Exercise 1.3 (Solutions)
Mathematics (Science Group): 10th
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Question # 1

$2x^4 - 11x^2 + 5 = 0$

Solution: $2x^4 - 11x^2 + 5 = 0$

Let $x^2 = y \Rightarrow \left(x^2\right)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ *and* $x^4 = y^2$

$2x^4 - 11x^2 + 5 = 0$

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5)$

$(y-5)(2y-1)$

$y - 5 = 0$ *or* $2y - 1 = 0$

$y = 5$ *or* $2y = 1$

$y = 5$ *or* $y = \frac{1}{2}$

Put in $x^2 = y$

$x^2 = 5$ *or* $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{5}$ $\quad \sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\sqrt{5}$ $\quad x = \pm\frac{1}{\sqrt{2}}$

Solution Set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

Question # 2

$2x^4 = 9x^2 - 4$

Solution: $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$ ......(1)

Let $x^2 = y$ ......(2)

Taking square on both sides

$\left(x^2\right)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ *and* $x^4 = y^2$ *in eq*(1)

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0$ $\quad 2y - 1 = 0$

$y = 4$ $\quad 2y = 1$

$y = 4$ $\quad y = \frac{1}{2}$

Put $y = x^2$

$x^2 = 4$ $\quad x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{4}$ $\quad \sqrt{x^2} = \sqrt{\frac{1}{2}}$

$x = \pm 2$ $\quad x = \pm\frac{1}{\sqrt{2}}$

Solution set is $\left\{\pm 2, \pm\frac{1}{\sqrt{2}}\right\}$

Question # 3

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution: $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$...(1)

Let $x^{\frac{1}{4}} = y$ .... (2)

Taking square on both sides

$\left(x^{\frac{1}{4}}\right)^2 = y^2$

$x^{\frac{1}{2}} = y^2$

Put $x^{\frac{1}{4}} = y$ *and* $x^{\frac{1}{2}} = y^2$

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

$5y^2 = 7y - 2$

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(y-1)(5y-2) = 0$

$y - 1 = 0$ *or* $5y - 2 = 0$

$y=1 \quad or \quad 5y=2$

$y=1 \quad or \quad y=\frac{2}{5}$

$Put\ \ y=x^{\frac{1}{4}}$

$x^{\frac{1}{4}}=1 \qquad or \quad x^{\frac{1}{4}}=\frac{2}{5}$

$\left(x^{\frac{1}{4}}\right)^4=(1)^4 \quad or \quad \left(x^{\frac{1}{4}}\right)^4=\left(\frac{2}{5}\right)^4$

$x=1 \qquad x=\frac{2^4}{5^4}$

$x=1 \qquad x=\frac{16}{625}$

$So,\ the\ solution\ set\ is\ \left\{1,\frac{16}{625}\right\}$

Question # 4

$x^{\frac{2}{3}}+54=15x^{\frac{1}{3}}$

$Solution: x^{\frac{2}{3}}+54=15x^{\frac{1}{3}}$ .....(1)

$Let\quad x^{\frac{1}{3}}=y$ ....(2)

$Taking\ square$

$\left(x^{\frac{1}{3}}\right)^2=y^2 \Rightarrow x^{\frac{2}{3}}=y^2\ in\,eq(1)$

$x^{\frac{2}{3}}+54=15x^{\frac{1}{3}}$

$y^2+54=15y$

$y^2-15y+54=0$

$y^2-9y-6y+54=0$

$y(y-9)-6(y-9)=0$

$(y-9)(y-6)$

$y-9=0 \quad y-6=0$

$y=9 \qquad y=6$

$Put\ \ y=x^{\frac{1}{3}}$

$x^{\frac{1}{3}}=9 \qquad x^{\frac{1}{3}}=6$

$\left(x^{\frac{1}{3}}\right)^3=(9)^3 \quad \left(x^{\frac{1}{3}}\right)=(6^3)$

$x=9^3 \qquad x=6^3$

$x=729 \qquad x=216$

$So\,the\,solution\,set\,is\ \{729,216\}$

Question # 5

$3x^{-2}+5=8x^{-1}$

$Solution: 3x^{-2}+5=8x^{-1}$........(1)

$x^{-1}=y \qquad x^{-2}=y^2$

$Put\ \ x^{-1}=y\ \ and\ \ x^{-2}=y^2\ in\,eq\,(1)$

$3y^2+5=8y$

$3y^2-8y+5=0$

$3y^2-3y-5y+5=0$

$3y(y-1)-5(y-1)=0$

$(3y-5)(y-1)=0$

$3y-5=0 \qquad y-1=0$

$3y=5 \qquad y=1$

$y=\frac{5}{3} \qquad y=1$

$From\ eq\,(2)\ put\ y=x^{-1}$

$x^{-1}=\frac{5}{3} \qquad x^{-1}=1$

$\frac{1}{x}=\frac{5}{3} \qquad \frac{1}{x}=1$

$x=\frac{3}{5} \qquad x=1$

$S.Set=\left\{\frac{3}{5},1\right\}$

Question # 6

$\left(2x^2+1\right)+\frac{3}{2x^2+1}=4$

$Solution:$

$\left(2x^2+1\right)+\frac{3}{2x^2+1}=4$ ....(1)

$Let\ 2x^2+1=y$

$y+\frac{3}{y}=4$

$Multiplying\ \ both\ \ sides\,by\ "y"$

$y^2+3=4y$

$y^2-4y+3=0$

$y^2-3y-1y+3=0$

$y(y-3)-1(y-3)=0$

$(y-3)(y-1)=0$

$y-1=0 \qquad y-3=0$

$y=1 \qquad y=3$

$Put\ \ y=2x^2+1$

$2x^2+1=3 \qquad 2x^2+1=1$

$2x^2=3-1 \qquad 2x^2=1-1$

$2x^2=2 \qquad 2x^2=0$

$x^2=\frac{2}{2} \qquad x^2=\frac{0}{2}$

$x^2=1 \qquad x^2=0$

$\sqrt{x^2}=\pm\sqrt{1} \qquad \sqrt{x^2}=\pm\sqrt{0}$

$x=\pm 1 \qquad x=0$

$S.Set=\{-1,1,0\}$

Question # 7

$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$

$Solution: \frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$

$Let\ \frac{x}{x-3}=y \Rightarrow \frac{x-3}{x}=\frac{1}{y}$

Equcaton become

$y+4\left(\frac{1}{y}\right)=4$

Multiplying both side by " y *"*

$y^2+4=4y$

$y^2-4y+4=0$

$y^2-2y-2y+4=0$

$y(y-2)-2(y-2)=0$

$(y-2)(y-2)$

$y-2=0 \Rightarrow y=2$

Put the value of y

$\frac{x}{x-3}=2 \Rightarrow x=2(x-3)$

$x=2x-6 \Rightarrow 6=2x-x$

$x=6$

$S.Set=\{6\}$

Question # 8

$\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$

$Solution: \frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}....(1)$

$\frac{4x+1}{4x-1}=y....(2) \Rightarrow \frac{4x-1}{4x+1}=\frac{1}{y}$

Equation (1) *become*

$y+\frac{1}{y}=2\frac{1}{6}$

$y+\frac{1}{y}=\frac{13}{6}$

Multiplying both sides by " $6y$ *"*

$6y^2+6=13y$

$6y^2-13y-6=0$

$6y^2-9y-4y-6=0$

$3y(2y-3)-2(2y-3)=0$

$(2y-3)(3y-2)$

$2y-3=0 \qquad 3y-2=0$

$2y=3 \qquad 3y=2$

$y=\frac{3}{2} \qquad y=\frac{2}{3}$

$\frac{4x+1}{4x-1}=\frac{3}{2}$

$2(4x+1)=3(4x-1)$

$8x+2=12x-3$

$2+3=12x-8x$

$5=4x$

$x=\frac{5}{4}$

$\frac{4x+1}{4x-1}=\frac{2}{3}$

$3(4x+1)=2(4x-1)$

$12x+3=8x-2$

$12x-8x=-2-3$

$4x=-5$

$x=-\frac{5}{4}$

$S.Set=\left\{\pm\frac{5}{4}\right\}$

<u>Question # 9</u>

$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$

$Solution: \frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$

$Let\ \ \frac{x-a}{x+a}=y\ \ or \frac{x+a}{x-a}=\frac{1}{y}$

$$y-\frac{1}{y}=\frac{7}{12}$$
$$\frac{y^2-1}{y}=\frac{7}{12}$$
$$12(y^2-1)=7y$$
$$12y^2-12=7y \Rightarrow 12y^2-7y-12=0$$
$$12y^2-16y+9y-12=0$$
$$4y(3y-4)+3(3y-4)$$
$$(3y-4)(4y+3)$$
$$3y-4=0 \quad 4y+3=0$$
$$3y=4 \qquad 4y=-3$$
$$y=\frac{4}{3} \qquad y=-\frac{3}{4}$$
$$\frac{x-a}{x+a}=\frac{4}{3} \qquad \frac{x+a}{x-a}=-\frac{3}{4}$$
$$3(x-a)=4(x+a)$$
$$3x-3a=4x+4a$$
$$-3a-4a=4x-3x$$
$$-7a=x$$
$$x=-7a$$
$$\frac{x+a}{x-a}=-\frac{3}{4}$$
$$4(x+a)=-3(x-a)$$
$$4x+4a=-3x+3a$$
$$4x+3x=3a-4a$$
$$7x=-a$$
$$x=\frac{a}{7}$$
$$S.Set=\left\{-7a,\frac{a}{7}\right\}$$

Question # 10

$$x^4-2x^3-2x^2+2x+1=0$$

Solution:

$$x^4-2x^3-2x^2+2x+1=0$$

Dividing both sides by "x^2"

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=\frac{0}{x^2}$$
$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$
$$\left(x^2+\frac{1}{x^2}\right)-2x+\frac{2}{x}-2=0$$
$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0 \quad ......(1)$$

Let $x-\frac{1}{x}=y$ ......(2)

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=(y)^2$$
$$x^2+\frac{1}{x^2}-2=y^2$$
$$x^2+\frac{1}{x^2}=y^2+2$$

Putting values in eq.(1)

$$y^2+2-2(y)-2=0$$
$$y^2-2y=0$$
$$y(y-2)=0$$
$$y=0 \qquad y-2=0$$
$$y=0 \qquad y=2$$

Put $y=x-\frac{1}{x}$ *from eq.*2

$$x-\frac{1}{x}=0 \qquad x-\frac{1}{x}=2$$
$$\frac{x^2-1}{x}=0 \qquad \frac{x^2-1}{x}=2$$
$$x^2-1=0 \qquad x^2-1=2x$$
$$x^2=1 \qquad x^2-2x-1=0$$
$$\sqrt{x^2}=\pm\sqrt{1} \quad 1x^2-2x-1=0$$
$$x=\pm 1$$

Solving $1x^2-2x-1=0$ *by quadratic formula*

$a=1 \;\; b=-2 \;\; c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4a(1)(-1)}}{2(1)}$$
$$x=\frac{+2\pm\sqrt{4+4}}{2}$$
$$x=\frac{2\pm\sqrt{8}}{2}$$
$$x=\frac{2\pm\sqrt{4\times 2}}{2}$$
$$x=\frac{2\pm 2\sqrt{2}}{2}$$

$x=\frac{2(1\pm\sqrt{2})}{2}$

$x=1\pm\sqrt{2}$

$S.Set=\left\{\pm1,1\pm\sqrt{2}\right\}$

Question # 11

$2x^4+x^3-6x^2+x+2=0$

$Solution:$

$2x^4+x^3-6x^2+x+2=0$

$Dividing\ each\ term\ by\ "x^2"$

$\frac{2x^4}{x^2}+\frac{x^3}{x^2}-\frac{6x^2}{x^2}+\frac{x}{x^2}+\frac{2}{x^2}=\frac{0}{x^2}$

$2x^2+x-6+\frac{1}{x}+\frac{2}{x^2}=0$

$2x^2+\frac{2}{x^2}+x+\frac{1}{x}-6=0$

$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0$ ......(1)

$Let\ x+\frac{1}{x}=y$ ......(2)

$Taking\ square\ on\ both\ side$

$\left(x+\frac{1}{x}\right)^2=(y)^2$

$x^2+\frac{1}{x^2}+2x\frac{1}{x}=y^2$

$x^2+\frac{1}{x^2}+2=y^2$

$x^2+\frac{1}{x^2}=y^2-2$

$Put\ x+\frac{1}{x}=y\ and\ x^2+\frac{1}{x^2}=y^2-2\ in\ eq\ (1)$

$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0$

$2y^2+5y-4y-10=0$

$y(2y+5)-2(2y+5)=0$

$2y+5=0 \quad y-2=0$

$2y=-5 \quad y=2$

$y=-\frac{5}{2} \quad y=2$

$Put\ x+\frac{1}{x}=y\ \ in\ eq\ (2)$

$x+\frac{1}{x}=-\frac{5}{2}$

$\frac{x^2+1}{x}=-\frac{5}{2}$

$2(x^2+1)=-5x$

$2x^2+2=-5x$

$2x^2+5x+2=0$

$2x^2+4x+x+2=0$

$2x(x+2)+1(x+2)=0$

$(x+2)(2x+1)$

$x+2=0 \quad 2x+1=0$

$x=-2 \quad 2x=-1$

$x=-2 \quad x=-\frac{1}{2}$

Also we have

$x+\frac{1}{x}=2$

$\frac{x^2+1}{x}=2$

$x^2+1=2x$

$x^2-2x+1=0$

$x^2-x-x+1=0$

$x(x-1)-1(x-1)=0$

$(x-1)(x-1)=0$

$x-1=0$

$x=1$

$S.Set=\left\{1,-2,-\frac{1}{2}\right\}$

Question # 12

$4.2^{2x+1}-9.2^x+1=0$

$Solution:\ 4.2^{2x+1}-9.2^x+1=0$

$4.2^{2x}.2^1-9.2^x+1=0$

$8.(2^x)^2.-9.2^x+1=0$

$Let\ 2^x=y \quad (2^x)^2=y^2$

$8y^2-9y+1=0$

$8y^2-8y-y+1=0$

$8y(y-1)-1(y-1)=0$

$(8y-1)(y-1)=0$

$8y-1=0 \quad y-1=0$

$8y=1 \quad y=1$

$y=\frac{1}{8} \quad y=1$

Put the value of y in above equation

$2^x = y \quad 2^x = 1$

$2^x = \frac{1}{8} \quad 2^x = 2^0$

$2^x = \frac{1}{2^3} \quad 2^x = 2^0$

$2^x = 2^{-3} \quad x = 0$

$x=-3$

$S.Set = \{-3, 0\}$

Question # 13

$3^{2x+2} = 12.3^x - 3$

$Solution: 3^{2x+2} = 12.3^x - 3$

$3^{2x}3^2 - 12.(3^x) + 3 = 0$

$9(3^x)^2 - 12.(3^x) + 3 = 0 \quad ......(1)$

$Let\ 3^x = y$

$Put\ 3^x = y\ in\ eq.(1)$

$9y^2 - 12y + 3 = 0$

$9y^2 - 9y - 3y + 3 = 0$

$9y(y-1) - 3(y-1) = 0$

$(y-1)(9y-3) = 0$

$y-1=0 \quad 9y-3=0$

$y=1 \quad 9y=3$

$y=\frac{3}{9}$

$Put\ y = 3^x\ from\ eq.(2)$

$3^x = 1 \quad 3^x = \frac{1}{3}$

$3^x = 3^0 \quad 3^x = 3^{-1}$

$x=0 \quad x=-1$

$S.Set = \{0, -1\}$

Question # 14

$2^x + 64.2^{-x} - 20 = 0$

$Solution:$

$2^x + 64.2^{-x} - 20 = 0$

$2^x + \frac{64}{2^x} - 20 = 0 \quad ......(1)$

$Let\ 2^x = y \quad ......(2)$

$Put\ 2^x = y\ in\ eq.(1)$

$y + \frac{64}{y} - 20 = 0$

Multiply both sides by "y"

$y^2 + 64 - 20y = 0$

$y^2 - 20y + 64 = 0$

$y^2 - 16y - 4y + 64 = 0$

$y(y-16) - 4(y-16) = 0$

$(y-16)(y-4) = 0$

$y-16=0 \quad y-4=0$

$y=16 \quad y=4$

$Put\ y = 2^x\ from\ eq.(2)$

$2^x = 16 \quad 2^x = 4$

$2^x = 2^4 \quad 2^x = 2^2$

$x=4 \quad x=2$

$S.Set = \{2, 4\}$

Question # 15:

$(x+1)(x+3)(x-5)(x-7) = 192$

$Solution:$

$(x+1)(x+3)(x-5)(x-7) = 192$

$\because \ a+b = c+d$

$1-5 = 3-7$

$-4 = -4$

$(x+1)(x-5)(x+3)(x-7) = 192$

$(x^2 - 5x + 1x - 5)(x^2 - 7x + 3x - 21) = 192$

$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \quad ......(1)$

$Let\ x^2 - 4x = y \quad ......(2)$

So, eq. (1) becomes

$(y-5)(y-21) = 192$

$y^2 - 21y - 5y + 105 = 192$

$y^2 - 26y + 105 - 192 = 0$

$y^2 - 26y - 87 = 0$

$y^2 - 29y + 3y - 87 = 0$

$y(y-29) + 3(y-29) = 0$

$(y-29)(y+3) = 0$

$y-29=0 \quad y+3=0$

$y=29 \quad y=-3$

Put $y=x^2-4x$ *in eq.*(2)

$x^2-4x=29 \quad x^2-4x=-3$

$x^2-4x-29=0 \quad x^2-4x+3=0$

Solve $x^2-4x-29=0$ *by quadratic formula*

$a=1 \quad b=-4 \quad c=-29$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16+116}}{2}$$

$$x=\frac{4\pm\sqrt{132}}{2}$$

$$x=\frac{4\pm\sqrt{4\times 33}}{2}$$

$$x=\frac{4\pm 2\sqrt{33}}{2}$$

$$x=\frac{2(2\pm\sqrt{33})}{2}$$

$x=2\pm\sqrt{33}$

Solve $x^2-4x+3=0$ *by factorization*

$x^2-4x+3=0$

$x^2-3x-x+3=0$

$x(x-3)-1(x-3)=0$

$(x-3)(x-1)=0$

$x-3=0 \quad x-1=0$

$x=3 \quad x=1$

$S.Set=\{1,3,2\pm\sqrt{33}\}$

Question # 16

$(x-1)(x-2)(x-8)(x+5)+360=0$

Solution:

$(x-1)(x-2)(x-8)(x+5)+360=0$

$[(x-1)(x-2)][(x-8)(x+5)]+360=0$

$[x^2-2x-1x+2][x^2+5x-8x-40]+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0 \quad \text{......(1)}$

Let $x^2-3x=y \quad \text{......(2)}$

Put in equation (1)

$(y+2)(y-40)+360=0$

$y^2-40y+2y-80+360=0$

$y^2-38y+280=0$

$y^2-28y-10y+280=0$

$y(y-28)-10(y-28)=0$

$(y-28)(y-10)=0$

$y-28=0 \quad y-10=0$

$y=28 \quad y=10$

Put $y=x^2-3x$ *from eq.*(2)

$x^2-3x=28$

$x^2-3x-28=0$

$x^2-7x+4x-28=0$

$x(x-7)+4(x-7)=0$

$(x-7)(x+4)=0$

$x-7=0 \quad x+4=0$

$x=7 \quad x=-4$

$x^2-3x=10$

$x^2-3x-10=0$

$x^2-5x+2x-10=0$

$x(x-5)+2(x-5)=0$

$(x-5)(x+2)=0$

$x-5=0 \quad x+2=0$

$x=5 \quad x=-2$

$S.Set=\{5,-2,7,-4\}$

Book: ***Exercise 1.3***
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