Question no1

if X=$\{1,3,5,7,.....19\}$ $Y=\{0,2,4,6,8,.....20\}$

$Z=\{2,3,5,7,11,13,17,19,23\}$ then find the following

(i) $X\cup(Y\cup Z)=(Y\cup Z)$

$Y\cup Z=\{0,2,4,6,8,.....20\}\cup\{2,3,5,7,11,13,17,19,23\}$

$Y\cup Z=\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$X\cup(Y\cup Z)=\{1,3,5,7,.....19\}\cup\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$X\cup(Y\cup Z)=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,23\}$

(ii) $(X\cup Y)\cup Z$

$=\left(\{1,3,5,7,.....\}\cup\{0,2,4,6,8,.....20\}\right)\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{0,1,2,3,4,5,6,7,....20\}\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{0,1,2,3,4,5,6,7,.....20,23\}$

(iii) $X\cap(Y\cap Z)$

$=\{1,3,5,7,.....19\}\cap\{(0,2,4,6,8,.....20)\cap(2,3,5,7,11,13,17,19,23)\}$

$=\{1,3,5,7,.....19\}\cap\{2\}$

$=\{\ \}$

(iv) $(X\cap Y)\cap Z$

$=\{(1,3,5,7,.....19)\cap(0,2,4,6,8,.....20)\}\cap(2,3,5,7,11,13,17,19,23)$

$=\{\ \}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{\ \}$

(v) $X\cup(Y\cap Z)$

$=\{1,3,5,7,.....19\}\cup\{(0,2,4,6,8,.....20)\cap(2,3,5,7,11,13,17,19,23)\}$

$=\{1,3,5,7,.....19\}\cup\{2\}$

$=\{1,2,3,5,7,.....19\}$

(vi) $=\{1,2,3,5,7,9,11,13,15,17,19,23\}$

$=(X\cup Y)\cap(X\cup Z)$

$=\{0,1,2,3,4,5,6,7,....20\}\cap\{1,2,3,5,7,9,11,13,15,17,19,23\}$

$=\{1,2,3,5,7,9,11,13,15,17,19\}$

(vii) $X \cap (Y \cap Z)$

$= \{1,3,5,7,.....19\} \cap \{(0,2,4,6,8,.....20) \cap (2,3,5,7,11,13,17,19,,23)\}$

$= \{1,3,5,7,.....19\} \cap \{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$= \{3,5,7,11,13,17,19\}$

(*Viii*) $(X \cap Y) \cup (X \cap Z)$

$X \cap Y = \{1,3,5,7,.....19\} \cap \{0,2,4,6,8,.....20\}$

$X \cap Y = \{\ \}$

$X \cap Z = \{1,3,5,7,.....19\} \cap \{2,3,5,7,11,13,17,19,,23\}$

$X \cap Z = \{3,5,7,11,13,17,19\}$

$(X \cap Y) \cup (X \cap Z) = \{\ \} \cup \{3,5,7,11,13,17,19\}$

$(X \cap Y) \cup (X \cap Z) = \{3,5,7,11,13,17,19\}$

Q.2 If $A = \{1,2,3,4,5,6\}$

$B = \{2,4,6,8\}$ $C = \{1,4,8\}$

*P*rove the following identites:

Solution:

L.H.S=A $\cap$ B

$= \{1,2,3,4,5,6\} \cap \{2,4,6,8\}$

$= \{2,4,6\}$

$R.H.S = B \cap A$

$= \{2,4,6,8\} \cap \{1,2,3,4,5,6\}$

$= \{2,4,6\}$

$L.H.S = R.H.S,$ *so*

$A \cap B = B \cap A$

(II) A $\cup$ B=B $\cup$ A

$LHS =$ A $\cup$ B

$= \{1,2,3,4,5,6\} \cup \{2,4,6,8\}$

$= \{1,2,3,4,5,6,7,8\}$

$RHS = \text{B} \cup \text{A}$

$=\{2,4,6,8\} \cup \{1,2,3,4,5,6\}$

$=\{1,2,3,4,5,6,7,8\}$

$LHS = RHS$

(iii) $\text{A} \cap (\text{B} \cup C) = (\text{A} \cap \text{B}) \cup (A \cap C)$

L.H.S=$\text{A} \cap (B \cup C)$

$= A \cap (\{2,4,6,8\} \cup \{1,4,8\})$

$= \{1,2,3,4,5,6\} \cap \{1,2,4,6,8\}$

$= \{1,2,4,6\}$

R.H.S=$(A \cap B) \cup (A \cap C)$

$A \cap B = \{1,2,3,4,5,6\} \cap \{2,4,6,8\}$

$= \{2,4,6\}$

$A \cap C = \{1,2,3,4,5,6\} \cap \{1,4,8\}$

$= \{1,4\}$

$(A \cap B) \cup (A \cap C) = \{2,4,6\} \cup \{1,4\}$

$= \{1,2,4,6\}$

L.H.S=R.H.S

So, $\text{A} \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) $\text{A} \cup (\text{B} \cap C) = (\text{A} \cup \text{B}) \cap (A \cup C)$

L.H.S=$\text{A} \cup (\text{B} \cap C)$

$= A \cup (\{2,4,6,8\} \cap \{1,4,8\})$

$= \{1,2,3,4,5,6\} \cap \{4,8\}$

$= \{1,2,3,4,5,6,8\}$

R.H.S=$(\text{A} \cup \text{B}) \cap (A \cup C)$

$A \cup B = \{1,2,3,4,5,6\} \cup \{2,4,6,8\}$

$= \{1,2,3,4,5,6,8\}$

$A \cup C = \{1,2,3,4,5,6\} \cup \{1,4,8\}$

$= \{1,2,3,4,5,6,8\}$

$(\text{A} \cup \text{B}) \cap (A \cup C) = \{1,2,3,4,5,6,8\} \cap \{1,2,3,4,5,6,8\}$

$= \{1,2,3,4,5,6,8\}$

L.H.S=R.H.S

So, $\text{A} \cup (\text{B} \cap C) = (\text{A} \cup \text{B}) \cap (A \cup C)$

Q.3 If U=$\{1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,3,5,7,9\}$, $B = \{2,3,5,7\}$ then verify the De Morgen's laws i.e, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

Solution :

$(A \cup B)' = A' \cap B'$

L.H.S = $(A \cup B)'$

$A \cup B = \{1,3,5,7,9\} \cup \{2,3,5,7\}$

$= \{1,2,3,5,7,9\}$

$(A \cup B)' = U - (A \cup B)$

$= \{1,2,3,...,10\} - \{1,2,3,5,7,9\}$

$= \{4,6,8,10\}$

R.H.S=$\text{A}' \cap \text{B}'$

$A' = U - A$

$= \{1,2,3,...,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B' = U - B$

$= \{1,2,3,...,10\} - \{2,3,5,7\}$

$= \{1,4,6,8,9,10\}$

$\text{A}' \cap \text{B}' = \{2,4,6,8,10\} \cap \{1,4,6,8,9,10\}$

$= \{4,6,8,10\}$

L.H.S=R.H.S

$(A \cup B)' = \text{A}' \cap \text{B}'$

$(ii)(A \cap B)' = \text{A}' \cup \text{B}'$

L.H.S = $(A \cap B)'$

$A \cap B = \{1,3,5,7,9\} \cap \{2,3,5,7\}$

$= \{3,5,7\}$

$(A \cap B)' = U - (A \cap B)$

$= \{1,2,3,...,10\} - \{3,5,7\}$

$= \{1,2,4,6,8,9,10\}$

R.H.S=$\text{A}' \cup \text{B}'$

$A' = U - A$

$= \{1,2,3,...,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B' = U - B$

$= \{1,2,3,...,10\} - \{2,3,5,7\}$

$= \{1,4,6,8,9,10\}$

$A' \cup B' = \{2,4,6,8,10\} \cup \{1,4,6,8,9,10\}$

$= \{1,2,4,6,8,9,10\}$

L.H.S=R.H.S

$(A \cap B)' = A' \cup B'$

Q.4 If U=$\{1,2,3,...,20\}$, $X = \{1,3,7,9,15,18,20\}$

$Y = \{1,3,5,...,17\}$ then show that

(i) $X - Y = X \cap Y'$

Solution:

L.H.S=$X - Y$

$= \{1,3,7,9,15,18,20\} - \{1,3,5,...,17\}$

$= \{18,20\}$

R.H.S=$X \cap Y'$

$Y' = U - Y$

$= \{1,2,3,...,20\} - \{1,3,5,...,17\}$

$= \{2,4,6,8,10,...,20\}$

$X \cap Y' = \{1,3,7,9,15,18,20\} \cap \{2,4,6,8,10,...,20\}$

$= \{18,20\}$

L.H.S=R.H.S

$X - Y = X \cap Y'$

(ii) $Y - X = Y \cap X'$

Solution:

L.H.S=$Y - X$

$= \{1,3,5,...,17\} - \{1,3,7,9,15,18,20\}$

$= \{5,11,13,17\}$

R.H.S=$Y \cap X'$

$X' = U - X$

$= \{1,2,3,...,20\} - \{1,3,7,9,15,18,20\}$

$= \{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$Y \cap X' = \{1,3,5,...,17\} \cap \{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$= \{5,11,13,17\}$

L.H.S=R.H.S

$Y - X = Y \cap X'$