UNIVERSITY OF GUJRAT

(B.A/B.Sc. Part-II)

Roll No: _____

Mathematics A-Course (Paper-IV)

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all, selecting **THREE** questions from **Section-A**, and **TWO** from **Section-B**.

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SECTION-A

1. a) Show that the tangents to the cardioids $r = a(1 + \cos\theta)$ at the points $\theta = \frac{\pi}{3}$ and $\theta = \frac{2\pi}{3}$ are respectively parallel and perpendicular to the initial line. *(6.6)* 5

   b) Find the asymptotes of the curve $x^2(x-y)^2 + a^2(x^2 - y^2) = a^2xy.$ *(7.1)* 5

2. a) Find the intervals in which the curve $y = 3x^5 - 40x^3 + 3x - 20$ faces i) upward ii) downward. Also find the points of inflection. *(7.2)* 5

   b) Show that the centre of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the folium $x^3 + y^3 = 3axy$ is $\left(\frac{21a}{16}, \frac{21a}{16}\right)$ *(7.8)* 5

3. a) If $V = \rho^m$ where $\rho^2 = x^2 + y^2 + z^2$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m+1)\rho^{m-2}$ *(9.1)* 5

   b) Find the percentage error in the area of an ellipse when an error of 1% is made in measuring each of the major and minor axes of the ellipse. *(9.2 · Exp)* 5

4. a) Find equations for the tangent plane and the normal line to the surface at point P

   $$x^2 + z^2 = \frac{a^2}{h^2}y^2 \qquad P\left(\frac{a}{\sqrt{2}}, h, \frac{a}{\sqrt{2}}\right) \quad (9.5)$$ 5

   b) Find the extrema of $f(x, y) = 2x^2 + xy^2 - 4x - 1.$ 5

5. a) Maximize $Z = 10x_1 + 11x_2$ with the conditions 5
   $$3x_1 + 4x_2 \leq 9$$
   $$5x_1 + 2x_2 \leq 8$$
   $$x_1 - 2x_2 \leq 1$$
   $$x_1 \geq 0 \text{ and } x_2 \geq 0$$

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   b) Solve the assignment model 5

1	4	6	3
9	7	10	9
4	5	4	7
8	7	8	5

SECTION – B

6. a) Find the area of region bounded by one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and its base. 5 *(7.5)*

   b) Show that the intrinsic equation of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $S = \frac{3a}{2}\sin^2\alpha$ *(7.6)* 5

7. a) Find the volume of a right circular cone having radius r and height h. 5

   b) Find the area of the surface of revolution generated by revolving about the x-axis the area bounded by an arc of the parabola $y^2 = 12x$ from $x = 0$ to $x = 3.$ 5

8. a) Evaluate: $\int_0^4 \int_{\sqrt{y}}^2 y \cos x^5 \, dx \, dy.$ 5

   b) Evaluate: $\int_0^4 \int_0^{4-x} \int_0^{4-x-y} dz \, dy \, dx.$ 5