1. The surface area A of a cube varies directly as the square of length (l) of an edge and $A = 27$square units when $l = 3$ units. Find $(i)A$ when $l = 4$ units $(ii)l$ when $A = 12$ square units.

Solution:

Given that

$A \propto l^2$

$A = kl^2$ ________(i)

Where k is the constant of variation.

Putting $A = 27\ sq.units$ & $l = 3\ units$ in eq(i)

$27 = k(3)^2$

$27 = 9k$

$k = \frac{27}{9}$

$k = 3$

Putting $k = 3$ in eq(i)

$A = 3l^2$ ________(ii)

$(i)A$ when $l = 4$ units

Putting $l = 4$ in eq(ii)

$A = (3)(4)^2$

$A = (3)(16)$

$A = 48\ sq.units$

$(ii)l$ when $A = 12$ square units.

Putting $A = 12$ eq(ii)

$12 = (3)l^2$

$l^2 = \frac{12}{3}$

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$l^2 = 4$

$l^2 = (2)^2$

Taking square root on both sides

$\sqrt{l^2} = \sqrt{(2)^2}$

$l = \pm 2$

Taking positive value of l

So, $l = 2\,units$

2. The surface area S of the sphere varies directly as square of radius r, and $S = 16\pi$ when $r = 2$. Find r when $S = 36\pi$.

Solution:

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Given that

$S \propto r^2$

$S = kr^2$ ________(i)

Where k is the constant of variation.

Putting $S = 16\pi$ & $r = 2$ in eq(i)

$16\pi = k(2)^2$

$16\pi = 4k$

$k = \frac{16\pi}{4}$

$k = 4\pi$

Putting $k = 4\pi$ in eq(i)

$S = 4r^2\pi$

Putting $S = 36\pi$ in above eq.

$36\pi = 4r^2\pi$

$r^2 = \frac{36\pi}{4\pi}$

$r^2 = 9$

$r^2 = (3)^2$

Taking square root on both sides

$\sqrt{r^2} = \sqrt{(3)^2}$

$r = \pm 3$

Taking positive value of r

So, $r = 3$

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3. In Hook's law the force F applied to stretch a string varies directly as the amount of elongation S and $F = 32lb$ when $S = 1.6in$. Find (i)S when $F = 50lb$ (ii)F when $S = 0.8in$.

Solution:

Given that

$F \propto S$

$F = kS$ ________(i)

Where k is the constant of variation.

Putting $F = 32lb$ & $S = 1.6in$ in eq(i)

$32lb = k(1.6in)$

$k = \frac{32lb}{1.6in}$

$k = 20lb/in$

Putting $k = 20lb/in$ in eq(i)

$F = 20S\ lb/in$ ________(ii)

(i)S when $F = 50lb$

Putting $F = 50lb$ in eq(ii)

$50lb = 20S\ lb/in$

$S = \frac{(50lb)(in)}{20lb}$

$S = \frac{5}{2}in$

$S = 2.5in$

(ii)F when $S = 0.8in$

Putting $S = 0.8in$ in eq(ii)

$F = (20)(0.8in)\left(\frac{lb}{in}\right)$

$F = 16lb$

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4. The intensity I of light from a given source varies directly as the square og distance d from it. If the intensity is 20 candlepower at a distance of 12 ft. from the source, find the intensity at a point 8ft. from the source.

Solution:

$I \propto \frac{1}{d^2}$

$I = \frac{k}{d^2}$ ________(i)

Where k is the constant of variation.

Putting $I = 20candlepower$ & $d = 12ft.$ In eq(i)

$20candlepower = \frac{k}{(12ft.)^2}$

$k = (20)(144)candlepower\ ft.^2$

$k = 2880candlepower\ ft.^2$

Putting $k = 2880candlepower\ ft.^2$ in eq(i)

$I = \frac{2880}{d^2}candlepower\ ft.^2$

Putting $d = 8ft.$ in above equation

$$I = \frac{2880}{(8ft.)^2} candlepower\ ft.^2$$

$$I = \frac{2880}{64ft.^2} candlepower ft.^2$$

$$I = 45\ candlepower$$

5. The pressure P in a body of fluid varies directly as the depth$d$. If the pressure exerted on the bottom of the tank by a column of fluid 5ft. high is 2.25lb/sq. in, how deep must the fluid be to exert a pressure of 9lb/sq. in?

Solution:

$P \propto d$

$P = kd$ ________(i)

Where k is the constant of variation.

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Putting $P = 2.25lb/sq.in$ & $d = 5ft.$ In eq(i)

$$2.25lb/sq.in = k(5ft.)$$

$$k = \frac{2.25lb}{5sq.in(ft.)}$$

$$k = 0.45\ lb/sq.in(ft.)$$

Putting $k = 0.45\ lb/sq.in(ft.)$ in eq(i)

$$P = 0.45d\ lb/sq.in(ft.)$$

Putting $P = 9\ lb/sq.in$ in eq(ii)

$$9\ lb/sq.in = (0.45)(d)\ lb/sq.in(ft.)$$

$$d = \frac{9lbsq.in(ft.)}{0.45lbsq.in}$$

$$d = 20ft.$$

6. Labour costs c varies jointly as the number of worker n and the average number of days$d$. If the cost of 800 workers for13 days is Rs.286000, then find the labour cost of 600 workers for 18 days.

Solution:

Given that

$c \propto nd$

$c = knd \qquad \_\_\_\_\_\_\_\_(i)$

Where k is the constant of variation.

Putting $c = 286000, n = 800$ & $d = 13$ in eq(i)

$286000 = k(800)(13)$

$286000 = 10400k$

$$k = \frac{286000}{10400}$$

$$k = \frac{55}{2}$$

Putting $k = \frac{55}{2}$ in eq(i)

$$c = \frac{55nd}{2}$$

Putting $n = 600$ & $d = 18$ in above equation

$$c = \frac{(55)(600)(18)}{2}$$

$c = 297{,}000\ Rs.$

7. The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length$l$. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4inch pillar must be to support a load of 28 tons?

Solution:

Given that

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$c \propto \frac{d^4}{l^2}$

$c = \frac{kd^4}{l^2}$ ________(i)

Where k is the constant of variation.

Putting $c = 63tons, d = 6inch$ & $l = 30\ feet$ in eq(i)

$63 = \frac{k(6)^4}{(30)^2}$

$63 = \frac{1296k}{900}$

$k = \frac{(63)(900)}{1296}$

$k = \frac{175}{4}$

Putting $k = \frac{175}{4}$ in eq(i)

$c = \frac{175d^4}{4l^2}$

Putting $c = 28tons$ & $d = 4in.$ In above equation

$28 = \frac{(175)(4)^4}{4l^2}$

$l^2 = \frac{(175)(256)}{(4)(28)}$

$l^2 = 400$

$l^2 = (20)^2$

Taking square root on both sides

$\sqrt{l^2} = \sqrt{(20)^2}$

$l = \pm 20$

Taking positive value of l

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So, $l = 20\ feet$

8. The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power p of the motor. If 25 sec. are required for a 4-hp motor to lift 500lb through 40ft, what power is required to lift 800lb, through 120ft in 40 sec.?

Solution:

Given that

$$T \propto \frac{wd}{p}$$

$$T = \frac{kwd}{p} \qquad \_\_\_\_\_\_(i)$$

Where k is the constant of variation.

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Putting $T = 25\ sec., w = 500lb, d = 40\ ft\ \&\ p = 4\ hp$ in eq(i)

$$25 = \frac{k(500)(40)}{4}$$

$$k = \frac{(25)(4)}{(500)(40)}$$

$$k = \frac{1}{200}$$

Putting $k = \frac{1}{200}$ in eq(i)

$$T = \frac{wd}{200p}$$

Putting $w = 800lb, T = 40sec., d = 120ft.$ In above equation

$$40 = \frac{(800)(120)}{200p}$$

$$p = \frac{96000}{(200)(40)}$$

$$p = \frac{96000}{8000}$$

$p = 12hp$

9. The kinetic energy (K.E) of a body varies jointly as the mass "m" of the body and the square of its velocity"v". If kinetic energy is 4320 ft./lb. when the mass is 45 lb. and the velocity is 24 ft./sec, determine the kinetic energy of a 3000lb automobile travelling 44 ft./sec.

Solution:

Given that

$K.E \propto mv^2$

$K.E = kmv^2$ ________(i)

Where k is the constant of variation.

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Putting $K.E = 4320\frac{ft}{lb}, m = 45\ lb$ & $v = 24\ ft/sec$ in eq(i)

$4320 = k(45)(24)^2$

$4320 = k(45)(576)$

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$k = \frac{4320}{25920}$

$k = \frac{27}{167}$

Putting $k = \frac{27}{167}$ in eq(i)

$K.E = \frac{27mv^2}{167}$

Putting $w = 3000lb, v = 44ft/sec$ in above equation

$K.E = \frac{(27)(3000)(44)^2}{167}$

$K.E = \frac{(27)(3000)(1936)}{167}$

$K.E = 968{,}000\ ft/lb$