# UNIVERSITY OF THE PUNJAB

MathCity.org
Merging Man and maths

Part – I A/2016
Examination:- B.A./B.Sc.

Roll No. .....

Subject: Mathematics A Course-I
PAPER: Calculus and Analytical Geometry

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

*NOTE: Attempt SIX questions by selecting TWO questions from Section-I, TWO questions from Section-II, ONE questions from Section-III and ONE question from Section-IV.*

## SECTION-I

Q.1. (a): Solve the inequality $\frac{2x}{x+2} \geq \frac{x}{x-2}$ 9

(b): Let $f(x)=\begin{cases} x\cos\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = o \end{cases}$ 8

Discuss the continuity of f at x=0

Q.2. (a): Find $\frac{dy}{dx}$ if $y = \arctan\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)$ 9

(b): Use differentials to find approximate value of $\tan 29^0$ 8

Q.3 (a): If $y = (arc\sin x)^2$, show that $(1-x^2)y'' - xy' - 2 = 0$, 9
Differentiate this equation n times and find the value of $y^n$ at x=0

(b): If $x > 0$, prove that $x - \ln(1+x) > \frac{x^2}{2(1+x)}$ 8

Q.4...(a): Find by Maclaurin formula, the first four terms of expansion of

$f(x) = e^{ax}\cos bx$ and remainder after n terms 8

(b):Use L' hospital ' rule to prove that $\lim_{x\to\infty}\left[\frac{a^{1/x}+b^{1/x}}{2}\right]^x = \sqrt{ab}$
a>0 .b>0 9

## SECTION-II

Q.5. (a) Integrate the following ? 5,5, 7

(i) $\int sec^3 x\, dx$

(ii) $\int \frac{dx}{(1+x)\sqrt{x^2-1}}$

(iii) $\int \frac{dx}{ax^n+bx}$

Q.6. (a): Prove that 8

$$\int_0^{\pi} \frac{xsinx}{1+sinx} dx = \frac{\pi^2}{2} - \pi$$

P.T.O.

(b): Obtain a reduction formula for $\int \frac{x^n}{\sqrt{1-x^2}}dx$ and hence

Evaluate $\int \frac{x^3}{\sqrt{1-x^2}}dx$ 9

Q.7 (a): Show that the pedal equation of the astroid x=a$cos^3\theta$, y=a$sin^3\theta$ is $r^2 = a^2 - 3p^2$ 9,8

(b): show that tangents to cardioids r = a ( 1+ cos $\theta$) at points $\theta = \frac{\pi}{3}$ and $\theta = \frac{2\pi}{3}$ are respectively parallel and perpendicular to initial line

Q.8 (a): If p = x cos$\theta$ + y sin $\theta$ touches the curve $\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$.

Prove that $p^n = (a\ cos\theta)^n + (b\ sin\theta)^n$ 9

(b): Prove that the area enclosed by parallelogram formed by the tangents at end of conjugate diameters of ellipse is constant 8

## SECTION-III

Q.9 (a): Find the asymptotes of the curve $x^2$y +x $y^2$+ xy +$y^2$+3x = 0 8

(b): Find the relative maxima and minima of r = 1 – cos $\theta$ 8

Q.10 (a): Show that the intrinsic equation of the parabola

$x^2$ =4ay is S=atan$\alpha$sec$\alpha$+aln(tan$\alpha$+sec$\alpha$) 8

(b): Prove that radius of curvature at point (2a,2a) on the curve $x^2$y = a($x^2$+$y^2$) is 2a 8

## SECTION-IV

Q.11 (a): The direction cosines of two straight lines are given by the equations l + m + n = 0 and $l^2$+$m^2$+ $n^2$=0. Find the angle between them.

(b): Find equation of the plane that passes through the points (3,2,-1) and (1,-3,4) and contains the line parallel to 2i – 4j + 3k 8,8

Q.12 (a): Find an equation of the sphere passing through the

Points (0,-2,-4) ,(2,-1,-1) and having its center on the st-line

2x - 3y=0 = 5y + 2z 8

(b): Find the direction of Qibla of Badshahi Mosque,Lahore,

Latitude = $32^0 35.4'$N and longitude = $74^0 18.7'$E 8