UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-II)

Mathematics A-Course (Paper-IV)

Roll No: 425684

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33

Attempt FIVE Questions in all, selecting THREE questions from Section-A, and TWO from Section-B.

SECTION-A

1. a) Show that the tangents to the cardioid $\gamma = a(1 + \cos\theta)$ at the points $\theta = \frac{\pi}{3}$ and $\theta = \frac{2\pi}{3}$ are respectively parallel and perpendicular to the initial line.

   b) Find the asymptotes of the curve $xy^2 - x^2y - 3x^2 - 2xy + y^2 + x - 2y + 1 = 0$ 5

2. a) Find the point on the straight line $2x - 7y + 5 = 0$ that is closest to the origin.

   b) Show that the centre of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the folium $x^3 + y^3 = 3axy$ is $\left(\frac{21a}{16}, \frac{21a}{16}\right)$ 5

3. a) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. 5

   b) If $f(x, y) = 0$ and $\phi(y, z) = 0$ show that $\frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} \cdot \frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} \cdot \frac{\partial \phi}{\partial y}$. 5

4. a) Find an equation of the tangent plane at any point $P(x_1, y_1, z_1)$ of the elliptic paraboloid $z = x^2 + 4y^2$. 5

   b) Find the extrema of $f(x, y) = e^{-(x^2 + y^2 + 2x)}$. 5

5. a) Maximize $z = 3x_1 + 2x_2$

   with $x_1 + 2x_2 \le 6,\quad 2x_1 + x_2 \le 8,\quad -x_1 + x_2 \le 1,\quad x_2 \le 2,\quad x_1 \ge 0,\ x_2 \ge 0.$ 5

   b) Solve the assignment model. 5

3	9	2	3	7
6	1	5	6	6
9	4	7	10	3
2	5	4	2	1
9	6	2	4	5

SECTION – B

6. a) Find the area of the smaller segment cut from a circular disc of radius 'a' by a chord at a distance 'b' from the centre $a > b$.

   b) Show that the length of the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between the points for which $\theta = 0$ and $\theta = 2\alpha$ is $S = 4a\sin\alpha$. 5

7. a) Find the volume of the solid of revolution generated by revolving the area enclosed by $y = x^2 - 1$ from $x = 1$ to $x = 3$ about the y-axis. 5

   b) Find the surface area generated by revolving the line segment between $(\gamma_1, 0)$ and (γ_2, h) about the y-axis. 5

8. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (y + y^3)\, dy\, dx$. 5

   b) $\int\int\int_S x^2y^2z\, dz\, dy\, dx$ S defined by $0 \le z \le x^2 - y^2$, $0 \le x \le 1$, $0 \le y \le 1$. 5