\frac{(1-x^2)^{1/2}}{1/\cancel{2}} + c$$

$$= x\,\mathrm{Sin}^{-1}x + \sqrt{1-x^2} + C.$$

$$\int u\,dv = uv - \int v\,du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

1 Evaluate the following integrals by parts.

(xiv) $\int x \mathrm{Sin}^{-1} x \, dx$

$$\int x \mathrm{Sin}^{-1}x \, dx = \frac{x^2}{2}\mathrm{Sin}^{-1}x + \frac{1}{2}\int\left(\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right)dx$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x + \frac{1}{2}\int\sqrt{1^2-x^2}\,dx - \frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x + \frac{1}{2}\left[\frac{1^2}{2}\mathrm{Sin}^{-1}\frac{x}{1} + \frac{x}{2}\sqrt{1^2-x^2}\right] - \frac{1}{2}\mathrm{Sin}^{-1}x + c$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x + \frac{1}{2}\left[\frac{1}{2}\mathrm{Sin}^{-1}x + \frac{x}{2}\sqrt{1-x^2}\right] - \frac{1}{2}\mathrm{Sin}^{-1}x + c.$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x + \frac{1}{4}\mathrm{Sin}^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{2}\mathrm{Sin}^{-1}x + c.$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x + \left(\frac{1}{4} - \frac{1}{2}\right)\mathrm{Sin}^{-1}x + \frac{x}{4}\sqrt{1-x^2} + c.$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x - \frac{1}{4}\mathrm{Sin}^{-1}x + \frac{x}{4}\sqrt{1-x^2} + c.$$

$$\int u\,dv = uv - \int v\,du$$

- **I** **Inverse**
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

1 Evaluate the following integrals by parts.

(xiv) $\int x\mathrm{Sin}^{-1}x\,dx$

$$\int u\,dv = uv - \int v\,du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

$$\int \underline{x}\ \underline{\mathrm{Sin}^{-1}x}\,dx$$

$u = \mathrm{Sin}^{-1}x, \quad dv = x\,dx$

$du = \frac{1}{\sqrt{1-x^2}}dx, \quad v = \frac{x^2}{2}$

Formula

$$\int u\,dv = uv - \int v\,du$$

$$\int x\,\mathrm{Sin}^{-1}x\,dx = (\mathrm{Sin}^{-1}x)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\cdot\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x + \frac{1}{2}\int\left(\frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right)dx$$

$$= \frac{x^2}{2}\mathrm{Sin}^{-1}x + \frac{1}{2}\int\left((1-x^2)^{1/2} - (1-x^2)^{-1/2}\right.$$

1 Evaluate the following integrals by parts.

(xix) $\int (\ln x)^2 \, dx$

$$\int \underline{(\ln x)^2} \, dx$$

$u = (\ln x)^2$, $dv = dx$

$du = 2(\ln x)\frac{1}{x}dx$, $v = x$

Formula $\int u\,dv = uv - \int v\,du$

$$\int (\ln x)^2 dx = (\ln x)^2 x - \int \cancel{x} \cdot 2(\ln x) \cdot \frac{1}{\cancel{x}} dx$$

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \underline{\ln x} \, dx$$

$u = \ln x$, $dv = dx$

$du = \frac{1}{x}dx$, $v = x$

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2\left[(\ln x)x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx\right]$$

$$= x(\ln x)^2 - 2x\ln x + 2\int dx$$

$$= x(\ln x)^2 - 2x\ln x + 2x + C.$$

$$\int u\,dv = uv - \int v\,du$$

- **I** Inverse
- **L** **Logarithmic**
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

1

Evaluate the following integrals by parts.

(xv) $\int e^{x} \sin x \cos x \, dx$

Chapter 3
INTEGRATION
Ex 3.4

$$2I = e^{x}\sin 2x - 2\int \underline{e^{x}}\,\underline{\cos 2x}\,dx$$

$$2I = e^{x}\sin 2x - 2\left[\cos 2x . e^{x} - \int e^{x}(-2\sin 2x\,dx)\right]$$

$u = \cos 2x$, $dv = e^{x}dx$
$du = -2\sin 2x\,dx$, $v = e^{x}$

$$\int u\,dv = uv - \int v\,du$$

$$2I = e^{x}\sin 2x - 2e^{x}\cos 2x - 4\underbrace{\int e^{x}\sin 2x\,dx}$$

$$2I = e^{x}\sin 2x - 2e^{x}\cos 2x - 4(2I) + c_1$$

$$2I = e^{x}\sin 2x - 2e^{x}\cos 2x - 8I + c_1$$

$$10I = e^{x}(\sin 2x - 2\cos 2x) + c_1$$

$$I = \frac{e^{x}}{10}(\sin 2x - 2\cos 2x) + \frac{c_1}{10}$$

$$I = \frac{e^{x}}{10}(\sin 2x - 2\cos 2x) + C$$

I Inverse
L Logarithmic
A Algebraic
T **Trigonometric**
E Exponential

1 Evaluate the following integrals by parts.

(xv) $\int e^x \sin x \cos x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int u\,dv = uv - \int v\,du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

Let $I = \int e^x \underline{\sin x}\, \underline{\cos x}\, dx$

$$I = \frac{1}{2}\int e^x \underline{2\sin x \cos x}\, dx$$

$$I = \frac{1}{2}\int e^x \sin 2x \, dx$$

$$2I = \int e^x \underline{\sin 2x}\, dx \quad \text{———} ①$$

$u = \sin 2x$, $dv = e^x dx$

$du = 2\cos 2x\, dx$, $v = e^x$

Formula

$$\int u\,dv = uv - \int v\,du$$

$$\int \sin 2x \cdot e^x dx = (\sin 2x)\, e^x - \int e^x \cdot 2\cos 2x \, dx$$

1 Evaluate the following integrals by parts.

(xvi) $\int x \sin x \cos x \, dx$

$$\int u\,dv = uv - \int v\,du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

Let $I = \int x \sin x \cos x \, dx$

$$I = \frac{1}{2}\int x(2\sin x \cos x)dx = \frac{1}{2}\int x \sin 2x \, dx$$

$$2I = \int \underline{x} \sin 2x \, dx \quad ✓$$

$u = x$, $dv = \sin 2x dx$

$du = dx$, $v = -\frac{\cos 2x}{2}$

Formula $\int u dv = uv - \int v du$

$$\int x \sin 2x dx = x\left(-\frac{\cos 2x}{2}\right) - \int\left(-\frac{\cos 2x}{2}\right) dx$$

$$2I = -\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2x \, dx$$

$$2I = -\frac{x}{2}\cos 2x + \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + C_1$$

$$I = -\frac{x}{4}\cos 2x + \frac{1}{8}\sin 2x + C$$

Ans.

1 Evaluate the following integrals by parts.

(xvii) $\int x \cos^2 x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

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$$\int u \, dv = uv - \int v \, du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

$$\int x \cos^2 x \, dx$$

$$= \int x\left(\frac{1+\cos 2x}{2}\right) dx = \frac{1}{2}\int (x + x\cos 2x)\, dx$$

$$= \frac{1}{2}\int x\, dx + \frac{1}{2}\int \underline{x}\,\underline{\cos 2x}\, dx$$

$u = x$, $dv = \cos 2x\,dx$

$du = dx$, $v = \frac{\sin 2x}{2}$

$$= \frac{1}{2}\left(\frac{x^2}{2}\right) + \frac{1}{2}\left[x\left(\frac{\sin 2x}{2}\right) - \int \frac{\sin 2x}{2} dx\right]$$

$$= \frac{x^2}{4} + \frac{1}{4} x \sin 2x - \frac{1}{4}\int \sin 2x \, dx$$

$$= \frac{x^2}{4} + \frac{x}{4}\sin 2x - \frac{1}{4}\left(-\frac{\cos 2x}{2}\right) + c$$

$$= \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + c.$$

1

Evaluate the following integrals by parts.

(xviii) $\int x \sin^2 x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

$$\int x \sin^2 x \, dx$$

$$= \int x \left(\frac{1 - \cos 2x}{2}\right) dx = \frac{1}{2} \int (x - x \cos 2x) \, dx$$

$$= \frac{1}{2} \int x \, dx - \frac{1}{2} \int \underline{\overset{\checkmark}{x}} \, \underline{\cos 2x} \, dx$$

$u = x$, $dv = \cos 2x \, dx$

$du = dx$, $v = \frac{\sin 2x}{2}$

$$= \frac{1}{2}\left(\frac{x^2}{2}\right) - \frac{1}{2}\left[x \cdot \frac{\sin 2x}{2} - \int \frac{\sin 2x}{2} \, dx\right]$$

$$= \frac{x^2}{4} - \frac{x}{4} \sin 2x + \frac{1}{4} \int \sin 2x \, dx$$

$$= \frac{x^2}{4} - \frac{x}{4} \sin 2x + \frac{1}{4}\left(-\frac{\cos 2x}{2}\right) + C$$

$$= \frac{x^2}{4} - \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + C.$$

$$\int u \, dv = uv - \int v \, du$$

- **I** Inverse
- **L** Logarithmic
- **A** **Algebraic**
- **T** Trigonometric
- **E** Exponential

1 Evaluate the following integrals by parts.

(xx) $\int \ln(\tan x) \sec^2 x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

$$\int \underline{\ln(\tan x)}\ \underline{\sec^2 x}\ dx$$

Formula

$$\int u\,dv = uv - \int v\,du$$

$u = \ln(\tan x)$, $dv = \sec^2 x\,dx$

$du = \frac{1}{\tan x} \cdot \sec^2 x\,dx$, $v = \tan x$

$$\int \ln(\tan x) \sec^2 x\,dx = \ln(\tan x) \cdot \tan x - \int \cancel{\tan x} \cdot \frac{1}{\cancel{\tan x}} \cdot \sec^2 x\,dx$$

$$= \ln(\tan x) \cdot \tan x - \int \sec^2 x\,dx$$

$$= \tan x . \ln(\tan x) - \tan x + C .$$

$$\int u\,dv = uv - \int v\,du$$

- **I** Inverse
- **L** **Logarithmic**
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

Chapter 3

INTEGRATION

Ex 3.4

1 Evaluate the following integrals by parts.

(xxi) $\int \frac{x\mathrm{Sin}^{-1}x}{\sqrt{1-x^2}}\,dx$

$$-\int \frac{x\,\mathrm{Sin}^{-1}x}{\sqrt{1-x^2}}\,dx$$

Formula $\int u\,dv = uv - \int v\,du$

$$\int \mathrm{Sin}^{-1}x\cdot\frac{x}{\sqrt{1-x^2}}\,dx = \mathrm{Sin}^{-1}x\cdot\left(-\sqrt{1-x^2}\right) - \int\left(-\cancel{\sqrt{1-x^2}}\right)\frac{1}{\cancel{\sqrt{1-x^2}}}\,dx$$

$$= -\sqrt{1-x^2}\,\mathrm{Sin}^{-1}x + \int dx$$

$$= -\sqrt{1-x^2}\,\mathrm{Sin}^{-1}x + x + c.$$

$u = \mathrm{Sin}^{-1}x$, $dv = \frac{x}{\sqrt{1-x^2}}\,dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$, $\int dv = -\frac{1}{2}\int (1-x^2)^{-1/2}\cdot -2x\,dx$

$$v = -\frac{1}{2}\,\frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

$$v = -\frac{1}{\cancel{2}}\,\frac{(1-x^2)^{1/2}}{\frac{1}{\cancel{2}}}$$

$$v = -\sqrt{1-x^2}$$

$$\int u\,dv = uv - \int v\,du$$

I Inverse
L Logarithmic
A Algebraic
T Trigonometric
E Exponential

2 Evaluate the following integral:

(i) $\int \tan^4 x \, dx$

$$\int u \, dv = uv - \int v \, du$$

$$\int \tan^4 x \, dx$$

$$= \int \tan^2 x \cdot \tan^2 x \, dx$$

$$= \int \tan^2 x \cdot (\sec^2 x - 1) \, dx$$

$$= \int (\tan^2 x \sec^2 x - \tan^2 x) \, dx$$

$$= \int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx$$

$$= \int (\underline{\tan x})^2 \sec^2 x \, dx - \int (\sec^2 x - 1) \, dx$$

$$= \frac{(\tan x)^3}{3} - \int \sec^2 x \, dx + \int dx$$

$$= \frac{\tan^3 x}{3} - \tan x + x + C.$$

2 Evaluate the following integral:

(ii) $\int \sec^4 x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int \sec^4 x \, dx$$

$$= \int \sec^2 x \cdot \sec^2 x \, dx$$

$$= \int (1+\tan^2 x) \cdot \sec^2 x \, dx$$

$$= \int \left(\sec^2 x + \tan^2 x \sec^2 x\right) dx$$

$$= \int \sec^2 x \, dx + \int (\underline{\tan x})^2 \sec^2 x \, dx$$

$$= \tan x + \frac{(\tan x)^3}{3} + C$$

$$= \tan x + \frac{\tan^3 x}{3} + C.$$

$$\boxed{\int u \, dv = uv - \int v \, du}$$

2 Evaluate the following integral:

(ii) $\int e^x \sin 2x \cos x \, dx$

$$\int u \, dv = uv - \int v \, du$$

Let $I = \int e^x \underline{\sin 2x} \, \underline{\cos x} \, dx$

$$I = \frac{1}{2} \int e^x (2 \sin 2x \cos x) \, dx$$

$$I = \frac{1}{2} \int e^x \left[\sin(2x + x) + \sin(2x - x) \right] dx$$

$$I = \frac{1}{2} \int \left[e^x \sin 3x + e^x \sin x \right] dx$$

$$2I = \int e^x \sin 3x \, dx + \int e^x \sin x \, dx$$

Let $I_1 = \int e^x \sin 3x \, dx$ and $I_2 = \int e^x \sin x \, dx$

$$2I = I_1 + I_2 \quad — ①$$

2 Evaluate the following integral:

(ii) $\int e^x \sin 2x \cos x \, dx$

$$\int u\, dv = uv - \int v\, du$$

$I_1 = \int \underline{e^x} \, \underline{\sin 3x} \, dx$ ✓

$u = \sin 3x, \quad dv = e^x dx$

$du = 3\cos 3x \, dx, \quad v = e^x$

$I_1 = (\sin 3x) e^x - \int e^x \cdot 3\cos 3x \, dx$

$I_1 = e^x \sin 3x - 3 \int \underline{e^x} \, \underline{\cos 3x} \, dx$

$u = \cos 3x, \quad dv = e^x dx$

$du = -3\sin 3x \, dx, \quad v = e^x$

$I_1 = e^x \sin 3x - 3\left[\cos 3x \cdot e^x - \int e^x(-3\sin 3x) dx\right]$

$I_1 = e^x \sin 3x - 3\left[e^x \cos 3x + 3\underline{\int e^x \sin 3x \, dx}\right]$

$I_1 = e^x \sin 3x - 3e^x \cos 3x - 9 I_1$

$10 I_1 = e^x (\sin 3x - 3 \cos 3x)$

$I_1 = \frac{e^x}{10} (\sin 3x - 3 \cos 3x).$

2 Evaluate the following integral:

(ii) $\int e^x \sin 2x \cos x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

$$I_2 = \int e^x \sin x \, dx$$

$u = \sin x$, $dv = e^x dx$

$du = \cos x \, dx$, $v = e^x$

$$I_2 = \sin x \cdot e^x - \int e^x \cdot \cos x \, dx$$

$$I_2 = e^x \sin x - \left[\cos x \cdot e^x - \int e^x(-\sin x) dx\right]$$

$u = \cos x$, $dv = e^x dx$

$du = -\sin x dx$, $v = e^x$

$$I_2 = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$I_2 = e^x \sin x - e^x \cos x - I_2$$

$$2I_2 = e^x(\sin x - \cos x) + C_1$$

$$I_2 = \frac{e^x}{2}(\sin x - \cos x) + C_2$$

$\left(C_2 = \frac{C_1}{2}\right)$

$$\int u \, dv = uv - \int v \, du$$

Put in ①

$$2I = \frac{e^x}{10}(\sin 3x - 3\cos 3x) + \frac{e^x}{2}(\sin x - \cos x) + C_2$$

$$I = \frac{e^x}{20}(\sin 3x - 3\cos 3x) + \frac{e^x}{4}(\sin x - \cos x) + C$$ Ans

2 Evaluate the following integral:

(iii) $\int \tan^3 \sec x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int \tan^3 x \sec x \, dx$$

$$= \int \tan^2 x \cdot \tan x \sec x \, dx$$

$$= \int (\sec^2 x - 1) \tan x \sec x \, dx$$

$$= \int \left(\sec^2 x \cdot \tan x \sec x - \tan x \sec x \right) dx$$

$$= \int (\underline{\sec x})^2 \cdot \tan x \sec x dx - \int \tan x \sec x \, dx$$

$$= \frac{\sec^3 x}{3} - \sec x + C.$$

$$\int u \, dv = uv - \int v \, du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

2 Evaluate the following integral:

(iv) $\int x^3 e^{5x}\, dx$

Chapter 3

INTEGRATION

Ex 3.4

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$$\int \underline{x}^3 \underline{e}^{5x} dx$$

$$= x^3\left(\frac{e^{5x}}{5}\right) - \int \frac{e^{5x}}{5} \cdot 3x^2 dx$$

$$= \frac{x^3}{5} e^{5x} - \frac{3}{5} \int \underline{x}^2 \underline{e}^{5x} dx$$

$$= \frac{x^3}{5} e^{5x} - \frac{3}{5}\left[x^2\left(\frac{e^{5x}}{5}\right) - \int \frac{e^{5x}}{5} \cdot 2x dx\right]$$

$$= \frac{x^3}{5} e^{5x} - \frac{3x^2}{25} e^{5x} + \frac{6}{25} \int \underline{x} \underline{e}^{5x} dx$$

$$= \frac{x^3}{5} e^{5x} - \frac{3x^2}{25} e^{5x} + \frac{6}{25}\left[x \frac{e^{5x}}{5} - \int \frac{e^{5x}}{5} dx\right]$$

$$= \frac{x^3}{5} e^{5x} - \frac{3x^2}{25} e^{5x} + \frac{6x}{125} e^{5x} - \frac{6}{625} e^{5x} + C.$$

$u = x^3$, $dv = e^{5x} dx$
$du = 3x^2 dx$, $v = \frac{e^{5x}}{5}$

$u = x^2$, $dv = e^{5x} dx$
$du = 2x dx$, $v = \frac{e^{5x}}{5}$

$u = x$, $dv = e^{5x} dx$
$du = dx$, $v = \frac{e^{5x}}{5}$

$$\int u\, dv = uv - \int v\, du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

2 Evaluate the following integral:

(vi) $\int e^{x} \cos 3x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

$$I = \int \underline{e}^{2x} \underline{\cos 3x} \, dx$$ ✓

$u = \cos 3x, \quad dv = e^{2x} dx$

$du = -3 \sin 3x \, dx, \quad v = \frac{e^{2x}}{2}$

$$I = \cos 3x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}(-3\sin 3x) dx$$

$$I = \frac{e^{2x}}{2} \cos 3x + \frac{3}{2} \int \underline{e}^{2x} \underline{\sin 3x} \, dx$$

$u = \sin 3x, \quad dv = e^{2x} dx$

$du = 3 \cos 3x \, dx, \quad v = \frac{e^{2x}}{2}$

$$I = \frac{e^{2x}}{2} \cos 3x + \frac{3}{2}\left[\sin 3x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \cdot 3 \cos 3x \, dx\right]$$

$$I = \frac{e^{2x}}{2} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} \int e^{2x} \cos 3x \, dx$$ ✓

$$I = \frac{e^{2x}}{2} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} I + C_1$$

$$\frac{13}{4} I = \frac{e^{2x}}{2}\left(\cos 3x + \frac{3}{2} \sin 3x\right) + C_1$$

$$I = \frac{\overset{2}{\cancel{4}}}{13} \cdot \frac{e^{2x}}{\cancel{2}}\left(\cos 3x + \frac{3}{2} \sin 3x\right) + \frac{4}{13} \cdot C_1 = \frac{2}{13} e^{2x}\left(\cos 3x + \frac{3}{2} \sin 3x\right) + C.$$

$$\int u \, dv = uv - \int v \, du$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

2 Evaluate the following integral:

(vii) $\int \csc^3 x \, dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int u\,dv = uv - \int v\,du$$

Let $I = \int \operatorname{cosec}^3 x \, dx = \int \operatorname{cosec} x \cdot \operatorname{cosec}^2 x \, dx$

$u = \operatorname{cosec} x$, $dv = \operatorname{cosec}^2 x \, dx$

$du = -\operatorname{cosec} x \cot x \, dx$, $v = -\cot x$

$$I = (\operatorname{cosec} x)(-\cot x) - \int (-\cot x)(-\operatorname{cosec} x \cot x) \, dx$$

$$I = -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \cot^2 x \, dx$$

$$I = -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x (\operatorname{cosec}^2 x - 1) \, dx$$

$$I = -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x \, dx + \int \operatorname{cosec} x \, dx$$

$$I = -\operatorname{cosec} x \cot x - I + \ln|\operatorname{cosec} x - \cot x| + c_1$$

$$2I = -\operatorname{cosec} x \cot x + \ln|\operatorname{cosec} x - \cot x| + c_1$$

$$I = \frac{1}{2}\left(-\operatorname{cosec} x \cot x + \ln|\operatorname{cosec} x - \cot x|\right) + C.$$

3 Show that $\int e^{ax}\sin bx\,dx = \frac{1}{\sqrt{a^2+b^2}}e^{ax}\sin(bx - \text{Tan}^{-1}\frac{b}{a}) + c$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{ax}\sin bx\,dx = \frac{1}{\sqrt{a^2+b^2}}e^{ax}\sin\left(bx - \text{Tan}^{-1}\frac{b}{a}\right) + c$$

Let $I = \int e^{ax}\underline{\sin bx}\,dx$

$$\boxed{\begin{matrix} u = \sin bx, & dv = e^{ax}dx \\ du = b\cos bx\,dx, & v = \frac{e^{ax}}{a} \end{matrix}}$$

$$I = (\sin bx)\left(\frac{e^{ax}}{a}\right) - \int \frac{e^{ax}}{a}\cdot b\cos bx\,dx$$

$$I = \frac{e^{ax}}{a}\sin bx - \frac{b}{a}\int \underline{e^{ax}}\,\underline{\cos bx}\,dx$$

$$\boxed{\begin{matrix} u = \cos bx, & dv = e^{ax}dx \\ du = -b\sin bx\,dx, & v = \frac{e^{ax}}{a} \end{matrix}}$$

$$I = \frac{e^{ax}}{a}\sin bx - \frac{b}{a}\left[\cos bx\cdot\frac{e^{ax}}{a} - \int \frac{e^{ax}}{a}\cdot(-b\sin bx)\,dx\right]$$

$$I = \frac{e^{ax}}{a}\sin bx - \frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}\int e^{ax}\sin bx\,dx$$

3 Show that $\int e^{ax}\sin bx\,dx = \frac{1}{\sqrt{a^2+b^2}}e^{ax}\sin(bx - \mathrm{Tan}^{-1}\frac{b}{a}) + c$

Chapter 3

INTEGRATION

Ex 3.4

$$\int u\,dv = uv - \int v\,du$$

$$I = \frac{e^{ax}}{a}\sin bx - \frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}I + c_1$$

$$I + \frac{b^2}{a^2}I = \frac{e^{ax}}{a}\sin bx - \frac{b}{a^2}e^{ax}\cos bx + c_1$$

$$\frac{a^2I + b^2I}{a^2} = \frac{a e^{ax}\sin bx - b e^{ax}\cos bx}{a^2} + c_1$$

$$(a^2+b^2)I = a e^{ax}\sin bx - b e^{ax}\cos bx + \frac{c_1}{a^2}$$

$$I = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + \frac{c_1}{(a^2+b^2)a^2}$$

$$I = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c \quad \text{—①}$$

3 **Show that** $\int e^{ax}\sin bx\,dx = \frac{1}{\sqrt{a^2+b^2}}e^{ax}\sin(bx - \mathrm{Tan}^{-1}\frac{b}{a}) + c$

Chapter 3

INTEGRATION

Ex 3.4

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$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Put $a = r\cos\theta$ ✓ , $b = r\sin\theta$ ✓

$$a^2+b^2 = (r\cos\theta)^2 + (r\sin\theta)^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta+\sin^2\theta) = r^2$$

$$r = \sqrt{a^2+b^2}$$ ✓

$$\frac{b}{a} = \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta} = \tan\theta$$

$$\theta = \mathrm{Tan}^{-1}\left(\frac{b}{a}\right).$$

$$a\sin bx - b\cos bx = r\cos\theta\sin bx - r\sin\theta\cos bx$$

$$= r(\cos\theta\sin bx - \sin\theta\cos bx)$$

$$= r(\sin bx\cos\theta - \cos bx\sin\theta)$$

$$= r\sin(bx - \theta)$$

3 Show that $\int e^{ax}\sin bx\,dx = \frac{1}{\sqrt{a^2+b^2}}e^{ax}\sin(bx - \mathrm{Tan}^{-1}\frac{b}{a}) + c$

$$a\sin bx - b\cos bx = \sqrt{a^2+b^2}\,\sin\left(bx - \mathrm{Tan}^{-1}\frac{b}{a}\right)$$

Put in ①

$$I = \frac{e^{ax}}{(a^2+b^2)^{1}}\left[\overset{\checkmark}{\sqrt{a^2+b^2}}\,\sin\left(bx - \mathrm{Tan}^{-1}\frac{b}{a}\right)\right] + C$$

$$I = \frac{e^{ax}}{(a^2+b^2)^{1-\frac{1}{2}}}\sin\left(bx - \mathrm{Tan}^{-1}\frac{b}{a}\right) + C$$

$$I = \frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx - \mathrm{Tan}^{-1}\frac{b}{a}\right) + C.$$

Hence proved

4 Evaluate the following indefinite integrals:

(i) $\int \sqrt{a^2 - x^2}\, dx$

Let $I = \int \sqrt{a^2 - x^2}\, dx$ ✓

$u = \sqrt{a^2-x^2}, \qquad dv = dx$

$du = \frac{1}{\cancel{2}}(a^2-x^2)^{-1/2}(-\cancel{2}x)dx, \qquad v = x$

$du = \frac{-x}{\sqrt{a^2-x^2}}dx$

$$I = \sqrt{a^2-x^2}\cdot x - \int x \cdot \left(\frac{-x}{\sqrt{a^2-x^2}}\right) dx$$

$$I = x\sqrt{a^2-x^2} - \int \frac{a^2-x^2-a^2}{\sqrt{a^2-x^2}} dx$$

$$I = x\sqrt{a^2-x^2} - \int\left(\frac{a^2-x^2}{\sqrt{a^2-x^2}} - \frac{a^2}{\sqrt{a^2-x^2}}\right) dx = x\sqrt{a^2-x^2} - \int\sqrt{a^2-x^2}\,dx + a^2\int\frac{dx}{\sqrt{a^2-x^2}}$$

$$I = x\sqrt{a^2-x^2} - I + a^2 \sin^{-1}\left(\frac{x}{a}\right) + C_1$$

$$2I = x\sqrt{a^2-x^2} + a^2 \sin^{-1}\frac{x}{a} + C_1$$

$$I = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

4 Evaluate the following indefinite integrals:

(ii) $\int \sqrt{x^2 - a^2}\, dx$

$U = \sqrt{x^2-a^2}$, $dv = dx$

$du = \frac{1}{\cancel{2}}(x^2-a^2)^{-1/2}(\cancel{2}x)dx$, $v = x$

$du = \frac{x}{\sqrt{x^2-a^2}}\, dx$

Let $I = \int \sqrt{x^2-a^2}\, dx$ ✓

$I = \sqrt{x^2-a^2}\cdot x - \int x \cdot \frac{x}{\sqrt{x^2-a^2}}\, dx$

$I = x\sqrt{a^2-x^2} - \int \frac{x^2-a^2+a^2}{\sqrt{x^2-a^2}}\, dx$

$I = x\sqrt{a^2-x^2} - \int\left(\frac{x^2-a^2}{\sqrt{x^2-a^2}} + \frac{a^2}{\sqrt{x^2-a^2}}\right)dx = x\sqrt{a^2-x^2} - \int\sqrt{x^2-a^2}\,dx - a^2\int\frac{dx}{\sqrt{x^2-a^2}}$ ✓

$I = x\sqrt{a^2-x^2} - I - a^2 \ln\left(x+\sqrt{x^2-a^2}\right) + C_1$

$2I = x\sqrt{a^2-x^2} - a^2 \ln\left(x+\sqrt{x^2-a^2}\right) + C_1$

$I = \frac{x}{2}\sqrt{a^2-x^2} - \frac{a^2}{2} \ln\left(x+\sqrt{x^2-a^2}\right) + C.$

4 Evaluate the following indefinite integrals:

(iii) $\int \sqrt{4-5x^2}\,dx$

Let $I = \int\sqrt{4-5x^2}\,dx = \int\sqrt{2^2-(\sqrt{5}x)^2}\,dx$ ✓

Let $\sqrt{5}x = t$ ✓ $\Rightarrow \sqrt{5}\,dx = dt \Rightarrow dx = \frac{1}{\sqrt{5}}dt$ ✓

$I = \int\sqrt{4-t^2}\cdot\frac{1}{\sqrt{5}}dt = \frac{1}{\sqrt{5}}\int\sqrt{4-t^2}\,dt$ ✓

$u = \sqrt{4-t^2}$, $dv = dt$

$du = \frac{1}{\cancel{2}}(4-t^2)^{-1/2}(-\cancel{2}t)dt$, $v = t$

$du = \frac{-t}{\sqrt{4-t^2}}dt$

$$I = \frac{1}{\sqrt{5}}\left[\sqrt{4-t^2}\cdot t - \int t\cdot\frac{-t}{\sqrt{4-t^2}}dt\right]$$

$$I = \frac{1}{\sqrt{5}}t\sqrt{4-t^2} - \frac{1}{\sqrt{5}}\int\frac{4-t^2-4}{\sqrt{4-t^2}}dt$$

4 Evaluate the following indefinite integrals:

(iii) $\int \sqrt{4-5x^2}\,dx$

$$I = \frac{1}{\sqrt{5}} t\sqrt{4-t^2} - \frac{1}{\sqrt{5}}\int\left(\frac{4-t^2}{\sqrt{4-t^2}} - \frac{4}{\sqrt{4-t^2}}\right)dt$$

$$I = \frac{1}{\sqrt{5}} t\sqrt{4-t^2} - \boxed{\frac{1}{\sqrt{5}}\int\sqrt{4-t^2}\,dt} + \frac{4}{\sqrt{5}}\int\frac{dt}{\sqrt{4-t^2}}$$

$$I = \frac{1}{\sqrt{5}} t\sqrt{4-t^2} - I + \frac{4}{\sqrt{5}}\sin^{-1}\left(\frac{t}{2}\right) + C_1$$

$$2I = \frac{1}{\cancel{\sqrt{5}}}\cancel{\sqrt{5}}x\sqrt{4-(\sqrt{5}x)^2} + \frac{4}{\sqrt{5}}\sin^{-1}\left(\frac{\sqrt{5}x}{2}\right) + C_1$$

$$I = \frac{x}{2}\sqrt{4-5x^2} + \frac{\overset{2}{\cancel{4}}}{\cancel{2}\sqrt{5}}\sin^{-1}\left(\frac{\sqrt{5}x}{2}\right) + \frac{C_1}{2}$$

$$I = \frac{x}{2}\sqrt{4-5x^2} + \frac{2}{\sqrt{5}}\sin^{-1}\left(\frac{\sqrt{5}x}{2}\right) + C.$$

4 Evaluate the following indefinite integrals:

(iv) $\int \sqrt{3-4x^2}dx$

Let $I = \int \sqrt{3-4x^2}\,dx = \int \sqrt{3-(2x)^2}\,dx$

Let $t = 2x$, $dt = 2\,dx$, $dx = \frac{1}{2}\,dt$

$I = \int \sqrt{3-t^2}\,\frac{1}{2}dt = \frac{1}{2}\int \sqrt{3-t^2}\,dt$

$u = \sqrt{3-t^2}$, $dv = dt$

$du = \frac{1}{\cancel{2}}(3-t^2)^{-1/2}(-\cancel{2}t)dt$, $v = t$

$du = \frac{-t}{\sqrt{3-t^2}}\,dt$

$I = \frac{1}{2}\left[\sqrt{3-t^2}\cdot t - \int t\left(\frac{-t}{\sqrt{3-t^2}}\right)dt\right]$

$I = \frac{t}{2}\sqrt{3-t^2} - \frac{1}{2}\int \frac{3-t^2-3}{\sqrt{3-t^2}}\,dt$

$I = \frac{t}{2}\sqrt{3-t^2} - \frac{1}{2}\int \left(\frac{3-t^2}{\sqrt{3-t^2}} - \frac{3}{\sqrt{3-t^2}}\right)dt$

4 Evaluate the following indefinite integrals:

(iv) $\int \sqrt{3-4x^2}\,dx$

$$-\frac{1}{2}\int\left(\frac{3-t^2}{\sqrt{3-t^2}}-\frac{3}{\sqrt{3-t^2}}\right)dt$$

$$I = \frac{t}{2}\sqrt{3-t^2} - \boxed{\frac{1}{2}\int\sqrt{3-t^2}\,dt} + \frac{3}{2}\int\frac{dt}{\sqrt{(\sqrt{3})^2-t^2}}$$

$$I = \frac{t}{2}\sqrt{3-t^2} - I + \frac{3}{2}\sin^{-1}\left(\frac{t}{\sqrt{3}}\right) + c_1$$

$$2I = \frac{t}{2}\sqrt{3-t^2} + \frac{3}{2}\sin^{-1}\left(\frac{t}{\sqrt{3}}\right) + c_1$$

$$I = \frac{\cancel{2}x}{\cancel{4}2}\sqrt{3-(2x)^2} + \frac{3}{2}\sin^{-1}\left(\frac{2x}{\sqrt{3}}\right) + c \qquad \boxed{c = \frac{c_1}{2}}$$

$$I = \frac{x}{2}\sqrt{3-4x^2} + \frac{3}{2}\sin^{-1}\left(\frac{2x}{\sqrt{3}}\right) + c.$$

4 Evaluate the following indefinite integrals:

(v) $\int \sqrt{x^2 + 4}dx$

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

Let $I = \int \sqrt{x^2+4}\, dx$ ✓

$U = \sqrt{x^2+4}$, $dv = dx$

$du = \frac{1}{\cancel{2}}(x^2+4)^{\frac{1}{2}-1}(\cancel{2}x)dx$, $v = x$

$du = \frac{x}{\sqrt{x^2+4}} dx$

$I = \sqrt{x^2+4}\cdot x - \int x \frac{x}{\sqrt{x^2+4}} dx$

$I = x\sqrt{x^2+4} - \int \frac{x^2+4-4}{\sqrt{x^2+4}} dx$

$I = x\sqrt{x^2+4} - \int\left(\frac{x^2+4}{\sqrt{x^2+4}} - \frac{4}{\sqrt{x^2+4}}\right)dx = x\sqrt{x^2+4} - \int\sqrt{x^2+4}\, dx + 4\int\frac{dx}{\sqrt{x^2+4}}$

$I = x\sqrt{x^2+4} - I + 4\ln\left(x+\sqrt{x^2+2^2}\right) + C_1$

$2I = x\sqrt{x^2+4} + 4\ln\left(x+\sqrt{x^2+4}\right) + C_1$

$I = \frac{x}{2}\sqrt{x^2+4} + 2\ln\left(x+\sqrt{x^2+4}\right) + C.$

4 Evaluate the following indefinite integrals:

(v) $\int x^2 e^{ax} dx$

Let $I = \int \underline{x^2}\, \underline{e^{ax}}\, dx$

$u = x^2, \quad dv = e^{ax} dx$
$du = 2x\,dx, \quad v = \frac{e^{ax}}{a}$

$$I = x^2 \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} \cdot 2x\, dx$$

$$I = \frac{x^2}{a} e^{ax} - \frac{2}{a} \int \underline{x}\, \underline{e^{ax}}\, dx$$

$u = x, \quad dv = e^{ax} dx$
$du = dx, \quad v = \frac{e^{ax}}{a}$

$$I = \frac{x^2}{a} e^{ax} - \frac{2}{a} \left[x \cdot \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} \cdot dx \right]$$

$$I = \frac{x^2}{a} e^{ax} - \frac{2}{a^2} x e^{ax} + \frac{2}{a^2} \int e^{ax} dx$$

$$I = \frac{x^2}{a} e^{ax} - \frac{2x}{a^2} e^{ax} + \frac{2}{a^2} \left(\frac{e^{ax}}{a} \right) + C.$$

$$I = \frac{x^2}{a} e^{ax} - \frac{2x}{a^2} e^{ax} + \frac{2}{a^3} e^{ax} + C.$$

5 Evaluate the following integrals:

(i) $\int e^x \left(\frac{1}{x} + \ln x\right) dx$

$$\int e^{x}\left(\frac{1}{\underline{x}} + \underline{\ln} x\right) dx$$

$$= \int e^{x}\left(\ln x + \frac{1}{x}\right) dx$$

$$= e^{x} \ln x + c$$

$a = 1$

$f(x) = \ln x$

$$\int e^{ax}[af(x) + f'(x)]\,dx = e^{ax} f(x) + c$$

5 Evaluate the following integrals:

(ii) $\int e^x(\cos x + \sin x)\,dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int e^x(\cos x + \sin x)\,dx$$

$$= \int e^x(\sin x + \cos x)\,dx$$

$$= e^x \sin x + C.$$

$a = 1$

$f(x) = \sin x$

$$\int e^{ax}[af(x) + f'(x)]\,dx = e^{ax}f(x) + c$$

5 Evaluate the following integrals:

(iv) $\int e^{3x}\left(\frac{3\sin x-\cos x}{\sin^2 x}\right)dx$

$$\int e^{ax}[af(x)+f'(x)]\,dx = e^{ax}f(x)+c$$

$$\int e^{3x}\left(\frac{3\sin x-\cos x}{\sin^2 x}\right)dx$$

$$= \int e^{3x}\left(\frac{3\cancel{\sin x}}{\sin^{\cancel{2}} x} - \frac{\cos x}{\sin^2 x}\right)dx$$

$$= \int e^{3x}\left(\frac{3}{\sin x} - \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\right)dx$$

$$= \int e^{3x}\left(3\,\underline{\text{cosec}\,x} - \cot x\,.\,\text{cosec}\,x\right)dx$$

$$= e^{3x}\,\text{cosec}\,x + C.$$

$a = 3$

$f(x) = \text{cosec}\,x$

$f'(x) = -\cot x\ \text{cosec}$

5 Evaluate the following integrals:

(v) $\int e^{2x}(-\sin x + \cos x)\,dx$

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

$$\int e^{2x}\left[-\sin x + 2\cos x\right] dx$$

$$= \int e^{2x}\left[2\cos x - \sin x\right] dx$$

$$= \int e^{2x}\left[2\cos x + (-\sin x)\right] dx$$

$$= e^{2x}\cos x + C$$

$a = 2$ ✓

$f(x) = \cos x$ ✓

$f'(x) = -\sin x$

$$\int e^{ax}[af(x) + f'(x)]\,dx = e^{ax}f(x) + c$$

5 **Evaluate the following integrals:**

(vi) $\int \frac{xe^x}{(1+x)^2}\,dx$

Chapter 3

INTEGRATION

Ex 3.4

$$\int \frac{x e^x}{(1+x)^2}\,dx$$

$$= \int e^x \cdot \frac{1+x-1}{(1+x)^2}\,dx$$

$$= \int e^x \left[\frac{1+x}{(1+x)^2} - \frac{1}{(1+x)^2} \right] dx$$

$$= \int e^{x} \left[\underline{(1+x)^{-1}} + \left\{ \underline{-(1+x)^{-2}} \right\} \right] dx$$

$$= e^x (1+x)^{-1} + C$$

$$= \frac{e^x}{1+x} + C.$$

$\alpha = 1$
$f(x) = (1+x)^{-1}$
$f'(x) = -(1+x)^{-2}$

$$\int e^{ax}[af(x) + f'(x)]\,dx = e^{ax} f(x) + c$$

5 Evaluate the following integrals:

(vii) $\int e^{-x}(\cos x - \sin x)\, dx$

$\int e^{-x}(\cos x - \sin x)\, dx$

$= \int e^{-x}\left[-\sin x + \cos x\right] dx$

$= \int e^{-x}\left[(-1)\sin x + \cos x\right] dx$

$= e^{-x}\sin x + C.$

$a = -1$

$f(x) = \sin x$

$f'(x) = \cos x$

$$\int e^{ax}[af(x) + f'(x)]\, dx = e^{ax} f(x) + c$$

5 Evaluate the following integrals:

(viii) $\int \frac{xe^{x}}{(1+x)^{2}}\,dx$

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

Let $I=\int \frac{e^{m\,\mathrm{Tan}^{-1}x}}{1+x^{2}}\,dx$

$I=\int e^{m\,\mathrm{Tan}^{-1}x}\cdot\frac{1}{1+x^{2}}\,dx$

Let $t=\mathrm{Tan}^{-1}x$

$dt=\frac{1}{1+x^{2}}\,dx$ ✓

$$\int e^{ax}[af(x)+f'(x)]\,dx=e^{ax}f(x)+c$$

$I=\int e^{mt}.\,dt$

$=\frac{e^{mt}}{m}+C$

$=\frac{e^{m\,\mathrm{Tan}^{-1}x}}{m}+C.$

5 Evaluate the following integrals:

(xi) $\int \frac{2x}{1-\sin x}\,dx$

Chapter 3

INTEGRATION

Ex 3.4

$$I = uv - \int v\,du$$

$$I = x\left(2\cot\left(45^\circ - \frac{x}{2}\right)\right) - \int 2\cot\left(45^\circ - \frac{x}{2}\right)dx$$

$$= 2x\cot\left(45^\circ - \frac{x}{2}\right) - 2\int \cot\left(45^\circ - \frac{x}{2}\right)dx$$

$$= 2x\cot\left(45^\circ - \frac{x}{2}\right) - 2\left[\frac{\ln\left(45^\circ - \frac{x}{2}\right)}{-1/2}\right] + c$$

$$= 2x\cot\left(45^\circ - \frac{x}{2}\right) + 4\ln\left(45^\circ - \frac{x}{2}\right) + C.$$

- **I** Inverse
- **L** Logarithmic
- **A** Algebraic
- **T** Trigonometric
- **E** Exponential

5 Evaluate the following integrals:

(x) $\int \frac{e^{x}(1+x)}{(2+x)^{2}}\,dx$

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

$$\int e^{ax}[af(x)+f'(x)]\,dx = e^{ax}f(x)+c$$

$$\int \frac{e^{x}(1+x)}{(2+x)^{2}}\,dx$$

$$= \int e^{x}\cdot\frac{1+x}{(2+x)^{2}}\,dx = \int e^{x}\cdot\frac{2+x-1}{(2+x)^{2}}\,dx$$

$$= \int e^{x}\left[\frac{2+x}{(2+x)^{2}} - \frac{1}{(2+x)^{2}}\right]dx$$

$$= \int e^{x}\left[\underline{(2+x)^{-1}} + \left\{\underline{-(2+x)^{-2}}\right\}\right]dx$$

$$= e^{x}(2+x)^{-1} + C$$

$$= \frac{e^{x}}{2+x} + C$$

$\alpha = 1$

$f(x) = (2+x)^{-1}$

$f'(x) = -(2+x)^{-2}$

5 Evaluate the following integrals:

(xi) $\int\left(\frac{1-\sin x}{(1-\cos x}\right)e^{x}\,dx$

Chapter 3

INTEGRATION

Ex 3.4

Ucademy

$$\int\left(\frac{1-\sin x}{1-\cos x}\right)e^{x}dx$$

$$= \int e^{x}\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^{2}\frac{x}{2}}\right)dx \;=\; \int e^{x}\left(\frac{1}{2\sin^{2}\frac{x}{2}} - \frac{\cancel{2}\cancel{\sin\frac{x}{2}}\cos\frac{x}{2}}{\cancel{2}\sin^{\cancel{2}}\frac{x}{2}}\right)d$$

$$= \int e^{x}\left(\frac{1}{2}\operatorname{cosec}^{2}\frac{x}{2} - \cot\frac{x}{2}\right)dx$$

$$= -\int e^{x}\left(\underline{\cot\frac{x}{2}} + \underline{\left(-\frac{1}{2}\operatorname{cosec}^{2}\frac{x}{2}\right)}\right)dx$$

$$= -\,e^{x}\cot\frac{x}{2} + c.$$

$$\int e^{ax}[af(x)+f'(x)]\,dx = e^{ax}f(x)+c$$

$a = 1$

$f(x) = \cot\frac{x}{2}$

$f'(x) = \left(-\operatorname{cosec}^{2}\frac{x}{2}\right)\frac{1}{2}$

$= -\frac{1}{2}\operatorname{cosec}^{2}\frac{x}{2}$

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