PPSC Paper 2015 (Lecturer in Mathematics)

Available at: http://www.mathcity.org/ppsc

Time Allowed: Two Hours Maximum Marks: 100

We are very thankful to *Kaushef Salamat* for providing this paper.

Instructions:

- Read QUESTION PAPER carefully and mark your answers on the ANSWER SHEET.
- Each question has four options. Fill only one box that you think is the correct answer. Each question carries 1 mark. 0.25 mark will be deducted for each incorrect answer.
- Use of calculator is NOT allowed.

1. $\int_{-4}^{0} \frac{t\,dt}{\sqrt{16-t62}}$

   (A) 0 (B) divergent (C) -4 (D) 4

2. The period of the function $A\cos wt + B\sin wt$ is

   (A) $\frac{\omega}{2\pi}$ (B) $2\pi\omega$ (C) $\frac{\omega}{2\pi}$ (D) $\frac{2\pi}{\omega}$

3. $A = (-4x-3y+az)\underline{i} + (bx+3y+5z)\underline{j} + (4x+cy+3z)\underline{k}$ is irrational when a, b, c are

   (A) $4, -3, 5$ (B) $4, 5, -3$ (C) $-3, 4, 5$ (D) $2, 3, 5$

4. $V = (-4x-6y+3z)\underline{i} + (-2x+y-5z)\underline{j} + (4x+6y+az)\underline{k}$ is is-solenoidal for $a =$ ———

   (A) 1 (B) 2 (C) 3 (D) 4

5. $\int_{(2,1)}^{(0,0)} (10x^4 - 2xy^3)dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$ is ———

   (A) 56 (B) 60 (C) 62 (D) 64

6. If S is the closed surface and V is the volume enclosed by S then $\iint_s \underline{r}\cdot\underline{n}ds =$ ———

   (A) v (B) $2v$ (C) $3v$ (D) $4v$

7. Centrifugal acceleration is ———

   (A) $-\omega\times(\omega\times r)$ (B) $\omega\times(\omega\times r)$ (C) $\omega.(\omega\times r)$ (D) $r\times(\omega\times r)$

8. Number of the degrees of freedom of two particles connected by a rigid road moving freely in the plane is ——

   (A) 2 (B) 3 (C) 4 (D) 5

9. The centroid of a uniform semicircular wire of radius a is

   (A) $\frac{2a}{\pi}$ (B) $\frac{4a}{\pi}$ (C) $\frac{a}{\pi}$ (D) $\frac{a}{2\pi}$

10. Moment of inertia of a rectangular plate with sides a, b about an axis $\perp$ to plate and passing through vertex is ———

    (A) $\frac{1}{3}Ma^2$ (B) $\frac{1}{3}Mb^2$ (C) $\frac{1}{3}M(a^2-b^2)$ (D) $\frac{1}{3}M(a^2+b^2)$

11. Every bounded infinite set has at least one limit point is the statement of ——

(A) Hein-Borel Theorem (B) welerstrass-Boizano theorem (C) Cantor's intersection theorem (D) none of these

12. $\lim_{x \longrightarrow 0} \frac{\bar{z}}{z} =$

(A) $\frac{1+i}{1-i}$ (B) 1 (C) Does not exist (D) -1

13. Cauchy-Riemann equations in polar form are ——

(A) $\frac{\delta u}{\delta r} = \frac{1}{r}\frac{\delta v}{\delta \theta}, \frac{\delta v}{\delta r} = -\frac{1}{r}\frac{\delta u}{\delta \theta}$ (B) $\frac{\delta u}{\delta r} = \frac{-1}{r}\frac{\delta v}{\delta \theta}, \frac{\delta v}{\delta r} = \frac{1}{r}\frac{\delta u}{\delta \theta}$ (C) $\frac{\delta u}{\delta r} = \frac{-1}{r}\frac{\delta v}{\delta \theta}, \frac{\delta v}{\delta r} = -\frac{1}{r}\frac{\delta u}{\delta \theta}$

(D) $\frac{\delta u}{\delta r} = \frac{1}{r}\frac{\delta v}{\delta \theta}, \frac{\delta v}{\delta r} = \frac{1}{r}\frac{\delta u}{\delta \theta}$

14. Evaluate $\int_C \frac{z^2 - z + 1}{z - 1} dz$, where C is the circle $|z| = \frac{1}{2}$:

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 0

15. The principal value of $(-i)^i$ is:

(A) $e^{\frac{\pi}{2}}$ (B) 1 (C) $e^{\frac{\pi}{2}}$ (D) e^{π}

16. The residue of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at $z = 2i$ is ———-

(A) $\frac{14}{25}$ (B) $\frac{7+i}{25}$ (C) $\frac{7-i}{25}$ (D) $\frac{-7-i}{25}$

17. Radius of convergence of $\sum (3+4i)^n z^n$ is ————

(A) $\frac{1}{5}$ (B) 5 (C) 7 (D) ∞

18. $\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$ is ————

(A) 1 (B) 0 (C) e^x (D) e^n

19. $U(x, y) = e^x \cos y$ is ———

(A) Harmonic (B) Analytic (C) Not harmonic (D) None of these

20. $\int_0^{\infty} \frac{\sin x}{x} dx =$ ————-

(A) 0 (B) $\frac{-\pi}{2}$ (C) $\frac{\pi}{2}$ (D) π

21. $\log(1+i) =$ ————-

(A) $\frac{1}{2}\ln 2 + \frac{\pi i}{4}$ (B) $\frac{1}{2}\ln 2 - \frac{\pi i}{4}$ (C) $\frac{1}{2}\ln 2 - \frac{3\pi i}{4}$ (D) $\frac{1}{2}\ln 2 + \frac{3\pi i}{4}$

22. Which of the following space is complete.

(A) Q (B) $]0, 1]$ (C) Z (D) R

23. Least upper bound of $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}$ is

(A) 0 (B) 1 (C) ∞ (D) $\frac{n}{n+1}$

24. $\lim_{x \to 1} \frac{x^3 - x}{1 - x + \ln x}$ is ————

(A) 2 (B) -2 (C) 1 (D) -1

25. $\lim_{x\to 0} x^{\sin x}$ is ————

(A) 0 (B) $n\frac{1}{2}$ (C) e (D) ∞

26. Minimum and maximum values of $f(x) = x^{\frac{2}{3}}(x^3 - 8)$ in interval $\left[-1, \frac{1}{2}\right]$ are

(A) $-7, 0$ (B) $0, 6$ (C) $1, 2$ (D) $-2, 3$

27. $\int_0^1 \frac{4}{1+x^2} dx =$ ————

(A) 0 (B) π (C) $\frac{4\pi}{3}$ (D) $-\pi$

28. $\int_0^{\pi} cosec^2 x dx =$ ————

(A) 0 (B) 1 (C) -1 (D) ∞

29. $\lim_{x\to 0} \sin\frac{1}{x} =$ ————-

(A) does not exist (B) 1 (C) 0 (D) -1

30. $\int_0^{\frac{3\pi}{4}} |\cos x| dx =$ ————

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{-1}{\sqrt{2}}$ (C) ∞ (D) $2 - \frac{1}{\sqrt{2}}$

31. $\sec\left(\tan^{-1}\frac{2}{3}\right) =$ ————

(A) $\frac{2}{\sqrt{13}}$ (B) $\frac{3}{\sqrt{13}}$ (C) $\frac{\sqrt{13}}{3}$ (D) $\frac{\sqrt{13}}{2}$

32. Which of the following is convergent series?

(A) $\sum \frac{1}{n^2}$ (B) $\sum \frac{1}{\sqrt{n}}$ (C) $\sum \frac{1}{n}$ (D) $\sum \frac{1}{n^{1/3}}$

33. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$ is the Maclaurin's series of ————

(A) $\cos x$ (B) $\sin x$ (C) $\sinh x$ (D) $\cosh x$

34. $\int_1^2 \frac{x}{y^2} dx dy =$————

(A) $\frac{3}{4}$ (B) $\frac{7}{8}$ (C) $\frac{3}{2}$ (D) $\frac{1}{2}$

35. Domain of $f(x) = \sqrt{1-x^2}$ is ————-

(A) $x < 1$ (B) $x > 1$ (C) $|x| \leq 1$ (D) $|x| \geq 1$

36. Domain of $f(x) = \frac{1}{\sqrt{(1-x)(2-x)}}$ is ————

(A) $\mathbb{R} \setminus [1, 2]$ (B) $\mathbb{R} \setminus \{1, 2\}$ (C) $[1, 2]$ (D) $]1, 2[$

37. $f : \mathbb{R} \to (-1, 1)$ defined by $f(x) =$————- is bijective.

(A) $\frac{x}{1-|x|}$ (B) $\frac{x}{1+|x|}$ (C) $\frac{1}{1+|x|}$ (D) $\frac{x}{-1+|x|}$

38. Interval of convergence of $\sum_{k=1}^{\infty} x^k$ is ————

(A) $]-1, 1[$ (B) $[-1, 1]$ (C) $(-\infty, +\infty)$ (D) $x = 0$

39. Which of the following are in the usual metric space (R, d)?
(A) Subset of $\mathbb{R}$ (B) Union of open interval (C) Intervals (D) Singleton subsets

40. Let $A = (0, 1] \cup (1, 3]$ and R with usual metric space. Then $A^\circ =$ ————-
(A) $A \setminus \{0\}$ (B) $A \setminus \{1\}$ (C) $A \setminus \{3\}$ (D) $(0, 1) \cup (1, 3)$

41. Let A be finite subset of a metric space X. Then $A^d =$ ————
(A) Singleton set 0 (B) ϕ (C) A (D) $X \setminus A$

42. Let A be a finite subset of X, d then A is ——
(A) Open set (B) Open as well as closed (C) Closed set (D) Neither open nor closed

43. If Y is a subset of (X, d) then ————
(A) Every open set in Y is open in X (B) Every open set in X is open in Y (C) O is open in $Y \iff O$ is open in X (D) O is open $\iff O = Y \cap G$ where G is open in X

44. Let $f(x) = 1 + x^3$. Then $(0, 0)$ is the point of ————
(A) Maximum value (B) Minimum value (C) Point of inflection (D) None of these

45. Number of elements in a co-finite topological space (X, τ) where $X = \{s, t, u\}$ is ————
(A) 2 (B) 3 (C) 4 (D) 8

46. The boundary of a subset $B = \left\{\frac{1}{n} : n \in N\right\}$ of $(\mathbb{R}, d)$ is ————
(A) B (B) $\{0\}$ (C) $B \cup \{0\}$ (D) ϕ

47. The real line $\mathbb{R}$ is a homeomorphic to ————-
(A) $(0, 4)$ (B) $\{-1, 1\}$ (C) Q (D) T_2-space

48. $\mathbb{R}$ with co-finite topology is ————
(A) T_0-space (B) T_1-space (C) T_1-space but not T_2-space (D) 3

49. Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then X is ——
(A) T_1-space (B) Regular space (C) T_2-space (D) Normal space

50. Which of the following is connected in $\mathbb{R}$ with usual topology?
(A) $\mathbb{N}$ (B) $\mathbb{Q}$ (C) $(0, 1]$ (D) $\mathbb{Z}$

51. Which of the following topology is not totally disconnected?
(A) $\{1\}$ (B) discrete space (C) $\mathbb{R}$ with usual topology (D) $\mathbb{Q}$

52. Which of the following is nowhere dense in $\mathbb{R}$:
(A) $\mathbb{R} \setminus \mathbb{Z}$ (B) $\mathbb{Z}$ (C) $\cup(n, n + 1), n \in \mathbb{Z}$ (D) $\mathbb{Q}$

53. Which of the following is dense in $\mathbb{R}$:
(A) $\mathbb{N}$ (B) $\mathbb{Z}$ (C) $\mathbb{R} \setminus \mathbb{Z}$ (D) $\mathbb{Q}$

54. $xy'' + y' = 0$ has a solution $y = \ln x$ on interval ————
(A) $(0, \infty)$ (B) $(-\infty, 0)$ (C) $(-\infty, \infty)$ (D) $[0, \infty[$

55. Which of the following is not linear?
(A) $y' = (\sin x)y$ (B) $y' = (\sin y)x + e^x$ (C) $y' + xy = e^x y$ (D) $y' = 5$

56. Solution of $y' = \dfrac{x+y}{x}$ is ————
(A) $y = \ln|kx|$ (B) $y = \ln|x|$ (C) $y = x\ln|kx|$ (D) $y = \ln|x| + k$

57. Which of the following differential equation is not exact?
(A) $2xydx + (1+x^2)dy = 0$ (B) $ydx - xdy = 0$ (C) $y' = \dfrac{2+ye^{xy}}{2y - xe^{xy}}$ (D) $x + \sin y\text{dx} + x\cos y - 2y\text{dy}$

58. Integrating factor for $y' + \left(\dfrac{4}{x}\right)y = x^4$ is ————
(A) x^4 (B) $\ln x^4$ (C) $4\ln|x|$ (D) $\ln|x|$

59. The area bounded by $y = 4 - x^2$ and x-axis is ————
(A) $\dfrac{4}{3}$ (B) $\dfrac{8}{3}$ (C) $\dfrac{16}{3}$ (D) $\dfrac{32}{3}$

60. Which of the following is scalar?
(A) $(\underline{a}\cdot\underline{b})\underline{c}$ (B) $\underline{a}\cdot(\underline{b}\times\underline{c})$ (C) $\underline{a}\times(\underline{b}\times\underline{c})$ (D) $(\underline{a}\cdot\underline{b})(\underline{a}-\underline{a})$

61. Projection of $\underline{a}$ on $\underline{b}$ is ————
(A) $\underline{a}\cdot\underline{b}$ (B) $\dfrac{\underline{a}}{|\underline{a}|}\cdot\underline{b}$ (C) $\underline{a}\cdot\dfrac{\underline{b}}{|\underline{b}|}$ (D) $\underline{a}\times\underline{b}$

62. Which of the following is scalar quantity?
(A) Momentum (B) Magnetic field intensity (C) Special heat (D) Moment of force

63. A vector lying in the plane of $\underline{a}$ and $\underline{b}$ is ———
(A) $(\underline{a}\times\underline{b})\times\underline{c}$ (B) $\underline{a}\times(\underline{b}\times\underline{c})$ (C) $(\underline{c}\times\underline{a})\times\underline{b}$ (D) $(\underline{c}\times\underline{b})\times\underline{a}$

64. Let $\underline{t}$, $\underline{n}$ and $\underline{b}$ denoted respectively the tangent, principal normal and binormal vector to the cure then osculating plane to the curve at P contains ———
(A) $\underline{t}$, $\underline{b}$ (B) $\underline{n}$, $\underline{b}$ (C) $\underline{t}$, $\underline{n}$ (D) $\underline{t}$, $\underline{n}$, $\underline{b}$

65. Let $\underline{t}, \underline{n}$ and $\underline{b}$ be as in the above question. Then $\tau\underline{b} - k\underline{t} =$ ———-
(A) $\dfrac{dt}{ds}$ (B) $\dfrac{dn}{ds}$ (C) $\dfrac{db}{ds}$ (D) $\dfrac{d}{ds}\left(\dfrac{\underline{t}\times\underline{n}}{ds}\right)$

66. Normal plane is perpendicular to ————
(A) $\underline{t}$ (B) $\underline{n}$ (C) $\underline{b}$ (D) $\underline{t}\times\underline{n}$

67. $\underline{t}\times\underline{b}=$ ————
(A) $\underline{n}$ (B) $-\underline{n}$ (C) $\underline{n}\times\underline{b}$ (D) none of these

68. $\{x|x\in\mathbb{C} : x^4 = 1\}$ is a ————
(A) Subgroup of $(\mathbb{C}\setminus\{0\},\cdot)$ (B) Subgroup of $(\mathbb{C},+)$ (C) None cyclic group (D) Subgroup of $(\mathbb{Q}\setminus\{0\},\cdot)$

69. $\mathbb{R}^3$ under vector product forms a ————
(A) group (B) monoid (C) semi-group (D) groupoid

70. An element x of group G satisfying $x^2 = x$ is called ————————
(A) Involution (B) Idempotent (C) Transposition (D) Cycle

71. $\frac{\mathbb{Z}}{\langle n \rangle}$ is isomorphic to ————————
(A) $n\mathbb{Z}$ (B) $\langle n \rangle$ (C) $\mathbb{Z}_n$ (D) $\{0, \pm 2n, \pm 4n, ...\}$

72. Let $G = \langle a : a^{12} = e \rangle$. Then $G =$ ————————
(A) $\langle a^5 \rangle$ (B) $\langle a^6 \rangle$ (C) $\langle a^2 \rangle$ (D) $\langle a^8 \rangle$

73. Let $G = \langle b : b^{17} = e \rangle$. Then G can be generated by ————————
(A) Any element of G (B) Any non-identity element of G (C) b, b^{-1} are the only generators of G (D) Identity

74. Let $G = \langle \alpha, \beta : \alpha^3 = \beta^2 = (\alpha\beta)^2 = e \rangle$. Then $N_G(\{e, \beta\}) =$ ————————
(A) $\{e\}$ (B) $\{e, \beta, \alpha\beta\}$ (C) G (D) $\{e, \beta\}$

75. Let $G = \langle \alpha, \beta : \alpha^4 = \beta^2 = (\alpha\beta)^2 = e \rangle$. Then $Z(G) =$ ————————
(A) $\{e\}$ (B) $\{e, \alpha^2\}$ (C) $\{e, \alpha, \alpha^2, \alpha^3\}$ (D) G

76. Which of the following is not true for an abelian group G?
(A) $[a, b] = e \, \forall \, a, b \in G$ (B) G is simple group of order 60 (C) $G' = \{0\}$
(D) $Z(G)) = G$

77. Inner automorphism of $Q = \{\pm 1, \pm i, \pm j\}$ is ————————
(A) $\{e\}$ (B) $C_2 \times C_2$ (C) Q (D) C_4

78. Number of conjugacy classes of a cyclic group of order 6 is ————————
(A) 1 (B) 2 (C) 3 (D) 6

79. Number of non-isomorphic abelian groups of order 12 is ————————
(A) 1 (B) 2 (C) 3 (D) 4

80. Order of sylow-2 subgroup of Q_8 is ————————
(A) 1 (B) 2 (C) 3 (D) 8

81. Which of the following is an ideal of $\mathbb{R}$?
(A) $\mathbb{Z}$ (B) $\{0\}$ (C) $\mathbb{C}$ (D) $\mathbb{Q}$

82. Which of the following is not an integral domain?
(A) $\mathbb{Z}$ (B) $\mathbb{Z}_7$ (C) $\mathbb{Q}$ (D) Set M_2 of 2×2 matrices with integer entries

83. Which of the following is a field?
(A) $\{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ (B) $\mathbb{Q} \setminus \{0\}$ (C) $\mathbb{Z}$ (D) $\mathbb{Z}_6$

84. Which of the following is not a vector space?
(A) $\mathbb{R}(\mathbb{R})$ (B) $\mathbb{R}(\mathbb{Q})$ (C) $\mathbb{R}(\mathbb{C})$ (D) $\mathbb{R}(\mathbb{Q})$

85. Let $\phi : \mathbb{Z} \to \mathbb{Z}_5$ be $\phi(a) = a \pmod 5$. Then $Ker(\phi) =$ ————————
(A) $\{0\}$ (B) $\{0, \pm 5, \pm 10, ...\}$ (C) $\mathbb{Z}_5$ (D) $\mathbb{Z}$

86. The number of proper ideals of $\mathbb{Z}_{17}$ is ————————
(A) 0 (B) 1 (C) 2 (D) 3

87. Which of the following is a division ring?
(A) $(\mathbb{Z}, +, \cdot)$ (B) $(\mathbb{E}, +, \cdot)$ (C) $(\mathbb{Q}, +, \cdot)$ (D) $(\mathbb{Z}_6, \oplus_6, \odot_6)$

88. $\int_{-1}^{2} (x + |x|)dx =$
(A) 0 (B) 4 (C) 2 (D) 6

89. $x = 6$ in $\mathbb{R}^3$ represents a
(A) Point (B) Line (C) Plane (D) Space

90. Kernel of $T : \mathbb{R}^3 \to \mathbb{R}^3$, where $T(x, y, z) = (x, y, 0)$, is
(A) Point (B) Line (C) Plane (D) Space

91. Dimension of $Hom(\mathbb{R}^3, \mathbb{R}^4) =$ ————————
(A) 3 (B) 4 (C) 7 (D) 12

92. Dimension of $Hom(M_{2,4}, P_2(t)) =$ ————————
(A) 4 (B) 8 (C) 16 (D) 24

93. A dice is thrown. The probability that the dots on the top are prime numbers or odd numbers is
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{5}{6}$

94. A coin is tossed 4 times in succession. The probability that at least one head occurs is
(A) $\frac{1}{16}$ (B) $\frac{4}{16}$ (C) $\frac{12}{16}$ (D) $\frac{15}{16}$

95. Number of necklaces made from 9 beads of different colours is ————————
(A) $\frac{8!}{2}$ (B) 8! (C) 7! (D) 9!

96. Period of $3\cos\frac{x}{5}$ is ————————
(A) 2π (B) $\frac{2\pi}{5}$ (C) 6π (D) 10π

97. Range of $\sec^{-1} x$ is ————————
(A) $[0, \pi]$ (B) $[0, \pi] \backslash \frac{\pi}{2}$ (C) $[\frac{-\pi}{2}, \frac{\pi}{2}]$ (D) $[\frac{-\pi}{2}, \frac{\pi}{2}] \backslash \{0\}$

98. Solution set of $\sin x \cos x = \frac{\sqrt{3}}{4}$ is ————————
(A) $\{\frac{\pi}{6} + n\pi\} \cup \{\frac{\pi}{3} + n\pi\}$ (B) $\{\frac{\pi}{3} + 2n\pi\} \cup \{\frac{2\pi}{3} + 2n\pi\}$ (C) $\{\frac{\pi}{6} + 2n\pi\} \cup \{\frac{5\pi}{6} + 2n\pi\}$
(D) $\{\frac{\pi}{12} + n\pi\} \cup \{\frac{5\pi}{12} + n\pi\}$

99. Which of the following is tautology?
(A) $p \to \sim q$ (B) $(p \to q) \cap (p\ q)$ (C) $p \to q \to \sim q \to \sim q$ (D) $p \cap \sim p$

100. $f(x) = \frac{1}{x}$ is not uniformly continuous in the region ————————

(A) $0 \leq |z| \leq 1$ (B) $0 \leq |z| < 1$ (C) $0 < |z| \leq 1$ (D) $0 < |z| < 1$
