

# Mathematics A-Course (Paper-II)

Attempt FIVE Questions in all. Select TWO Questions from Section-A and THREE from Section-B.

## Section-A

1. a) Prove that  $\left| \frac{az+b}{bz+a} \right| = 1$  for  $|z| = 1$ . 1.1 5
- b) State and prove the DE-MOIVRES theorem on complex numbers. 5
2. a) Prove that  $\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$ . 1.3 5
- b) If  $\sin(\theta + i\phi) = \cos \alpha + i \sin \alpha$  Prove that  $\cos^2 \theta = \pm \sin \alpha$ . 1.3 5
3. a) Separate into real and imaginary parts  $\sin^{-1}(\cos \theta + i \sin \theta)$  5
- b) Find the sum of the infinite series  $\sin \theta + \frac{1}{2} \sin 3\theta + \frac{1.3}{2.4} \sin 5\theta + \dots$  1.5 5

## Section-B

4. a) Find equation of tangent and normal of the curve  $x(x^2 + y^2) - ay^2 = 0$  at  $x = \frac{a}{2}$ . 6.2 5
- b) Sketch the graph of the curve  $r = a(1 + \cos \theta)$ ,  $a > 0$ . 6.5 5
5. a) Show that the pedal equation of the curve  $c^2(x^2 + y^2) = x^2 y^2$  is  $\frac{1}{p^2} + \frac{3}{r^2} = \frac{1}{c^2}$  6.2 5
- b) Express the following equation in polar form and find the eccentricity and equation of directrix  $8x^2 + 9y^2 + 4x - 4 = 0$ . 6.6 5
6. a) Under what condition on  $x, y$  and  $z$  is the point  $P(x, y, z)$  is equidistant from the points  $(3, -1, 4)$  and  $(-1, 5, 0)$ . 8.1 5
- b) Find an equation of the sphere passing through the points  $(0, -2, -4)$ ,  $(2, -1, -1)$  and having its centre on the straight line.  $2x - 3y = 0 = 5y + 2z$ . 5
7. a) If measures of two directional angles of a straight line are  $45^\circ$  and  $60^\circ$  find measure of the third directional angle. 5
- b) Find an equation of plane through the three given points  $A(2, 1, 1)$ ,  $B(6, 3, 1)$ ,  $C(-2, 1, 2)$ . 5
8. a) Express the equation in cylindrical and spherical coordinates  $x^2 - y^2 - z^2 = 1$  5
- b) Discuss the surface of Ellipsoid. 5