UNIVERSITY OF THE PUNJAB

Part – I A/2016
Examination:- B.A./B.Sc.

Roll No. .....

Subject: Mathematics B Course-I **TIME ALLOWED: 3 hrs.**
PAPER: Vector **MAX. MARKS: 100**

Note: Attempt SIX questions by selecting ONE question from SECTION-I, TWO questions from Section-II, TWO questions from SECTION-III and ONE question from SECTION-IV.

SECTION-I

Q1 (a) Verify the formula $\underline{a} \times (\underline{b} \times \underline{c}) = \underline{a}.\underline{cb} - \underline{a}.\underline{bc}$ 8

by taking $\underline{a} = \hat{i}+\hat{j}$, $\underline{b} = -\hat{i}+2\hat{k}$ and $\underline{c} = \hat{j}+\hat{k}$

(b) Show that for a vector function $\vec{f}(t)$, $\frac{d}{dt}(\vec{f}.\vec{f}' \times \vec{f}'') = \vec{f}.\vec{f}'' \times f'''$ 8

Q2 (a) Define the gradient scalar point function. If 8

$\phi = 2x^3y^2z^4$ then find divgradϕ.

(b) If $\vec{A} = x^2z\underline{i}+yz^3\underline{j}-3xy\underline{k}$, 8

$\phi = x^2yz$, find curl $(\phi\vec{A})$ at (1,1,1)

SECTION-II

Q3 (a) Forces of magnitude P,2P.3P,4P act respectively along the sides AB,BC,CD,DA of a square ABCD. of side a, and forces each of magnitude $(8\sqrt{2})P$ act along the diagonals BD, AC. Find the magnitude of the resultant force and the distance of its line of action from A. 9

(b) A couple of moment G acts on a square board ABCD of side a. Replace the couple by forces acting along AB, BD and CA. 8

Q4 (a) A triangular lamina ABC, right-angled at A, rests with its plane vertical, and with the sides AB,AC supported by smooth fixed pegs D,E in a horizontal line. Prove that the inclination Θ of AC to the horizontal is given by 9

$AC \cos\Theta - AB \sin\Theta = 3DE \cos 2\Theta$

(b) The radius of the faces of a frustum of a solid cone are 2 ft. and 3 ft. and the height of the frustum is 4 ft. find the distance of the c.g from the larger face. 8

Q5 (a) Find the position of the centroid of a quadrant of an elliptic lamina. 9

(b) A uniform ladder, of length 70 feet, rests against a vertical wall with which it makes an angle of 45°, the coefficient of friction between the ladder and the wall and the ground respectively being 1/3 and ½. If a man, whose weight is one half that of the ladder, ascends the ladder, where will he be when the ladder slips? 8

Q6 (a) A regular octahedron formed of twelve equal rods, each of weight w, freely joined together is suspended from one corner.Show that the thrust in each horizontal rod is $3/2\sqrt{2}w$. 9

(b) A rod 4 ft. long rests on a rough floor against the smooth edge of a table of height 3 ft. If the rod is on the point of slipping when inclined at an angle of 60° to the horizontal, find the coefficient of friction. 8

P.T.O.

SECTION-III

Q7 (a) Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

With uniform speed v when the particle is at (0,b). 9

(b) A particle is projected vertically upwards. After a time t, another particle is sent up from the same point with the same velocity and meet the first at height h during the downwards flight of the first. Find the velocity of projection. 8

Q8 (a) A particle moving in a straight line starts with a velocity u and has acceleration v^3, where v is the velocity of the particle at time t. Find the velocity and the time as functions of the distance travelled by the particle. 9

(b) A gun of mass M fires a shell of mass m horizontally, and the energy of the explosion is such that it would be sufficient to project the shell vertically to a height h. Show that the velocity of recoil of the gun has magnitude. 8

$$\sqrt{\frac{2m^2gh}{M(M+m)}}$$

Q9 (a) A shall bursts on contact with the ground and pieces from it fly in all directions with all speeds upto 80feet per second. Prove that a man 100 feet away is in danger for $\frac{5}{\sqrt{2}}$ seconds. 9

(b) A projectile is launched at an angle α from a cliff of height H above sea level. If it falls into the sea at a distance D from the base of the cliff, prove that the maximum height above sea level is $H+\frac{D^2\tan^2\alpha}{4(H+D\tan\alpha)}$ 8

Q10 (a) Two particles of masses 7m and 3m are fastened to the end A and B respectively of a weightless rigid rod of length 3a, which is freely hinged at a point O distance from A. If the rod is just disturbed from its position of unstable equilibrium, prove that the speed with which A will pass through its position of stable equilibrium is $\sqrt{\frac{4ga}{19}}$. Determine also the angular velocity when the rod has turned through an angle Θ with the vertical. 9

(b) A particle falls under gravity in a medium which opposes the motion with a force proportional to the velocity of the body. The limiting velocity acquired by the particle is such that it would be attained in time T if the body were falling in vacuum. Show that the medium being present, the body acquires half the limiting velocity in a time 0.693 T. 8

SECTION-IV

Q11 (a) A particle describes a curve $r^nCosn\Theta=a^n$. Find law of force? 8

(b) Three perfectly elastic balls of masses m, 2m and 3m are placed in a straight line. The first impinges directly on the second with a velocity u and then the second impinges on the third. Find the velocity of the third ball after impact. 8

Q12 (a) Two equal spheres of mass m are in contact on a smooth horizontal table. A third equal ball of m′ impinges symmetrically on them and is reduced to rest. Prove that $e=\frac{2m'}{3m}$ and find the loss of K.E due to impact. 8

(b) A planet is describing an ellipse about the sun as focus. Show that its velocity away from the sun is greatest when the radius vector to the planet is at right angles to the major axis of the path, and that it then is $\frac{2\pi ae}{T(1-e^2)}$, where 2 a the major axis, e the eccentricity, and T is the periodic time. 8