Exercise 4.3

Resolve into partial fractions

1. $\frac{3x-11}{(x+3)(x^2+1)}$

Let $\frac{3x-11}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+C}{(x^2+1)}$ - - - - - - -(i)

Multiplying eq(i) by $(x+3)(x^2+1)$ b/s

$(x+3)(x^2+1)\frac{3x-11}{(x+3)(x^2+1)} = \frac{A}{(x+3)}(x+3)(x^2+1) + \frac{Bx+C}{(x^2+1)}(x+3)(x^2+1)$

$3x - 11 = A(x^2+1) + (Bx+C)(x+3)$ - - - - - - - -(ii)

$3x - 11 = Ax^2 + A + Bx^2 + 3Bx + Cx + 3C$

$3x - 11 = x^2(A+B) + x(3B+C) + (A+3C)$ - - - - - - - -(iii)

Eq(i) is an identity which holds good for all x and hence for x=-3

Put x=-3 i.e. , x+3=0 in eq (ii)

$3(-3) - 11 = A((-3)^2+1) + (Bx+C)(0)$

$-9 - 11 = A(9+1)$

$-20 = 10A$

$A = -2$

Equating coefficients of x^2 & x on b/s of eq(iii)

$0 = A + B$; $3 = 3B + C$

$B = -A$; $3 = 3(2) + C$

$B = -(-2)$; $3 - 6 = C$

$B = 2$; $C = -3$

Put values of A, B & C in eq (i)

Thus, $\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{(x+3)} + \frac{2x-3}{(x^2+1)}$ are the required partial fractions.

2. $\frac{3x+7}{(x^2+1)(x+3)}$

Let $\frac{3x+7}{(x^2+1)(x+3)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x+3)}$ - - - - - - - - - - - (i)

Multiplying eq (i) by $(x^2+1)(x+3)$ on b/s

$$(x^2+1)(x+3)\frac{3x+7}{(x^2+1)(x+3)} = \frac{Ax+B}{(x^2+1)}(x^2+1)(x+3) + \frac{C}{(x+3)}(x^2+1)(x+3)$$

$3x+7 = (Ax+B)(x+3) + C(x^2+1)$ - - - - - - - -(ii)

$3x+7 = Ax^2 + 3Ax + Bx + 3B + Cx^2 + C$

$3x+7 = x^2(A+C) + x(3A+B) + (3B+C)$ - - - - - - - - - (iii)

Eq (ii) is an identity which holds good for all x and hence for x=-3

Put x=-3 i.e. , x+3=0 in eq (ii)

$3(-3)+7 = (Ax+B)(0) + B((-3)^2+1)$

$-9+7 = C(9+1)$

$-2 = 10C$

$C = -\frac{2}{10}$

$C = -\frac{1}{5}$

Equating coefficients of x^2 & x on b/s of eq(iii)

$0 = A + C$; $3 = 3A + B$

$0 = A - \frac{1}{5}$; $3 - \frac{3}{5} = B$

$A = \frac{1}{5}$; $\frac{15-3}{5} = B$

; $B = \frac{12}{5}$

Put values of A, B & C in eq (i)

$$\frac{3x+7}{(x^2+1)(x+3)} = \frac{\frac{1}{5}x+\frac{12}{5}}{(x^2+1)} + \frac{-\frac{1}{5}}{(x+3)}$$

$$\frac{3x+7}{(x^2+1)(x+3)} = \frac{\frac{x+12}{5}}{(x^2+1)} - \frac{1}{5(x+3)}$$

$\frac{3x+7}{(x^2+1)(x+3)} = \frac{x+12}{5(x^2+1)} - \frac{1}{5(x+3)}$ are the required partial fractions.

3. $\frac{1}{(x+1)(x^2+1)}$

Let $\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)}$ - - - - - - - - - -(i)

Multiplying eq(i) by $(x+1)(x^2+1)$ on b/s

$(x+1)(x^2+1)\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)}(x+1)(x^2+1) + \frac{Bx+C}{(x^2+1)}(x+1)(x^2+1)$

$1 = A(x^2+1) + (Bx+C)(x+1)$ - - - - - - - - - -(ii)

$1 = Ax^2 + A + Bx^2 + Bx + Cx + C$

$1 = x^{2}(A+B) + x(B+C) + (A+B+C)$ - - - - - - - - - -(iii)

Eq(ii) is an identity which holds good for all x and hence for x=-1

Put x=-1 i.e. ,x+1=0 in eq (ii)

$1 = A((-1)^2+1) + (Bx+C)(0)$

$1 = A(1+1)$

$1 = 2A$

$A = \frac{1}{2}$

Equating coefficients of x^2 & x on b/s of eq(iii)

$0 = A + B$; $0 = B + C$

$B = -A$; $C = -B$

$B = -\frac{1}{2}$; $C = -\left(-\frac{1}{2}\right)$

; $C = \frac{1}{2}$

Put values of A,B &C in eq (i)

$\frac{1}{(x+1)(x^2+1)} = \frac{\frac{1}{2}}{(x+1)} + \frac{-\frac{1}{2}x+\frac{1}{2}}{(x^2+1)}$

$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} + \frac{-\frac{x-1}{2}}{(x^2+1)}$

$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} - \frac{(x-1)}{2(x^2+1)}$ are the required partial fractions.

4. $\frac{9x-7}{(x+3)(x^2+1)}$

Let $\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+C}{(x^2+1)}$ - - - - - - - - - - (i)

Multiplying eq(i) by $(x+3)(x^2+1)$ on b/s

$(x+3)(x^2+1)\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{(x+3)}(x+3)(x^2+1) + \frac{Bx+C}{(x^2+1)}(x+3)(x^2+1)$

$9x-7 = A(x^2+1) + (Bx+C)(x+3)$- - - - - - - -(ii)

$9x-7 = Ax^2 + A + Bx^2 + 3Bx + Cx + 3C$

$9x-7 = x^2(A+B) + x(3B+C) + (A+3C)$- - - - - - - - - -(iii)

Eq(ii) is an identity which holds good for all x and hence for x=-3

Put x=-3 i.e. , x+3=0 on eq(ii)

$9(-3) - 7 = A((-3)^2 + 1) + (Bx+C)(0)$

$-27 - 7 = A(10)$

$A = -\frac{34}{10}$

$A = -\frac{17}{5}$

Equating coefficients of x^2 & x on b/s of eq(iii)

$0 = A + B$; $9 = 3B + C$

$B = -A$; $C = 9 - \frac{3(17)}{5}$

$B = -\left(-\frac{17}{5}\right)$; $C = \frac{45-51}{5}$

$B = \frac{17}{5}$; $C = -\frac{6}{5}$

Put values of A, B &C in eq (i)

$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-\frac{17}{5}}{(x+3)} + \frac{\frac{17}{5}x-\frac{6}{5}}{(x^2+1)}$

$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{\frac{17x-6}{5}}{(x^2+1)}$

$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{17x-6}{5(x^2+1)}$ are the required partial fractions.

5. $\frac{3x+7}{(x+3)(x^2+4)}$

Let $\frac{3x+7}{(x+3)(x^2+4)} = \frac{A}{(x+3)} + \frac{Bx+C}{(x^2+4)}$ - - - - - - - - - -(i)

Multiplying eq(i) by $(x+3)(x^2+4)$ on b/s

$(x+3)(x^2+4)\ \frac{3x+7}{(x+3)(x^2+4)} = \frac{A}{(x+3)}(x+3)(x^2+4)\ + \frac{Bx+C}{(x^2+4)}(x+3)(x^2+4)$

$3x+7 = A(x^2+4) + (Bx+C)(x+3)$ - - - - - - - - - - -(ii)

$3x+7 = Ax^2 + 4A + Bx^2 + 3Bx + Cx + 3C$

$3x+7 = x^2(A+B) + x(3B+C) + (4A+3C)$- - - - - - - - - -(iii)

Eq(ii) is an identity which holds good for all x and hence for x=-3

Put x=-3 i.e. , x+3=0 in eq(ii)

$3(-3)+7 = A((-3)^2+4) + (Bx+C)(0)$

$-9+7 = A(9+4)$

$-2 = 13A$

$A = -\frac{2}{13}$

Equating coefficients of x^2 & x on b/s pf eq(iii)

$0 = A + B$; $3 = 3B + C$

$0 = -\frac{2}{13} + B$; $C = 3 - \frac{3(2)}{13}$

$B = \frac{2}{13}$; $C = \frac{39-6}{13}$

; $C = \frac{33}{13}$

Put values of A, B & C in eq (i)

$\frac{3x+7}{(x+3)(x^2+4)} = \frac{-\frac{2}{13}}{(x+3)} + \frac{\frac{2}{13}x+\frac{33}{13}}{(x^2+4)}$

$\frac{3x+7}{(x+3)(x^2+4)} = \frac{-2}{13(x+3)} + \frac{2x+33}{13(x^2+4)}$ are the required partial fractions.

6. $\frac{x^2}{(x+2)(x^2+4)}$

Let $\frac{x^2}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+4)}$ ---------- (i)

Multiplying eq(i) by $(x+2)(x^2+4)$ on b/s

$$(x+2)(x^2+4)\frac{x^2}{(x+2)(x^2+4)} = \frac{A}{(x+2)}(x+2)(x^2+4) + \frac{Bx+C}{(x^2+4)}(x+2)(x^2+4)$$

$x^2 = A(x^2+4) + (Bx+C)(x+2)$ -------- (ii)

$x^2 = Ax^2 + 4A + Bx^2 + 2Bx + Cx + 2C$

$x^2 = x^2(A+B) + x(2B+C) + (4A+2C)$ ----------- (iii)

Eq(ii) is an identity which holds good for all x and hence for x=-2

Put x=-2 i.e. , x+2=0 in eq (ii)

$(-2)^2 = A((-2)^2+4) + (Bx+C)(0)$

$4 = A(4+4)$

$4 = 8A$

$$A = \frac{4}{8}$$

$$A = \frac{1}{2}$$

Equating coefficients of x^2 & x on b/s of eq (iii)

$1 = A + B$; $0 = 2B + C$

$B = 1 - \frac{1}{2}$; $C = -2B$

$B = \frac{2-1}{2}$; $C = -2(\frac{1}{2})$

$B = \frac{1}{2}$; $C = -1$

Put values of A, B & C in eq (i)

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{\frac{1}{2}}{(x+2)} + \frac{\frac{1}{2}x-1}{(x^2+4)}$$

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{\frac{x-2}{2}}{(x^2+4)}$$

$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{(x-2)}{2(x^2+4)}$ are the required partial fractions.

7. $\frac{1}{x^3+1}$

Let $\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)}$

$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-x+1)}$ ----------- (i)

Multiplying eq(i) by $(x+1)(x^2-x+1)$ on b/s

$$(x+1)(x^2-x+1)\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)}(x+1)(x^2-x+1) + \frac{Bx+C}{(x^2-x+1)}(x+1)(x^2-x+1)$$

$1 = A(x^2-x+1) + (Bx+C)(x+1)$ ---------- (ii)

$1 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$

$1 = x^2(A+B) + x(-A+B+C) + (A+C)$ --------- (iii)

Eq(ii) is an identity which holds good for all x and hence for x=-1

Put x=-1 i.e. , x+1=0 in eq (ii)

$1 = A((-1)^2 - (-1) + 1) + (Bx+C)(0)$

$1 = A(1+1+1)$

$1 = 3A$

$A = \frac{1}{3}$

Equating coefficients of x^2 & x on b/s of eq (iii)

$0 = A + B$; $0 = -A + B + C$

$B = -A$; $C = A - B$

$B = -\frac{1}{3}$; $C = \frac{1}{3} - (-\frac{1}{3})$

; $C = \frac{1+1}{3}$

; $C = \frac{2}{3}$

Put values of A, B & C in eq(i)

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{\frac{1}{3}}{(x+1)} + \frac{-\frac{1}{3}x+\frac{2}{3}}{(x^2-x+1)}$$

$\frac{1}{x^3+1}=\frac{1}{(x+1)(x^2-x+1)}=\frac{1}{3(x+1)}+\frac{-\frac{x-2}{3}}{(x^2-x+1)}$

$\frac{1}{x^3+1}=\frac{1}{3(x+1)}-\frac{(x-2)}{3(x^2-x+1)}$ are the required partial fractions.

8. $\frac{x^2+1}{x^3+1}$

Let $\frac{x^2+1}{x^3+1}=\frac{x^2+1}{(x+1)(x^2-x+1)}$ by long division

$\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{A}{(x+1)}+\frac{Bx+C}{(x^2-x+1)}$ - - - - - - - - - - (i)

Multiplying eq (i) by $(x+1)(x^2-x+1)$ on b/s

$(x+1)(x^2-x+1)\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{A}{(x+1)}(x+1)(x^2-x+1)+\frac{Bx+C}{(x^2-x+1)}(x+1)(x^2-x+1)$

$x^2+1=A(x^2-x+1)+(Bx+C)(x+1)$ - - - - - - - - - (ii)

$x^2+1=Ax^2-Ax+A+Bx^2+Bx+Cx+C$

$x^2+1=x^2(A+B)+x(-A+B+C)+(A+C)$ - - - - - - - - - (iii)

Eq(ii) is an identity which holds good for all x and hence for x=-1

Put x=-1 i.e. , x+1=0 in eq (ii)

$(-1)^2+1=A((-1)^2-(-1)+1)+(Bx+C)(0)$

$1+1=A(1+1+1)$

$2=3A$

$A=\frac{2}{3}$

Equating coefficients of x^2 & x on b/s of eq(iii)

$1=A+B$; $0=-A+B+C$

$B=1-A$; $C=A-B$

$B=1-\frac{2}{3}$; $C=\frac{2}{3}-\frac{1}{3}$

$B=\frac{3-2}{3}$; $C=\frac{2-1}{3}$

$B=\frac{1}{3}$; $C=\frac{1}{3}$

Put values of A, B & C in eq (iii)

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{\frac{2}{3}}{(x+1)} + \frac{\frac{1}{3}x+\frac{1}{3}}{(x^2-x+1)}$$

$$\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3(x+1)} + \frac{\frac{x+1}{3}}{(x^2-x+1)}$$

$\frac{x^2+1}{x^3+1} = \frac{2}{3(x+1)} + \frac{(x+1)}{3(x^2-x+1)}$ are the required partial fractions.