THEOREMS CH#12

9th class Math Science (English medium)

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Merging man and math

by

Bahadar Ali Khan

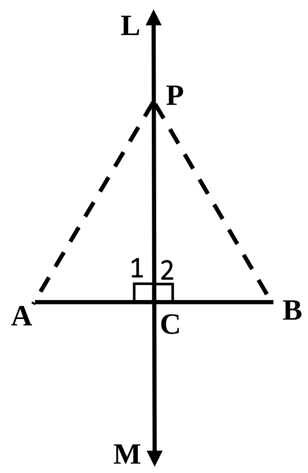
Prepared by: BAHADAR ALI KHAN

Civil Engineer from University of Engineering & Technology Lahore (Narowal Campus)

Available at https://www.mathcity.org

For Home Tuition Contact at 0305-8069878

Theorem #1: Any Point on the right bisector of a line segment is equidistant from its end points.



Given : $\overleftrightarrow{LM}$ intersects $\overline{AB}$ at point C such that $\overleftrightarrow{LM} \perp \overline{AB}$ and $\overline{AC} \cong \overline{BC}$. Point P is on $\overleftrightarrow{LM}$

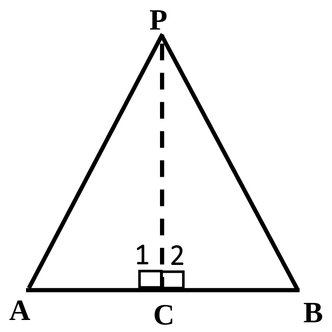
To Prove : $\overline{PA} \cong \overline{PB}$

Construction : Join P to A and B.

Proof

Statements	Reasons
$\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{AC} \cong \overline{BC}$	*Given*
$\angle 1 \cong \angle 2 = 90^o$	$\overleftrightarrow{PC} \perp \overline{AB}$
$\overline{PC} \cong \overline{PC}$	*Common*
$\Delta ACP \cong \Delta BCP$	S.A.S Postulate
$\overline{PA} \cong \overline{PB}$	*Corresponding* sides of congruent triangles

Theorem #2: Any Point equidistant from the end points of a line segment is on the right bisector of it.



Given : $\overline{AB}$ is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$

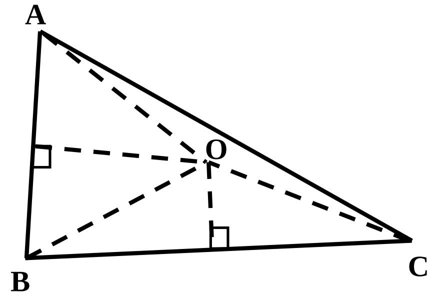
To Prove : Point P is on the right bisector of $\overline{AB}$

Construction : Join P with C, that is mid-point of $\overline{AB}$

Proof

Statements	Reasons
$\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{PA} \cong \overline{PB}$	*Given*
$\overline{PC} \cong \overline{PC}$	*Common*
$\overline{AC} \cong \overline{BC}$	*Construction*
$\Delta ACP \cong \Delta BCP$	S.S.S Postulate
$\angle 1 \cong \angle 2$ ……………..(1)	*Corresponding* angles of congruent triangles
$\angle 1 + \angle 2 = 180$ ……….(2)	*Supplementary angles*
$\angle 1 = \angle 2 = 90^o$	From (1) and (2)
$\overline{PC} \perp \overline{AB}$……..(3)	$\angle 1 = \angle 2 = 90^o$
$\overline{AC} \cong \overline{BC}$………(4)	*Construction*
Point P is on the right bisector of $\overline{AB}$	

Theorem #3: The right bisectors of the sides of a triangle are concurrent.



***Given*:** ΔABC

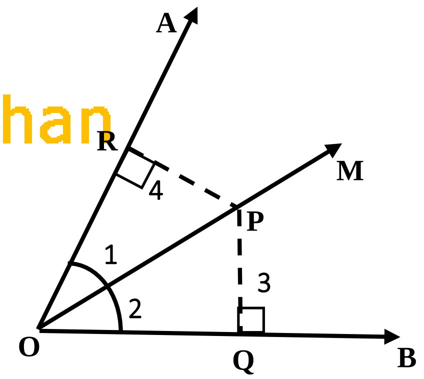
***To Prove*:** The right bisectors of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ are concurrent.

***Construction*:** Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at point O. Join O to A,B and C.

Pr*oof*

Statements	Re*asons*
$\overline{OA} \cong \overline{OB}$ ....(1)	Any Point on the right bisector of a line segment is equidistant from its end points
$\overline{OB} \cong \overline{OC}$ ....(2)	*Same* as (1) .
$\overline{OA} \cong \overline{OC}$ ....(3)	From (1) and (2)
$\therefore$ *Po*int O is on the right bisector on $\overline{CA}$ ...(4)	
But point O is also on the right bisector of $\overline{AB}$ and $\overline{BC}$ ...(5)	*Construction*
Hence the right bisectors $\overline{AB}, \overline{BC}$ and $\overline{CA}$ are concurrent	From (4) and (5)

Theorem #4: Any point on the bisector of an angle is equidistant from its arms.



***Given*:** A point P is on $\overrightarrow{OM}$, the bisector of $\angle AOB$

***To Prove*:** $\overline{PQ} \cong \overline{PR}$

***Construction*:** Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$

Pr*oof*

Statements	Re*asons*
$\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	*Common*
$\angle 1 \cong \angle 2$	*Given*
$\angle 3 \cong \angle 4$	*Construction*
$\Delta POQ \cong \Delta POR$	S.A.A Postulate
$\overline{PQ} \cong \overline{PR}$	*Corresponding* sides of congruent triangles

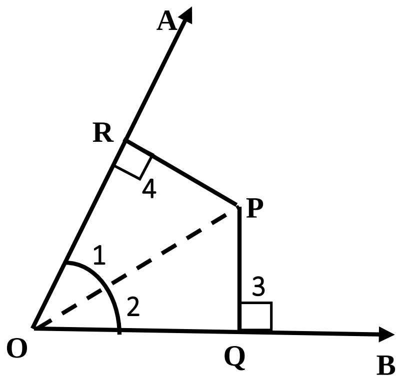
Theorem #5: Any point inside and angle, equidistant from its arms, is on the bisector of it.

Given: A point P lies inside $\angle AOB$ such that

$\overline{PQ} \cong \overline{PR}$, where $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$

To Prove: Point P is on the bisector of $\angle AOB$

Construction: Join P to O



Proof

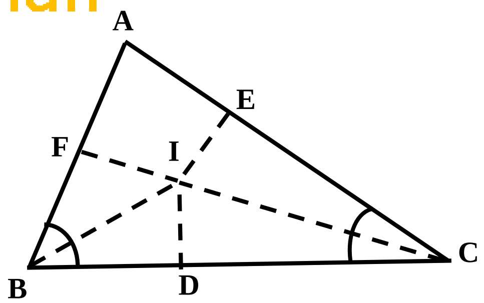
Statements	***Reasons***
$\Delta POQ \leftrightarrow \Delta POR$	
$\angle 3 \cong \angle 4 = 90^o$	*Given*
$\overline{PO} \cong \overline{PO}$	*Common*
$\overline{PQ} \cong \overline{PR}$	*Given*
$\Delta POQ \cong \Delta POR$	H.S ≅ H.S
$\angle 1 \cong \angle 2$	*Corresponding* angles of congruent triangles
Hence point P is on the bisector of $\angle AOB$	

Theorem #6: The bisectors of angles of a triangle are concurrent.

Given: ΔABC

To Prove: The bisectors of $\angle A, \angle B$ and $\angle C$ are concurrent.

Construction: Draw the bisectors of $\angle A$ and $\angle C$ which intersect at I. From I, Draw $\overline{IF} \perp \overline{AB}, \overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$



Proof

Statements	***Reasons***
$\overline{ID} \cong \overline{IF}$ $\overline{ID} \cong \overline{IE}$	(any point on bisector of an angle is equidistant from its arms)
$\overline{IE} \cong \overline{IF}$	Pr*oved*
So, the point I is on the bisector of $\angle A$ ...(1)	
Also, the point I is on the bisector of $\angle B$ and $\angle C$ ...(2)	*Construction*
bisectors of $\angle A, \angle B$ and $\angle C$ are concurrent	From (1) and (2)