Exercise 4.2

Resolve into partial fractions.

1. $\frac{x^2-3x+1}{(x-1)^2(x-2)}$

Let $\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-2)}$ - - - - - - - - - - - - (i)

Multiplying eq(i) by $(x-1)^2(x-2)$ on b/s

$(x-1)^2(x-2)\frac{x^2-3x+1}{(x-1)^2(x-2)} = (x-1)^2(x-2)\ \frac{A}{(x-1)} + (x-1)^2(x-2)\ \frac{B}{(x-1)^2}$

$+ \frac{C}{(x-2)}(x-1)^2(x-2)$

x^2-3x+1 = A $(x-2)(x-1)$ + B $(x-2)$ + C $(x-1)^2$ - - - - - - - - -(ii)

x^2-3x+1 = A (x^2-3x+2) + Bx -2B + C(x^2-2x+1)

x^2-3x+1 = Ax^2 -3Ax +2A + Bx -2B +Cx^2-2Cx +C

x^2-3x+1 = x^2(A +C) + x (-3A +B -2C) + (2A-2B+C) - - - - - - - - (iii)

Eq(ii) is an identity which holds good for all x and hence for x=1 & x=2

Put x=1 i.e. , x-1=0 in eq(ii)

$(1)^2-3(1)+1$ = A (0) +B (1-2) + C(0)

1-3+1 = B (-1)

-1= -B

B=1

Put x=2 i.e. , x-2=0 in eq (ii)

$(2)^2-3(2)+1$ = A(0) + B(0) +C (2-1)

4-6+1 = C

C=-1

Equating the coefficients of x^2 on b/s of eq(iii)

1 = A + C

A=1- C

A= 1 – (-1)

A= 1+1

A=2

Put values of A, B, C in eq(i)

Thus, $\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{2}{(x-1)} + \frac{1}{(x-1)^2} - \frac{1}{(x-2)}$ are the required partial fractions.

2. $\frac{x^2+7x+11}{(x+2)^2(x+3)}$

$\frac{x^2+7x+11}{(x+2)^2(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x+3)}$ - - - - - - - - - - - - - - (i)

Multiplying eq(ii) by $(x+2)^2(x+3)$ on b/s

$(x+2)^2(x+3)\frac{x^2+7x+11}{(x+2)^2(x+3)} = (x+2)^2(x+3)\frac{A}{(x+2)} + (x+2)^2(x+3)\frac{B}{(x+2)^2} + \frac{C}{(x+3)}(x+2)^2(x+3)$

$x^2+7x+11$ = A (x+2) (x+3) +B (x+3) + C $(x+2)^2$ - - - - - - - - - - - (ii)

$x^2+7x+11$ = A (x^2+5x+6) + B (x+3) +C (x^2+4x+4)

$x^2+7x+11$ = A$x^2+5Ax+6A+Bx+3B+Cx^2+4Cx+4C$

$x^2+7x+11 = x^2(A+C)+x(5A+B+4C)+(6A+3B+4C)$- - - - - - - - (iii)

Eq(ii) is an identity which holds good for all x and hence for x=-2 & x=-3

Put x=-2 i.e. , x+2=0 in eq(ii)

$(-2)^2+7(-2)+11$= A(0) +B(-2+3) + C(0)

4-14+11 = B

B=1

Put x=-3 i.e. , x+3=0 in eq (ii)

$(-3)^2+7(-3)+11$ = A(0) + B(0) + C $(-3+2)^2$

9-21+11 = C$(-1)^2$

-1=C

C=-1

Equating the coefficients of $(x)^2$ on b/s of eq (iii)

1= A+ C

A= 1-C

A= 1 – (-1)

A = 1+1

A=2

Put values of A, B & C in eq(i)

Thus, $\frac{x^2+7x+11}{(x+2)^2(x+3)} = \frac{2}{(x+2)} + \frac{1}{(x+2)^2} - \frac{1}{(x+3)}$ are the required partial fractions.

3. $\frac{9}{(x-1)(x+2)^2}$

Let $\frac{9}{(x-1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$ - - - - - - - - (i)

Multiplying eq(i) by $(x-1)(x+2)^2$ on b/s

$(x-1)(x+2)^2 \frac{9}{(x-1)(x+2)^2} = (x-1)(x+2)^2 \frac{A}{(x-1)} + (x-1)(x+2)^2 \frac{B}{(x+2)} + \frac{C}{(x+2)^2}(x-1)(x+2)^2$

$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$ - - - - - - - -(ii)

$9 = A(x^2+4x+4) + B(x^2+x-2) + C(x-1)$

$9 = Ax^2 + 4Ax + 4A + Bx^2 + Bx - 2B + Cx - C$

$9 = x^2(A+B) + x(4A+B+C) + (4A-2B-C)$ - - - - - - - - (iii)

Eq (ii) is an identity which holds good for all x and hence for x=1 & x=-2

Put x=1 i.e. , x-1=0 in eq (ii)

$9 = A(1+2)^2 + B(0) + C(0)$

9=9A

A=1

Put x=-2 i.e. , x+2=0 in eq (ii)

$9 = A(0) + B(0) + C(-2-1)$

9=-3C

C=-3

Equating coefficients of x^2 on b/s of eq (iii)

0=A+B

B=-A

B=-1

Put values of A, B & C in eq (i)

Thus, $\frac{9}{(x-1)(x+2)^2} = \frac{1}{(x-1)} - \frac{1}{(x+2)} - \frac{3}{(x+2)^2}$ are the required partial fractions.

4. $\frac{x^4+1}{x^2(x-1)}$

By long division,

$\frac{x^4+1}{x^2(x-1)} = x + 1 + \frac{x^2+1}{x^2(x-1)}$ - - - - - - - - - (A)

Consider ,

$\frac{x^2+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)}$ - - - - - - - - -(i)

Multiplying eq (ii) by $x^2(x-1)$ on b/s

$x^2(x-1)\,\frac{x^2+1}{x^2(x-1)} = x^2(x-1)\,\frac{A}{x} + x^2(x-1)\,\frac{B}{x^2} + x^2(x-1)\,\frac{C}{(x-1)}$

$x^2 + 1 = x(x-1)A\ + (x-1)B\ + x^2C$ - - - - - - - - (ii)

$x^2 + 1 = x^2A - Ax + Bx - \mathrm{B} + x^2C$

$x^2 + 1 = x^2(A+C) + x(-A+B) - \mathrm{B}$ - - - - - - - - -(iii)

Eq (ii) is an identity which holds good for all x and hence for x=0 & x=1

Put x=0 in eq (ii)

$(0)^2+1=A(0)\ +(0-1)B\ +C(0)$

1=-B

B= -1

Put x=1 i.e. x-1=0 in eq (ii)

$(1)^2+1=A(0)\ +B(0)\ +(1)^2C$

2=C

C=2

Equating coefficients of x^2 on b/s of eq (iii)

1= A+C

A=1-C

A=1-2

A=-1

Put values of A, B & C in eq (i)

$\frac{x^2+1}{x^2(x-1)}=-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{(x-1)}$

Eq (A) ⇒

$\frac{x^4+1}{x^2(x-1)}=x+1-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{(x-1)}$ are the required partial fractions.

5.$\frac{7x+4}{(3x+2)(x+1)^2}$

$\frac{7x+4}{(3x+2)(x+1)^2}=\frac{A}{(3x+2)}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$ - - - - - - - - - -(i)

Multiplying eq(i) by $(3x+2)(x+1)^2$ on b/s

$(3x+2)(x+1)^2\frac{7x+4}{(3x+2)(x+1)^2}=\frac{A}{(3x+2)}(3x+2)(x+1)^2+\frac{B}{(x+1)}(3x+2)(x+1)^2+$
$\frac{C}{(x+1)^2}(3x+2)(x+1)^2$

$7x+4=A(x+1)^2+B(3x+2)(x+1)+C(3x+2)$ - - - - - - -(ii)

$7x+4=A(x^2+2x+1)+B(3x^2+5x+2)+C(3x+2)$

$7x+4=Ax^2+2Ax+A+3Bx^2+5Bx+2B+3Cx+2C$

$7x+4=x^2(A+3B)+x(2A+5B+3C)+(A+2B+2C)$ - - - - - - -(iii)

Eq(ii) is an identity which holds good for all x and hence for x=-2/3 & x=-1

Put x=-2/3 i.e. 3x+2=0 in eq (ii)

$7(-\frac{2}{3})+4=A(-\frac{2}{3}+1)^2+B(0)+C(0)$

$\frac{-14+12}{3}=A(\frac{-2+3}{3})^2$

$\frac{-2}{3}=A(\frac{1}{9})$

$A=\frac{(-2)(9)}{3}$

$A=\frac{-18}{3}$

A=-6

Put x=-1 i.e x+1=0 in eq(ii)

$7(-1)+4=A(0)+B(0)+C(3(-1)+2)$

$-7+4=C(-3+2)$

$-3=-C$

$C=3$

Equating coefficients of x^2 on b/s of eq (iii)

0 = A + 3B

3B = -A

3B = - (-6)

B = 6/3

B=2

Put values of A, B & C in eq (i)

Thus, $\frac{7x+4}{(3x+2)(x+1)^2} = \frac{-6}{(3x+2)} + \frac{2}{(x+1)} + \frac{3}{(x+1)^2}$ are the required partial fractions.

6. $\frac{1}{(x-1)^2(x+1)}$

Let $\frac{1}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$ - - - - - - - - -(i)

Multiplying eq (i) by $(x-1)^2(x+1)$ on b/s

$(x-1)^2(x+1)\frac{1}{(x-1)^2(x+1)} = \frac{A}{(x-1)}(x-1)^2(x+1) + \frac{B}{(x-1)^2}(x-1)^2(x+1) + \frac{C}{(x+1)}(x-1)^2(x+1)$

$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$ - - - - - - - - -(ii)

$1 = A(x^2-1) + B(x+1) + C(x^2-2x+1)$

$1 = Ax^2 - A + Bx + B + Cx^2 - 2Cx + C$

$1 = x^2(A+C) + x(B-2C) + (-A+B+C)$ - - - - - - - - - -(iii)

Eq(ii) is an identity which holds good for all x and hence for x=1 & x=-1

Put x=1 i.e. , x-1=0 in eq(ii)

$1 = A(0) + B(1+1) + C(0)$

$1 = 2B$

$B = \frac{1}{2}$

Put x=-1 i.e. , x+1=0 in eq (ii)

$1 = A(0) + B(0) + C(-1-1)^2$

$1 = C(-2)^2$

$1 = 4C$

$C = \frac{1}{4}$

Equating coefficients of x^2 on b/s of eq(iii)

$0 = A + C$

$A = -C$

$A = -\frac{1}{4}$

Put values of A, B & C in eq (i)

Thus, $\frac{1}{(x-1)^2(x+1)} = \frac{-1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)}$

Or $\frac{1}{(x-1)^2(x+1)} = \frac{1}{4(x+1)} - \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$ are the required partial fractions.

7. $\frac{3x^2+15x+16}{(x+2)^2}$

$\frac{3x^2+15x+16}{(x+2)^2} = \frac{3x^2+15x+16}{x^2+4x+4}$

By long division,

$\frac{3x^2+15x+16}{(x+2)^2} = 3 + \frac{3x+4}{(x+2)^2}$ - - - - - - - -(A)

Consider , $\frac{3x+4}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$ - - - - - - - - - - (i)

Multiplying eq (i) by $(x+2)^2$ on b/s

$(x+2)^2 \frac{3x+4}{(x+2)^2} = \frac{A}{x+2}(x+2)^2 + \frac{B}{(x+2)^2}(x+2)^2$

$3x + 4 = A(x+2) + B$ - - - - - - - - -(ii)

$3x + 4 = Ax + 2A + B$ - - - - - - - - - -(iii)

Eq(ii) is an identity which holds good for all x and hence for x=-2

Put x=-2 i.e. , x+2=0 in eq(ii)

$3(-2) + 4 = A(0) + B$

$-6 + 4 = B$

$B = -2$

Equating coefficients of x on b/s of eq(iii)

$A = 3$

Put values of A & B in eq(i)

$\frac{3x+4}{(x+2)^2} = \frac{3}{x+2} - \frac{2}{(x+2)^2}$

Hence , eq(A) ⇒

$\frac{3x^2+15x+16}{(x+2)^2} = 3 + \frac{3}{x+2} - \frac{2}{(x+2)^2}$ are the required partial fraction.

8. $\frac{1}{(x^2-1)(x+1)}$

$\frac{1}{(x^2-1)(x+1)} = \frac{1}{(x+1))(x-1)(x+1)}$

$\frac{1}{(x^2-1)(x+1)} = \frac{1}{(x-1)(x+1)^2}$

Let $\frac{1}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$ - - - - - - -(i)

Multiplying eq(i) by$(x-1)(x+1)^2$ on b/s

$(x-1)(x+1)^2\frac{1}{(x-1)(x+1)^2} = \frac{A}{(x-1)}(x-1)(x+1)^2 + \frac{B}{(x+1)}(x-1)(x+1)^2 + \frac{C}{(x+1)^2}(x-1)(x+1)^2$

$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$ - - - - - - - - - - -(ii)

$1 = A(x^2+2x+1) + B(x^2-1) + C(x-1)$

$1 = Ax^2 + 2Ax + A + Bx^2 - B + Cx - C$

$1 = x^2(A+B) + x(2A+C) + (A-B-C)$ - - - - - - - - -(iii)

Eq(ii) is an identity which holds good for all x and hence for x=1 & x=-1

Put x=1 i.e. , x-1=0 in eq (ii)

$1 = A(1+1)^2 + B(0) + C(0)$

$1 = A(2)^2$

$1 = 4A$

$A = \frac{1}{4}$

Put x=-1 i.e. , x+1=0 in eq(ii)

$1 = A(0) + B(0) + C(-1-1)$

$1 = -2C$

$C = -\frac{1}{2}$

Equating coefficients of x^2 on b/s of eq(iii)

$0 = A + B$

$A = -B$

$B = -\frac{1}{4}$

Put values of A, B & C in eq (i)

Thus, $\frac{1}{(x-1)(x+1)^2} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}$ are the required partial fractions.