

UNIVERSITY OF THE PUNJAB



MathCity.org
Merging Man and maths

Part - I A/2016
Examination:- B.A./B.Sc.

Roll No.

Subject: Mathematics General-I
PAPER: Calculus (Differential and Integral Calculus)

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Attempt any **SIX** questions by selecting **TWO** questions from **Section-I**, **TWO** questions from **Section-II**, **ONE** question from **Section-III** and **ONE** question from **Section-IV**.

Section-I

Q. No. 1(a)

i. Solve the inequality $|x^2 - x + 1| > 1$

4+4

ii. Examine the continuity of

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & \text{if } 0 \leq x < a \\ a & \text{if } x = a \\ 2a & \text{if } x > a \end{cases}$$

at $x = a$

(b)

4+5

i. Evaluate $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x}$

ii. Let $\delta > 0$ and $a \in \mathbb{R}$. Show that $a - \delta < x < a + \delta$ if and only if $|x - a| < \delta$

Q. No. 2(a) Let $f(x) = \begin{cases} \sin 2x & \text{if } 0 < x \leq \frac{\pi}{6} \\ ax + b & \text{if } \frac{\pi}{6} < x \leq 1 \end{cases}$

8+9

Derive the values of a and b if $f(x)$ is continuous and differentiable at $x = \frac{\pi}{6}$.

(b) If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)y'' - xy' - 2 = 0$

Differentiate this equation n times and find $y^{(n)}(0)$.

Q. No. 3(a) If $f(x) = -\frac{\cos x}{2\sin^2 x} + \frac{1}{2} \ln \tan \left(\frac{x}{2}\right)$

8+9

Then show that $f'(x) = \operatorname{cosec}^3 x$

(b) The side of a cube is measured with a possible error of $\pm 2\%$. Find the percentage error in the surface area of one the cube.

Q. No. 4(a) Evaluate the given limits

4+4

i. $\lim_{x \rightarrow 0} \left(\frac{1}{x \sin^{-1} x} - \frac{1}{x^2} \right)$

ii. $\lim_{x \rightarrow 0} (\tan x)^{\sin 2x}$

(b) Prove that $f(x) = 2x - \tan^{-1} x - \ln(x + \sqrt{x^2 + 1})$ is an increasing function on $[0, \infty[$

9

P.T.O.

Section-II

Q. No. 5(a) Evaluate $\int \frac{1}{a+b \cosh x} dx$ 8+9

(b) Evaluate $\int \frac{1}{(x^2+4x+5)\sqrt{x+2}} dx$

Q. No. 6(a) Evaluate $\int_0^\pi \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$ 8+9

(b) Obtain a reduction formula for $\int \frac{x^n}{\sqrt{1-x^2}} dx$ and hence evaluate $\int \frac{x^3}{\sqrt{1-x^2}} dx$

Q. No. 7(a) Analyze and graph the conics represented by $xy = 1$ 8+9

(b) If $PF P'$ and $QF Q'$ are two perpendicular focal chords of a conic, prove that

$$\frac{1}{|PF| \cdot |FP'|} + \frac{1}{|QF| \cdot |FQ'|}, \text{ is constant.}$$

Q. No. 8(a) Find the pedal equation of the curve $\frac{l}{r} = 1 + e \cos \theta$ 8+9

(b) If $x = a \cos g(t)$, $y = b \sin g(t)$, prove that: $xy^2 \frac{d^2 y}{dx^2} = b^2 \frac{dy}{dx}$

Section- III

Q. No. 9(a) Find the relative maxima and minima of y if $r = 1 - \cos \theta$ 8+8

(b) Find equations of the tangents at the multiple points of the curve

$$(y-2)^2 = x(x-1)^2$$

Q. No. 10(a) Find the point on the curve $y = \ln x$ where the curvature K is maximum

(b) Find the envelope of the family of lines $y = mx + \sqrt{a^2 m^2 + b^2}$, m being the parameter. 8+8

Section- IV

Q. No. 11(a) Verify that $f_{xy} = f_{yx}$ 8+8

$$f(x, y) = x^y + y^x$$

(b) Find $\frac{d^2 y}{dx^2}$ if $x^3 + y^3 = 3axy$

Q. No. 12(a) Show that the ellipsoid $\frac{x^2}{12} + \frac{y^2}{16} + \frac{z^2}{12} = 1$ and the hyperboloid 8+8

$$\frac{y^2}{3} - x^2 - z^2 = 1 \text{ intersect orthogonally.}$$

(b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} \int_0^{\sqrt{a^2-x^2-y^2}} x dz dx dy$