# UNIVERSITY OF THE PUNJAB



A/2015

**Examination:- B.A./B.Sc.**

Roll No. .....

**Subject: Mathematics General**
**PAPER: A**

**TIME ALLOWED: 3 hrs.**
**MAX. MARKS: 100**

***NOTE: Attempt any SIX questions by selecting TWO questions from Section – I, TWO questions from Section – II, ONE question from Section – III and ONE question from Section – IV.***

## Section-I

Q. 1. (a) Solve the inequality 8+9

$$\frac{2x}{x+2} \geq \frac{x}{x-2}$$

(b) If $y = \sin(a \, arc \sin x)$, prove that

$$(1-x^2)y^{(n+2)} = (2n+1)xy^{(n+1)} + (n^2 - a^2)y^{(n)}$$

Q. 2. (a) Discuss the validity of Rolle's Theorem for the function $f(x) = x(x+3)e^{\frac{-x}{2}}$ on [-3, 0] and also find c such that $f'(c) = 0$ 8+9

(b) Evaluate: $\lim_{x\to 0} \frac{sinx - \ln(e^x cosx)}{xsinx}$

Q. 3. (a) Find equations of the "asymptotes" of the curve $r \sin n\theta = a$ 8+9

(b) Find the positional nature of the multiple points on the curve $x^2(x-y) + y^2 = 0$

Q. 4. (a). Find the radius of curvature at any point on the curve $r^n = a^n \sin n\theta$

(b) Find the intervals in which the curve: 8+9

$y = (x^2 + 4x + 5)e^{-x}$

Faces upward or downward. Also find its points of inflection.

## Section-II

Q. 5. Evaluate the integrals: 5, 6, 6

i. $\int \frac{sinx}{sin3x} dx$ ii. $\int \frac{x\sqrt{1+x}}{\sqrt{1-x}} dx$ iii. $\int \frac{1}{tanx - sinx} dx$

Q. 6. (a) Show that $\int_0^{\pi} \frac{x}{1+\sin x} dx = \pi$ 8+9

(b) Use Simpson's rule to evaluate : $\int_0^1 \frac{1}{1+x^2} dx \quad with \quad n = 4$

Q. 7. (a) Find the area of the smaller segment cut from a circular disc of radius "a" by a chord at a distance "b" from the centre, $(a > b)$. 8+9

(b) Sketch the graph of the curve $r = -a(1 + \cos\theta)$ , a > 0

Q. 8. (a) If $\frac{b}{a}$ is small, then calculate the perimeter of the limacon 8+9
$r = a + b\cos\theta$

(b) Find the surface area generated by revolving the line segment between $(r_1, 0)$ and $(r_2, h)$ $about\ the\ y - axis$

## Section-III

Q. 9. (a) Test the series $\sum_2^{\infty} \frac{1}{n(\ln n)^p}$ converges or diverges. 8+8

(b) Test the series for absolute convergence, conditional convergence or divergence
$\sum_1^{\infty} \frac{(-1)^n(n+2)}{n(n+1)}$

Q. 10. (a) If $x > 0$ , prove that the series $\sum_1^{\infty} \frac{1.3.5.....(2n-1)}{1.4.7.....(3n-2)} x^n \ converges\ for\ x < \frac{3}{2}$ 8+8

(b) Find the interval and radius of convergence of $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n(\ln n)^2}$

## Section-IV

Q. 11. (a) Find $\frac{d^2y}{dx^2}$ if $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$ 8+8

(b) Use differentials to approximate $\sqrt{49.5}$

Q. 12. (a) If $f(x,y) = x^y + y^x$, $then\ verify\ that \quad f_{xy} = f_{yx}$ 8+8

(b) Find the extreme values of the function
$f(x,y) = \frac{1}{x} + xy - \frac{8}{y}$