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Exercise 2.3 (Solutions)

Mathematics (Science Group): 10th

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Q.1 Without solving, find the sum and product of the roots of following quadratic equations:

(i) $x^2-5x+3=0$

$$Solution: x^2-5x+3=0$$
$$ax^2+bx+c=0$$
$$a=1,\ b=-5,\ c=3$$
$$Sum\ of\ roots=S=\frac{-b}{a}=-\left(\frac{-5}{1}\right)=5$$
$$\text{Product of roots}=P=\frac{c}{a}=\frac{3}{1}=3$$

(ii) $3x^2+7x-11=0$

$$Solution: 3x^2+7x-11=0$$
$$ax^2+bx+c=0$$
$$a=3,\ b=7,\ c=-11$$
$$Sum\ of\ roots=S=\frac{-b}{a}=\frac{-7}{3}$$
$$\text{Product of roots}=P=\frac{c}{a}=\frac{-11}{3}$$

(iii) $px^2-qx+r=0$

$$Solution: px^2-qx+r=0$$
$$ax^2+bx+c=0$$
$$a=p,\ b=-q,\ c=r$$
$$Sum\ of\ roots=S=\frac{-b}{a}=-\left(\frac{-q}{p}\right)=\frac{q}{p}$$
$$\text{Product of roots}=P=\frac{c}{a}=\frac{r}{p}$$

(iv) $(a+b)x^2-ax+b=0$

$$Solution: (a+b)x^2-ax+b=0$$
$$Ax^2+Bx+C=0$$
$$A=(a+b),\ B=-a,\ C=b$$
$$Sum\ of\ roots=S=\frac{-b}{a}=-\left(\frac{-a}{a+b}\right)=\frac{-a}{a+b}$$
$$\text{Product of roots}=P=\frac{c}{a}=\frac{b}{a+b}$$

(v) $(l+m)x^2+(m+n)x+n-l=0$

$$Solution: (l+m)x^2+(m+n)x+n-l=0$$
$$ax^2+bx+c=0$$
$$a=(l+m),\ b=(m+n),\ c=n-l$$
$$Sum\ of\ roots=S=\frac{-b}{a}=\frac{-(m+n)}{l+m}$$
$$\text{Product of roots}=P=\frac{c}{a}=\frac{n-l}{l+m}$$

(vi) $7x^2-5mx+9n=0$

$$Solution: 7x^2-5mx+9n=0$$
$$ax^2+bx+c=0$$
$$a=7,\ b=-5m,\ c=9n$$
$$Sum\ of\ roots=S=\frac{-b}{a}=-\left(\frac{-5m}{7}\right)=\frac{5m}{7}$$
$$\text{Product of roots}=P=\frac{c}{a}=\frac{9n}{7}$$

Q.2 Find the value of k if.

(i) Sum of the roots of equation $2kx^2-3x+4k=0$ is twice the product of roots.

$$Solution: 2kx^2-3x+4k=0$$

$$ax^2+bx+c=0$$

$$a=2k,\ b=-3,\ c=4k$$

Let α,β *be the roots of equation*

$$Sum\ of\ roots=\alpha+\beta=\frac{-(-3)}{2k}=\frac{3}{2k}$$

$$\text{Productof roots}=\alpha\beta=\frac{c}{a}=\frac{4k}{2k}=2$$

According to given conditions:

$$S=2P$$

$$\frac{3}{2k}=2(2)$$

$$\frac{3}{2k}=4$$

$$3=4(2k)$$

$$3=8k$$

$$\frac{3}{8}=k$$

$$k=\frac{3}{8}$$

(ii) Sum of the roots of the equation $x^2+(3k-7)x+5k=0$ *is* $\frac{3}{2}$ *times the products of roots.*

Solution: $1x^2+(3k-7)x+5k=0$

$$ax^2+bx+c=0$$

$$a=1,\ b=(3k-7),\ c=5k$$

Let α,β *be the roots of equation*

$$Sum\ of\ roots=\alpha+\beta=\frac{-b}{a}=\frac{-(3k-7)}{1}=-3k+7$$

$$\text{Productof roots}=\alpha\beta=\frac{c}{a}=\frac{5k}{1}=5k$$

According to given conditions:

$$S=\frac{3}{2}P$$

$$-3k+7=\frac{3}{2}(5k)$$

$$2(-3k+7)=3(5k)$$

$$-6k+14=15k$$

$$14=15k+6k$$

$$14=21k$$

$$k=\frac{14}{21}=\frac{2}{3}$$

$$k=\frac{2}{3}$$

Q.3 Find k, if

(ii) Sum of the squares of the roots of the equation $4kx^2+3kx-8=0$ *is* 2

Solution: $x^2+3kx-8=0$

$$ax^2+bx+c=0$$

$$a=4k,\ b=3k,\ c=-8$$

$$Sum\ of\ roots=\alpha+\beta=\frac{-b}{a}$$

$$=\frac{-3k}{4k}=\frac{-3}{4}$$

$$\text{Product of roots}=P=\alpha\beta=\frac{c}{a}$$

$$=\frac{-8}{4k}=\frac{-2}{k}$$

Given that sum of square of roots is 2

$$\alpha^2+\beta^2=2$$

$$\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta=2$$

$$(\alpha+\beta)^2-2\alpha\beta=2$$

$$Put\ \alpha+\beta=\frac{-3}{4}\quad \alpha\beta=\frac{-2}{k}$$

$$\left(\frac{-3}{4}\right)^2-2\left(\frac{-2}{k}\right)=2$$

$$\frac{9}{16}+\frac{4}{k}=2\quad\Rightarrow\frac{9k+64}{16k}=2$$

$$9k+64=2(16k)$$

$$9k+64=32k$$

$$64=32k-9k$$

$$64 = 23k \Rightarrow k = \frac{64}{23}$$

$$Hence\ \ k = \frac{64}{23}$$

(ii) Sum of the square of the roots of the equation $x^2 - 2kx + (2k+1) = 0$ is 6

Solution: $x^2 - 2kx + (2k+1) = 0$

$$ax^2 + bx + c = 0$$

$$a = 1,\ b = -2k,\ c = 2k+1$$

α, β are the roots of equation

$$Sum\ of\ roots = \alpha + \beta = \frac{-b}{a}$$

$$= \frac{-(-2k)}{1} = 2k$$

$$\text{Product of roots} = P = \alpha\beta = \frac{c}{a}$$

$$= \frac{2k+1}{1} = 2k+1$$

According to given condition:

$$\alpha^2 + \beta^2 = 6$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = 6$$

$$(\alpha + \beta)^2 - 2\alpha\beta = 6$$

$$Put\ \alpha + \beta = 2k \quad \alpha\beta = 2k+1$$

$$(2k)^2 - 2(2k+1) = 6$$

$$4k^2 - 4k - 2 = 6$$

$$4k^2 - 4k - 2 - 6 = 0$$

$$4k^2 - 4k - 8 = 0$$

$$4(k^2 - k - 2) = 0$$

$$\because k^2 - k - 2 = 0$$

$$k^2 - 2k + k - 2 = 0$$

$$k(k-2) + 1(k-2) = 0$$

$$(k-2)(k+1) = 0$$

$$k - 2 = 0 \qquad k + 1 = 0$$

$$k = 2 \qquad k = -1$$

$$k = -1, 2$$

Q.4 Find p if

(i) The roots of the equation $x^2 - x + p^2 = 0$ differ by unity.

Solution: $x^2 - x + p^2 = 0$

$$ax^2 + bx + c = 0$$

$$a = 1,\ b = -1,\ c = p^2$$

Let the roots are 'α' and '$\alpha - 1$'

$$\text{Sum of roots} = S = \alpha + \alpha - 1$$

$$= \frac{-b}{a} = -\left(\frac{-1}{1}\right) = 1$$

$$2\alpha - 1 = 1$$

$$2\alpha = 1 + 1$$

$$2\alpha = 2$$

$$\alpha = 1$$

$$\text{Product of roots} = P = \alpha(\alpha - 1)$$

$$= \frac{c}{a} = \frac{p^2}{1} = p^2$$

$$\alpha(\alpha - 1) = p^2$$

Putting value of $\alpha = 1$

$$1(1-1) = p^2$$

$$p^2 = 0 \Rightarrow p = 0$$

(ii) Find p if the roots of the equation $x^2 + 3x + p - 2 = 0$ differ by 2.

Solution: $x^2 + 3x + p - 2 = 0$

$$ax^2 + bx + c = 0$$

$$a = 1,\ b = 3,\ c = p - 2$$

Let the roots are 'α' and '$\alpha - 2$'

$$\text{Sum of roots} = S = \alpha + \alpha - 2$$

$$= \frac{-b}{a} = \left(\frac{-3}{1}\right) = -3$$

$$2\alpha-2=-3$$
$$2\alpha=-3+2$$
$$2\alpha=-1$$
$$\alpha=\frac{-1}{2}$$

Product of roots $=P=\alpha(\alpha-2)$

$$=\frac{c}{a}=\frac{p-2}{1}=p-2$$
$$\alpha(\alpha-2)=p-2$$

Putting value of $\alpha=\frac{-1}{2}$

$$\frac{-1}{2}\left(\frac{-1}{2}-2\right)=p-2$$
$$\frac{-1}{2}\left(\frac{-1-4}{2}\right)=p-2$$
$$\frac{-1}{2}\left(\frac{-5}{2}\right)=p-2$$
$$\frac{5}{4}=p-2$$
$$p=\frac{5}{4}+2$$
$$p=\frac{5+8}{4}$$
$$p=\frac{13}{4}$$

Q.5 Find m if

(i) The roots of the equation $x^2-7x+3m-5=0$ satisfy the relation $3\alpha+2\beta=4$.

Solution: $x^2-7x+3m-5=0$

Let α,β be the roots of given equation

$$1x^2-7x+3m-5=0$$
$$ax^2+bx+c=0$$
$$a=1,\ b=-7,\ c=3m-5$$

Sum of roots $=S=\alpha+\beta=\frac{-b}{a}$

$$=-\left(\frac{-7}{1}\right)=7 \quad ...(i)$$

Product of roots $=\alpha\beta=\frac{c}{a}=\frac{3m-5}{1}$

$$=3m-5 \quad ...(ii)$$

Since $3\alpha+2\beta=4 \quad ...(iii)$

From equation (i)

$$\alpha+\beta=7$$
$$\beta=7-\alpha$$

Put $\beta=7-\alpha$ in equation (iii)

$$3\alpha+2\beta=4$$
$$3\alpha+2(7-\alpha)=4$$
$$3\alpha+14-2\alpha=4$$
$$\alpha+14=4$$
$$\alpha=4-14$$
$$\alpha=-10$$

Put $\alpha=-10$ in $\beta=7-\alpha$

$$\beta=7-(-10)$$
$$\beta=7+10 \Rightarrow \beta=17$$

Put $\alpha=-10$ and $\beta=17$ in equation (ii)

$$\alpha\beta=3m-5$$
$$(-10)(17)=3m-5$$
$$-170=3m-5$$
$$-170+5=3m$$
$$-165=3m$$
$$\frac{-165}{3}=m$$
$$-55=m \Rightarrow m=-55$$

(ii) Find m if the roots of the equation $x^2-7x+3m-5=0$ satisfy the relation $3\alpha-2\beta=4$.

Solution: $x^2-7x+3m-5=0$

Let α,β be the roots of given equation

$$1x^2+7x+3m-5=0$$

$ax^2+bx+c=0$

$a=1,\ b=7,\ c=3m-5$

Sum of roots $= S = \alpha+\beta = \dfrac{-b}{a}$

$= \dfrac{-7}{1} = -7 \quad ...(i)$

Product of roots$=\alpha\beta = \dfrac{c}{a} = \dfrac{3m-5}{1}$

$= 3m-5 \quad ...(ii)$

Since $3\alpha+2\beta=4 \quad ...(iii)$

From equation (i)

$\alpha+\beta=-7$

$\beta=-7-\alpha$

Put $\beta=-7-\alpha$ in equation(iii)

$3\alpha-2\beta=4$

$3\alpha-2(-7-\alpha)=4$

$3\alpha+14+2\alpha=4$

$5\alpha+14=4$

$5\alpha=4-14$

$5\alpha=-10$

$\alpha=\dfrac{-10}{5}$

$\alpha=-2$

Put $\alpha=-2$ in equation (i)

$\alpha+\beta=-7$

$-2+\beta=-7$

$\beta=-7+2 \quad \Rightarrow \beta=-5$

Put $\alpha=-2$ and $\beta=-5$ in equation (ii)

$\alpha\beta=3m-5$

$(-2)(-5)=3m-5$

$10=3m-5$

$10+5=3m$

$15=3m$

$\dfrac{15}{3}=m \quad \Rightarrow m=5$

(iii) Find m if the roots of the equation $3x^2-2x+7m+2=0$ satisfy the relation $7\alpha-3\beta=18$.

Solution: $3x^2-2x+7m+2=0$

Let α,β be the roots of given equation

$3x^2-2x+7m+2=0$

$ax^2+bx+c=0$

$a=3,\ b=-2,\ c=7m+2$

Sum of roots $= S = \alpha+\beta = \dfrac{-b}{a}$

$= \dfrac{-(-2)}{3} = \dfrac{2}{3} \quad ...(i)$

Product of roots$=\alpha\beta = \dfrac{c}{a}$

$= \dfrac{7m+2}{3} \quad ...(ii)$

Since $7\alpha-3\beta=18 \quad ...(iii)$

From equation (i)

$\alpha+\beta=\dfrac{2}{3}$

$\beta=\dfrac{2}{3}-\alpha$

Put $\beta=\dfrac{2}{3}-\alpha$ in equation(iii)

$7\alpha-3\beta=18$

$7\alpha-3\left(\dfrac{2}{3}-\alpha\right)=18$

$7\alpha-\dfrac{6}{3}+3\alpha=18$

$7\alpha-2+3\alpha=18$

$10\alpha=18+2$

$10\alpha=20$

$\alpha=\dfrac{20}{10}$

$\alpha=2$

Put $\alpha=2$ in equation (i)

$$\alpha+\beta=\frac{2}{3}$$

$$2+\beta=\frac{2}{3}$$

$$\beta=\frac{2}{3}-2=\frac{-4}{3} \quad \Rightarrow \beta=\frac{-4}{3}$$

Put $\alpha=2$ and $\beta=\frac{-4}{3}$ in equation (ii)

$$\alpha\beta=\frac{7m+2}{3}$$

$$2\left(\frac{-4}{3}\right)=\frac{7m+2}{3}$$

$$\frac{-8}{3}\times 3=7m+2$$

$$7m+2=-8$$

$$7m=-8-2$$

$$7m=-10 \Rightarrow m=\frac{-10}{7}$$

Q.6 Find m if sum and product of the roots of the following equations is equal to given number λ.

(i) $(2m+3)x^2+(7m-5)x+(3m-10)=0$

Solution:

$$(2m+3)x^2+(7m-5)x+(3m-10)=0$$

$$ax^2+bx+c=0$$

$$a=(2m+3),\ b=(7m-5),\ c=(3m-10)$$

Let α,β are the roots of the equation, then

$$\text{Sum of roots}=S=\alpha+\beta=\frac{-b}{a}$$

$$=-\frac{(7m-5)}{2m+3}=\frac{5-7m}{2m+3}$$

$$\text{Product of roots}=P=\frac{c}{a}=\frac{3m-10}{2m+3}$$

As given that

$$\alpha+\beta=\lambda \ ...(i) \quad \alpha\beta=\lambda...(ii)$$

From (i) and (ii)

$$\alpha+\beta=\alpha\beta$$

$$\frac{5-7m}{2m+3}=\frac{3m-10}{2m+3}$$

$$(2m+3)(5-7m)=(2m+3)(3m-10)$$

$$5-7m=3m-10$$

$$5+10=3m+7m$$

$$15=10m$$

$$\frac{15}{10}=m \quad \Rightarrow m=\frac{3}{2}$$

(ii) $4x^2-(3+5m)x-(9m-17)=0$

Solution: $4x^2-(3+5m)x-(9m-17)=0$

Let α,β are the roots of the equation, then

$$4x^2-(3+5m)x-(9m-17)=0$$

$$ax^2+bx+c=0$$

$$a=4,\ b=-(3+5m),\ c=-(9m-17)$$

$$\text{Sum of roots}=S=\alpha+\beta=\frac{-b}{a}$$

$$=-\left(-\frac{(3+5m)}{4}\right)=\frac{3+5m}{4}$$

$$\text{Product of roots}=P=\alpha\beta=\frac{c}{a}=-\left(\frac{9m-17}{4}\right)$$

Let $\alpha+\beta=\lambda \ ...(i) \quad \alpha\beta=\lambda...(ii)$

From (i) and (ii)

$$\alpha+\beta=\alpha\beta$$

$$\frac{3+5m}{4}=\frac{-(9m-17)}{4}$$

$$3+5m=-9m+17$$

$$9m+5=17-3$$

$$14m=14$$

$$\frac{14}{14}=m \quad \Rightarrow m=1$$
