1. Find a third proportional to

(i). $\mathbf{6, 12}$

Solution:

Let c be the third proportional

$6:12 :: 12:c$

As, product of extremes= product of means

Therefore, $6c = (12)(12)$

$6c = 144$

$c = \frac{144}{6}$

$c = 24$

(ii). $\boldsymbol{a^3, 3a^2}$

Solution:

Let c be the third proportional

$a^3:3a^2 :: 3a^2:c$

As, product of extremes= product of means

So, $(a^3)(c) = (3a^2)(3a^2)$

$(a^3)(c) = 9a^4$

$c = \frac{9a^4}{a^3}$

$c = 9a^{4-3}$

$c = 9a$

(iii). $\boldsymbol{a^2 - b^2, a - b}$

Solution:

Let c be the third proportional

$a^2 - b^2 : a - b :: a - b : c$

As, product of extremes= product of means

So, $(a^2 - b^2)(c) = (a - b)(a - b)$

$$c = \frac{(a-b)(a-b)}{a^2 - b^2}$$

$$c = \frac{(a-b)(a-b)}{(a+b)(a-b)}$$

$$c = \frac{a-b}{a+b}$$

(iv). $(x-y)^2, x^3 - y^3$

Solution:

Let c be the third proportional

$(x-y)^2 : x^3 - y^3 :: x^3 - y^3 : c$

As, product of extremes= product of means

So, $(x-y)^2(c) = (x^3 - y^3)(x^3 - y^3)$

$$c = \frac{(x-y)(x^2 + xy + b^2)(x-y)(x^2 + xy + y^2)}{(x-y)(x-y)}$$

$$c = (x^2 + xy + y^2)^2$$

(v). $(x+y)^2, x^2 - xy - 2y^2$

Solution:

Let c be the third proportional

$(x+y)^2 : x^2 - xy - 2y^2 :: x^2 - xy - 2y^2 : c$

As, product of extremes= product of means

$(x+y)^2(c) = (x^2 - xy - 2y^2)(x^2 - xy - 2y^2)$

$(x+y)^2(c) = (x^2 - xy - 2y^2)^2$

$$c = \frac{(x^2 - 2xy + xy - 2y^2)^2}{(x+y)^2}$$

$$c = \frac{(x(x-2y) + y(x-2y))^2}{(x+y)^2}$$

$$c = \frac{((x-2y)(x+y))^2}{(x+y)^2}$$

$$c = \frac{(x-2y)^2(x+y)^2}{(x+y)^2}$$

$$c = (x-2y)^2$$

(vi). $\frac{p^2-q^2}{p^3+q^3}, \frac{p-q}{p^2-pq+q^2}$

Solution:

Let c be the third proportional

$$\frac{p^2-q^2}{p^3+q^3} : \frac{p-q}{p^2-pq+q^2} :: \frac{p-q}{p^2-pq+q^2} : c$$

As, product of extremes= product of means

So,

$$\left(\frac{p^2-q^2}{p^3+q^3}\right)(c) = \left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)$$

$$\left(\frac{p^2-q^2}{p^3+q^3}\right)(c) = \frac{(p-q)^2}{(p^2-pq+q^2)^2}$$

$$c = \left(\frac{(p-q)^2}{(p^2-pq+q^2)^2}\right)\left(\frac{p^3+q^3}{p^2-q^2}\right)$$

$$c = \left(\frac{(p-q)^2}{(p^2-pq+q^2)^2}\right)\left(\frac{(p+q)(p^2-pq+q^2)}{(p+q)(p-q)}\right)$$

$$c = \frac{p-q}{p^2-pq+q^2}$$

2. Find a fourth proportional to

(i). $\mathbf{5, 8, 15}$

Solution:

Let d be the fourth proportional

$5:8 :: 15:d$

As, product of extremes= product of means

So, $(5)(d) = (8)(15)$

$5d = 120$

$d = \frac{120}{5}$

$d = 24$

(ii). $\mathbf{4x^4, 2x^3, 18x^5}$

Solution:

Let d be the fourth proportional

$4x^4: 2x^3 :: 18x^5: d$

As, product of extremes= product of means

So, $(4x^4)(d) = (2x^3)(18x^5)$

$(4x^4)(d) = 36x^8$

$d = \frac{36x^8}{(4x^4)}$

$d = 9x^{8-4}$

$d = 9x^4$

(iii). $\mathbf{15a^5b^6, 10a^2b^5, 21a^3b^3}$

Solution:

Let d be the fourth proportional

$15a^5b^6: 10a^2b^5 :: 21a^3b^3: d$

As, product of extremes= product of means

So, $(15a^5b^6)(d) = (10a^2b^5)(21a^3b^3)$

$$d = \frac{210a^{2+3}b^{5+3}}{15a^5b^6}$$

$$d = \frac{14a^5b^8}{a^5b^6}$$

$$d = 14a^{5-5}b^{8-6}$$

$$d = 14a^0b^2$$

$$d = 14b^2$$

(iv). $\boldsymbol{x^2 - 11x + 24, (x-3), 5x^4 - 40x^3}$

Solution:

Let d be the fourth proportional

$x^2 - 11x + 24 : (x-3) :: 5x^4 - 40x^3 : d$

As, product of extremes= product of means

So, $(x^2 - 11x + 24)(d) = (x-3)(5x^4 - 40x^3)$

$$d = \frac{5x^3(x-8)(x-3)}{x^2 - 3x - 8x + 24}$$

$$d = \frac{5x^3(x-8)(x-3)}{x(x-3) - 8(x-3)}$$

$$d = \frac{5x^3(x-8)(x-3)}{(x-3)(x-8)}$$

$$d = 5x^3$$

(v). $\boldsymbol{p^3 + q^3, p^2 - q^2, p^2 - pq + q^2}$

Solution:

Let d be the fourth proportional

$p^3 + q^3 : p^2 - q^2 :: p^2 - pq + q^2 : d$

As, product of extremes= product of means

So, $(p^3+q^3)(d)=(p^2-q^2)(p^2-pq+q^2)$

$$d=\frac{(p^2-q^2)(p^2-pq+q^2)}{p^3+q^3}$$

$$d=\frac{(p+q)(p-q)(p^2-pq+q^2)}{(p+q)(p^2-pq+q^2)}$$

$d=p+q$

(vi). $\boldsymbol{(p^2-q^2)(p^2+pq+q^2), (p^3+q^3), (p^3-q^3)}$

Solution:

Let d be the fourth proportional

$(p^2-q^2)(p^2+pq+q^2) : p^3+q^3 :: p^3-q^3 : d$

As, product of extremes= product of means

So, $(p^2-q^2)(p^2+pq+q^2)(d)=(p^3+q^3)(p^3-q^3)$

$$d=\frac{(p^3+q^3)(p^3-q^3)}{(p^2-q^2)(p^2+pq+q^2)}$$

$$d=\frac{(p+q)(p^2-pq+q^2)(p-q)(p^2+pq+q^2)}{(p+q)(p-q)(p^2+pq+q^2)}$$

$d=p^2-pq+q^2$

3. Find a mean proportional between

(i). $\mathbf{20, 45}$

Solution:

Let m be the mean proportional

$20 : m :: m : 45$

As, product of means = product of extremes

So, $(m)(m)=(20)(45)$

$m^2=900$

Taking square root on both sides

$\sqrt{m^2} = \sqrt{900}$

$m = \pm 30$

(ii). $\boldsymbol{20x^3y^5, 5x^7y}$

Solution:

Let m be the mean proportional

$20x^3y^5 : m :: m : 5x^7y$

As, product of means = product of extremes

So, $(m)(m) = (20x^3y^5)(5x^7y)$

$m^2 = 100x^{3+7}y^{5+1}$

$m^2 = 100x^{10}y^6$

$m^2 = (10)^2(x^5)^2(y^3)^2$

Taking square root on both sides

$\sqrt{m^2} = \sqrt{(10)^2(x^5)^2(y^3)^2}$

$m = \pm 10x^5y^3$

(iii). $\boldsymbol{15p^4qr^3, 135q^5r^7}$

Solution:

Let m be the mean proportional

$15p^4qr^3 : m :: m : 135q^5r^7$

As, product of means = product of extremes

So, $(m)(m) = (15p^4qr^3)(135q^5r^7)$

$m^2 = 2025p^4q^{1+5}r^{3+7}$

$m^2 = 2025p^4q^6r^{10}$

$m^2 = (45)^2(p^2)^2(q^3)^2(r^5)^2$

Taking square root on both sides

$\sqrt{m^2} = \sqrt{(45)^2(p^2)^2(q^3)^2(r^5)^2}$

$m = \pm 45p^2q^3r^5$

(iv). $\boldsymbol{x^2 - y^2, \frac{x-y}{x+y}}$

Solution:

Let m be the mean proportional

$x^2 - y^2 : m :: m : \dfrac{x-y}{x+y}$

As, product of means = product of extremes

So, $(m)(m) = (x^2 - y^2)\left(\frac{x-y}{x+y}\right)$

$m^2 = \dfrac{(x-y)(x+y)(x-y)}{x+y}$

$m^2 = (x-y)^2$

Taking square root on both sides

$\sqrt{m^2} = \sqrt{(x-y)^2}$

$m = \pm(x-y)$

4. Find the value of the letter involved in the following continued proportions

(i). $\boldsymbol{5, p, 45}$

Solution:

Since $5, p, 45$ are in continued proportion

$5 : p :: p : 45$

As, product of means = product of extremes

So, $(p)(p) = (5)(45)$

$p^2 = 225$

$p^2 = (15)^2$

Taking square root on both sides

$\sqrt{p^2} = \sqrt{(15)^2}$

$p = \pm 15$

(ii). $8, x, 18$

Solution:

Since $8, x \; and \; 18$ are in continued proportion

$8: x :: x: 18$

As, product of means = product of extremes

$(x)(x) = (8)(18)$

$x^2 = 144$

$x^2 = (12)^2$

Taking square root on both sides

$\sqrt{x^2} = \sqrt{(12)^2}$

$x = \pm 12$

(iii). $12, 3p - 6, 27$

Solution:

Since $12, 3p - 6 \; and \; 27$ are in continued proportion

$12: 3p - 6 :: 3p - 6: 27$

As, product of means = product of extremes

$(3p - 6)(3p - 6) = (12)(27)$

$(3p - 6)^2 = 324$

$(3p - 6)^2 = (18)^2$

Taking square root on both sides

$\sqrt{(3p-6)^2} = \sqrt{(18)^2}$

$3p - 6 = \pm 18$

$3p - 6 = 18$; $3p - 6 = -18$

$3p = 18 + 6$; $3p = -18 + 6$

$3p = 24$; $3p = -12$

$p = \frac{24}{3}$; $p = -\frac{12}{3}$

$p = 8$; $p = -4$

(iv). $\mathbf{7, m-3, 28}$

Solution:

Since $7, m-3$ *and* 28 are in continued proportion

$7: m-3 :: m-3: 28$

As, product of means = product of extremes

$(m-3)(m-3) = (7)(28)$

$(m-3)^2 = 196$

$(m-3)^2 = (14)^2$

Taking square root on both sides

$\sqrt{(m-3)^2} = \sqrt{(14)^2}$

$m - 3 = \pm 14$

$m - 3 = 14$; $m - 3 = -14$

$m = 14 + 3$; $m = -14 + 3$

$m = 17$; $m = -11$