UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I)

Roll No: A.8

Pass Marks : 33%

Mathematics A-Course (Paper-I)

Attempt **FIVE** Questions in all. Select **THREE** Questions from **Section-A** and **TWO** from **Section-B.**

SECTION-A

1. a) Solve $\frac{x^2 - 2}{1 - 2x} > 1$. (1ch) 5

b) Examine whether the given function is continuous at $x = 0$ $f(x) = \begin{cases} (1+3x)^{1/x} & \text{if } x \neq 0 \\ e^2 & \text{if } x = 0 \end{cases}$ Ch1 5

2. a) Define continuous function, if f is differentiable at a point $a \in \text{Dom } f$, then f is continuous at a. 47Ch2

b) Find $f'(x)$, where $f(x) = \frac{-\text{Cos } x}{2\text{Sin}^2 x} + \frac{1}{2}\ \ell n \tan\left(\frac{x}{2}\right)$ Ch2 5

3. a) Show that $\frac{d^n}{dx^n}\left[\frac{\ell n\, x}{x}\right] = \frac{(-1)^n n!}{x^{n+1}}\left[\ell n\, x - 1 - \frac{1}{2} - \frac{1}{3} \ldots\ldots\ldots - \frac{1}{n}\right]$ Ch2 5

b) Find $\frac{dy}{dx}$ by using partial derivatives $x^3 + x^2 + xy^2 + \text{Sin } y = 0$ Ch2 5

4. a) State and prove the CAUCHY MEAN VALUE THEOREM? Ch3 5

b) Let $f(x) = x^2$, $g(x) = x^3$, verify Cauchy Mean Value Theorem on [1, 2]. Also find C. Ch3 5

5. a) Show that, under certain condition to be stated, $f(a + h) = f(a) + h f'(a + \theta h)$ where $0 < \theta < 1$, Prove also that, the limiting value of θ, When h decreases definitely is $\frac{1}{2}$. Ch3 5

b) Evaluate $\lim\limits_{x \to \frac{\pi}{2}} \frac{\tan x}{\tan 3x}$ Ch3 5

SECTION-B

6. a) Evaluate $\int \text{Sin}^{-1}\left(\sqrt{\frac{x}{x+a}}\right) dx.$ 5

b) Evaluate $\int \frac{\text{Sec } x}{1 + \text{Cosec } x}\, dx$ 5

7. a) Evaluate (by definition) $\int_0^{\pi/2} \text{Cos } x\, dx.$ 5

b) Evaluate $\int_0^{\pi/4} \ell n\,(1 + \tan x)\, dx.$ Example 10, 180 5

8. a) Determine where the following improper integral converges. Evaluate the integral that converge

$$\int_0^{\infty} \frac{\ell n\,(1 + x^2)}{1 + x^2}\, dx.$$ 5

b) Show that $\int \text{Sec}^{2n+1} x\, dx = \frac{\text{Sec}^{2n-1} x \tan x}{2n} + \left(1 - \frac{1}{2n}\right) \int \text{Sec}^{2n-1} x\, dx.$ 5