UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I)

Mathematics A-Course (Paper-II)

Roll No: ____

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**

Section-A

1. a) State and prove De Movre's Theorem. 5

   b) Find the square of all the 5^{th} roots of $\frac{1}{2} + \frac{\sqrt{3}}{2} i$. 5

2. a) If $\text{Sin}(\theta + \phi i) = \text{Cos}\,\alpha + i\,\text{Sin}\,\alpha$ Prove that $\text{Cos}^2\theta = \pm\,\text{Sin}\,\alpha$ 5

   b) Show that $\text{Log}(1 + \text{Cos}\,\theta + i\,\text{Sin}\,\theta) = \ell n\,(2\,\text{Cos}\frac{\theta}{2}) + i\frac{\theta}{2}$ 5

3. a) Sum the series $\text{Cos}^2\theta + \text{Cos}^2 2\theta + \text{Cos}^2 3\theta + \ldots\ldots\ldots\ldots + \text{Cos}^2 n\theta$. 5

   b) Separate into real and imaginary parts $\text{Tan}^{-1}(x + i\,y)$ 5

Section-B

4. a) Find the condition that the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ should intersect orthogonally. 5

   b) Express $y^2 = 4 - 4x$ in polar form and find the eccentricity and equation of the directrix. 5

5. a) Sketch the graph of the Curve $r = a + b\,\text{Cos}\,\theta$. 5

   b) Prove that the pedal equation of the asteroid $x = a\,\text{Cos}^3\theta$, $y = a\,\text{Sin}^3\theta$ is $r^2 = a^2 - 3p^2$ 5

6. a) A variable line in two adjacent positions has direction cosines ℓ, m, n and $\ell + \delta\ell$, $m + \delta m$, $n + \delta n$. Show that measure of the small angle $\delta\theta$ between the two positions is given by $(\delta\theta)^2 = (\delta\ell)^2 + (\delta m)^2 + (\delta n)^2$ 5

   b) The vertices of tetrahedron are (0, 0, 0), (3, 0,0), (0, -4, 0) and (0, 0, 5). Find equations of the plane of its faces. 5

7. a) Find equation of the perpendicular from the origin to the line. $x + 2y + 3z + 4 = 0 = 2x + 3y + 4z + 5$. Also find the coordinates of the foot of the perpendicular. 5

   b) Find the coordinates of the point on the join of (-3, 7, -13) and (-6, 1, -10) which is nearest to the intersection of the planes $2x - y - 3z + 32 = 0$ and $3x + 2y - 15z - 8 = 0$. 5

8. a) Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ 5

   Find equations of the straight line perpendicular to both the given straight lines and also its points of intersection with the given straight lines.

   b) Find and equation of the sphere circumscribing the tetrahedron whose faces are $x = 0$, $y = 0$, $z = 0$ and $\ell x + my + nz + p = 0$ 5